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# Market competition, R&D spillovers, and firms' cost asymmetry

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**ABSTRACT**

We examine the effects of firms' cost asymmetry on R&D investments. We obtain five novel results. First, the social preference between noncooperative and cooperative R&D investments is independent of the degree of firms' cost asymmetry. Second, R&D investments of low-cost firms are larger than those of high-cost firms. Third, for a small spillover, noncooperative R&D investments have a U-shaped curve with the degree of market competition. Fourth, as market competition intensifies, a low-cost (high-cost) firm may decrease (increase) its noncooperative R&D investment. Fifth, the difference in profit between low-cost and high-cost firms increases with the degree of market competition.

**KEYWORDS**

Asymmetric costs; oligopoly; two-stage Cournot; spillover.

**JEL CLASSIFICATION**

L13; L22; O32

## 1. Introduction

Strategic research and development (R&D) investments have long been examined using the two-stage Cournot model. In the first stage, firms decide on their cost-reducing R&D investments and, in the second stage, they compete in a quantity-setting game. Most prior studies assume that firms are symmetric, which implies that, for example, firms have identical cost functions. Such studies include those of Brander and Spencer (1983), d'Aspremont and Jacquemin (1988), Henriques (1990), De Bondt and Veugelers (1991), De Bondt et al. (1992), Kamien et al. (1992), Suzumura (1992), Salant and Shaffer (1998), Amir (2000), Hinloopen (2000), Matsumura et al. (2013), and Shibata (2014).

However in practice, production costs between firms are often asymmetric. For example, large/mature (small/young) firms can be viewed as having a cost advantage (disadvantage). Because a patent provides an inventor with the opportunity to reap a reward from his or her investment in research and development, the patent-holding firm enjoys a cost advantage over other firms, which do not have the patent. In liberalized industries, such as gas, electricity, telecommunications, and so on, an incumbent has a cost advantage over any entrant. From this viewpoint, economists and policymakers agree that cost asymmetry exists between firms.

Several studies incorporate cost asymmetries. Spencer and Brander (1983), Lahiri and Ono (1999), Kitahara and Matsumura (2006), and Ishida et al. (2011) assume there are two types of firms that differ in terms of their production costs. Under such asymmetry of firms' production costs, we consider the following questions. How does the asymmetry of production costs influence the social preference for noncooperative and cooperative R&D investments? How does asymmetry affect market R&D investments? Are the R&D investments of low-cost firms larger than those of high-cost firms? How does the intensity of market competition affect low-cost and high-cost firms' R&D investments? How does the intensity of market competition affect the difference between the profits of low-cost and high-cost firms? The main contribution of this study is to examine the effects of production cost asymmetry on R&D investments by answering the above questions.

In this study, we extend the oligopoly model developed by Shibata (2014) to include production cost asymmetries between firms. Shibata (2014) extends the work of Matsumura et al. (2013) by incorporating spillovers from cost-reducing R&D investments. However, Shibata (2014) assumes that the production costs between firms are symmetric (identical). In contrast, in this study, Firm $i$'s production cost is defined as

$$c_i(x_i, x_j) := c + z_i - x_i - \beta x_j,$$

where $c > 0$ and $\beta \in [0,1]$ are constants, and $x_i$ represents Firm $i$'s cost-reducing R&D investment ($i, j \in \{1,2\}, i \neq j$). Here, the most important assumption is that $z_1 = z_2$ does not always hold, where $z_i \in [-b, b]$ and $b \geq 0$ is constant. That is, Firm $i$'s production cost is asymmetric (i.e., $c_i(x_i, x_j)$), whereas in Shibata (2014), the production cost is symmetric; that is, $c(x_i, x_j)$, because $z_i = 0$. In particular, without loss of generality, we assume $z_1 \leq z_2$, which implies that Firms 1 and 2 are low-cost and high-cost firms, respectively.[1] In this study, we consider how cost asymmetries (i.e., $z_1 \leq z_2$) affect firms' strategic R&D investments.

We obtain five important results. First, which of noncooperative and cooperative R&D investments is socially preferred is independent of the degree of firms' production cost asymmetry. This implies that the coexistence of large/mature and small/young firms in a market does not influence the social preference between R&D competition and R&D cooperation strategies. In addition, the fact that a low-cost firm's noncooperative R&D investment is larger than its cooperative R&D investment is equivalent to the fact that a high-cost firm's noncooperative R&D investment is larger than its cooperative R&D investment. Second, the noncooperative and cooperative R&D investments of a low-cost firm are larger than those of a high-cost firm. This result is consistent with the findings of empirical studies by Shefer and Frenkel (2005) and Manez et al. (2009). Third, noncooperative R&D investments have a U-shaped curve with the degree of market competition for a small spillover, but are decreasing for a large spillover. The finding of nonlinearity for a small spillover fits well with the empirical results of Aghion et al. (2005) and Sacco and Schmutzler (2011). Fourth, as the market competition intensifies, a low-cost firm's noncooperative R&D investment may decrease, whereas that of a high-cost firm may increase. Fifth, the difference in profit between low-cost and high-cost firms is increasing with the degree of market competition, although the profits of the low-cost and high-cost firms are decreasing with the degree of market competition.

The remainder of this paper is organized as follows. Section 2 describes the model setup, and provides the solution to the model. Section 3 discusses the properties of the solution, and describes the implications of the model. Section 4 concludes the paper. The derivations of noncooperative and cooperative R&D are presented in the Appendix. Note that for results that follow immediately from Shibata (2014), the proofs are omitted.

[1] The asymmetric production costs between firms are the same as those in Spencer and Brander (1983), Lahiri and Ono (1999), Kitahara and Matsumura (2006), and Ishida et al. (2011).

## 2. Model

In this section, we begin with a description of the model. We then formulate an optimization problem and derive the solution. As a benchmark, we provide the model implications for the two extreme cases in our model, namely, a duopoly market and a perfectly competitive market.

### 2.1. *Setup*

Consider a market in which there are two firms (Firms 1 and 2)[2] that face a linear inverse demand function:

$$p(q_1, q_2) := a - q_1 - q_2 \geq 0, \tag{1}$$

where $q_i \geq 0$ is Firm $i$'s quantity produced ($i \in \{1, 2\}$), and $a > 0$ is some constant.

The cost function of Firm $i$'s production is given by

$$c_i(x_i, x_j)q_i := (c + z_i - x_i - \beta x_j)q_i \geq 0, \tag{2}$$

for any $i$ ($i, j \in \{1, 2\}, i \neq j$), where $c > 0$ and $z_i \in [-b, b]$ are some constants ($b \geq 0$), $x_i \geq 0$ is Firm $i$'s R&D investment, and $\beta \in [0, 1]$ measures R&D investment "spillovers."[3] We assume that $a > c + (1/2)(z_1 + z_2)$.[4] When $\beta = 0$, Firm $i$'s R&D reduces only its own marginal cost, whereas when $\beta = 1$, spillovers are perfect. Thus, $c_i(0, 0) = c + z_i > 0$ represents Firm $i$'s ex ante marginal cost before R&D investment, whereas $c_i(x_i, x_j) \geq 0$ represents Firm $i$'s ex post marginal cost after R&D investment. The crucial assumption of this study is stated as follows.

**Assumption 1.** $z_1 \leq z_2$

Assumption 1 means that Firms 1 and 2 are "low-cost" and "high-cost" firms, respectively.

The cost function of Firm $i$'s R&D investment is given by

$$g(x_i) := \frac{1}{2}\gamma x_i^2 \geq 0, \tag{3}$$

where $\gamma \geq 1$ represents the efficient measure of the cost ($i \in \{1, 2\}$). Note that the cost of R&D investment in (3) is symmetric, although the cost of production in (2) is asymmetric. Thus, the asymmetry between firms is defined by the production cost function. This assumption is the same as that made by Ishida et al. (2011).

Firm $i$'s profit is given by

$$\hat{\pi}_i(q_i, q_j, x_i, x_j) - g(x_i) \geq 0, \quad i \neq j, \tag{4}$$

[2]The model assumption is the same as that of Shibata (2014), with the exception of the asymmetry of the firms' production costs.

[3]See d'Aspremont and Jacquemin (1988) for spillovers.

[4]This is one of the conditions that ensures that market R&D investments are strictly positive. See Equations (23) and (34) for further detail.

where $\hat{\pi}_i(q_i, q_j, x_i, x_j)$ is defined as

$$\hat{\pi}_i(q_i, q_j, x_i, x_j) := (p(q_1, q_2) - c_i(x_i, x_j))q_i \geq 0. \tag{5}$$

Following Matsumura et al. (2013) and Shibata (2014), we assume that a constant $\lambda$ ($\lambda \in [0, 1]$) exists, such that Firm $i$ maximizes

$$\hat{\pi}_i(q_i, q_j, x_i, x_j) - \lambda\hat{\pi}_j(q_i, q_j, x_i, x_j) - g(x_i) \geq 0, \quad i \neq j. \tag{6}$$

We assume that (6) represents the utility function (relative profit) of Firm $i$.[5] If $\lambda = 0$, (6) is simply the profit of Firm $i$. If $\lambda \neq 0$, (6) is Firm $i$'s profit, less the gross profit of Firm $j$. Thus, the parameter $\lambda$ can be viewed as a continuous measure of the intensity of competition (i.e., market structure). The extreme case of $\lambda = 0$ is a duopoly market, corresponding to the d'Aspremont and Jacquemin (1988) model. The extreme case of $\lambda \to 1$ is approximately a perfectly competitive market.[6] Thus, the case of $\lambda \in (0, 1)$ represents an oligopoly market.[7]

The structure of the game is as follows. First, each firm simultaneously chooses the level of R&D investment. Second, each firm simultaneously chooses the quantity of output produced, based on the choice of R&D investment. That is, the optimization problem is a two-stage optimization problem. We analyze two games in the two-stage problem. In the first game, the firm acts *noncooperatively* in terms of R&D and the quantity produced. In the second game, the firm acts *cooperatively* in terms of R&D, and *noncooperatively* in terms of the quantity produced. These two games are the same as those examined in d'Aspremont and Jacquemin (1988).

## 2.2. *Output strategies*

We now solve the firm's decision problem by working backward. This subsection derives the optimal quantity strategies for given R&D investment strategies.

Given R&D investment strategies $x_i \geq 0$ and $x_j \geq 0$, the optimization problem is formulated as

$$\max_{q_i \geq 0}\{\hat{\pi}_i(q_i, q_j, x_i, x_j) - \lambda\hat{\pi}_j(q_i, q_j, x_i, x_j)\} \geq 0, \tag{7}$$

for all $i$ $(i, j \in \{1, 2\}, i \neq j)$. The optimal output is given by

$$q_i(x_i, x_j) = \frac{h_{i1}(x_i, x_j)}{(3 - \lambda)(1 + \lambda)} \geq 0, \tag{8}$$

where

$$h_{i1}(x_i, x_j) := (1 + \lambda)(a - c) - 2z_i + (1 - \lambda)z_j + (2 - \beta + \beta\lambda)x_i + (2\beta - 1 + \lambda)x_j, \tag{9}$$

[5]Following Matsumura et al. (2013), we call (6) the relative profit.

[6]Specifically, the extreme case of $\lambda \to 1$ is a perfect competitive market for $z_1 = z_2$, but is not for $z_1 > z_2$. However, because we assume that $z_i$ is very small, the extreme case of $\lambda \to 1$ can approximate a perfectly competitive market.

[7]As shown in Shibata (2014), $\lambda$ is justified as a measure of the intensity of competition because the equilibrium price and quantity are monotonically decreasing and increasing with $\lambda$, respectively, whereas an increase in competitiveness increases the consumer surplus and decreases the producer surplus.

for all $i$ $(i,j \in \{1,2\}, i \neq j)$. Firm $i$'s gross profit is given by

$$\begin{aligned}\pi_i(x_i,x_j) :=& \hat{\pi}_i(q_i(x_i,x_j), q_j(x_i,x_j), x_i, x_j)\\ =& \frac{h_{i1}(x_i,x_j)h_{i2}(x_i,x_j)}{(3-\lambda)^2(1+\lambda)} \geq 0,\end{aligned} \tag{10}$$

where

$$h_{i2}(x_i,x_j) := (1-\lambda)(a-c) - (2-\lambda)z_i + z_j + (2-\beta-\lambda)x_i + (2\beta-1-\beta\lambda)x_j. \tag{11}$$

### 2.3. *R&D competition strategies*

In this subsection, we derive the optimal noncooperative R&D investment strategy.

The optimization problem is formulated as

$$\max_{x_i \geq 0} f_i(x_i,x_j) \geq 0, \tag{12}$$

where

$$f_i(x_i,x_j) := \pi_i(x_i,x_j) - \lambda\pi_j(x_i,x_j) - \frac{1}{2}\gamma x_i^2, \tag{13}$$

for all $i$ $(i,j \in \{1,2\}, i \neq j)$. Differentiating (12) with respect to $x_i$ gives

$$\frac{\partial f_i(x_i,x_j)}{\partial x_i} = \frac{1}{(3-\lambda)^2(1+\lambda)} w_i(x_i,x_j) - \gamma x_i = 0, \tag{14}$$

where

$$\begin{aligned}w_i(x_i,x_j) :=& h_{i1}(x_i,x_j)(2-\beta-\lambda) + h_{i2}(x_i,x_j)(2-\beta+\beta\lambda)\\ &- \lambda\big(h_{j1}(x_i,x_j)(2\beta-1-\beta\lambda) + h_{j2}(x_i,x_j)(2\beta-1+\lambda)\big),\end{aligned} \tag{15}$$

for all $i$ $(i,j \in \{1,2\}, i \neq j)$.[8] As shown in the Appendix, if $f_i^{\mathrm{n}} := f_i(x_1^{\mathrm{n}}, x_2^{\mathrm{n}}) \geq 0$ for all $i$, the noncooperative equilibrium is obtained as $(x_1^{\mathrm{n}}, x_2^{\mathrm{n}})$, where

$$\begin{aligned}x_i^{\mathrm{n}} =& \frac{1}{(\delta_6-\delta_4)^2-\delta_5^2}\\ &\times \big((\delta_6-\delta_4+\delta_5)\delta_1(a-c) + [(\delta_6-\delta_4)\delta_2+\delta_5\delta_3]z_i + [(\delta_6-\delta_4)\delta_3+\delta_5\delta_2]z_j\big),\end{aligned} \tag{16}$$

[8]The noncooperative equilibrium $(x_1^{\mathrm{n}}, x_2^{\mathrm{n}})$ is obtained by solving the simultaneous equations, $\partial f_1/\partial x_1 = 0$ and $\partial f_2/\partial x_2 = 0$.

where

$$\delta_1 :=2(2-\beta)+\lambda\big(1-3\beta-2\lambda+(1+\beta)\lambda^2\big)\geq 0, \tag{17}$$
$$\delta_2 :=-4(2-\beta)-\lambda\big(-6+7\beta-4\beta\lambda+2\lambda+\beta\lambda^2\big)\leq 0, \tag{18}$$
$$\delta_3 :=2(2-\beta)-\lambda\big(7-10\beta+4\beta\lambda-4\lambda+\lambda^2\big)\geq 0, \tag{19}$$
$$\delta_4 :=2\big((2-\beta+\beta\lambda)(2-\beta-\lambda)-\lambda(2\beta-1+\lambda)(2\beta-1-\beta\lambda)\big), \tag{20}$$
$$\delta_5 :=(1-\lambda)\big((2\beta-1+\lambda)(2-\beta-\lambda)+(2\beta-1-\beta\lambda)(2-\beta+\beta\lambda)\big), \tag{21}$$
$$\delta_6 :=\gamma(3-\lambda)^2(1+\lambda)\geq 0. \tag{22}$$

Note that $(x_1^{\mathrm{n}}, x_2^{\mathrm{n}})$ is obtained as a unique solution for the region defined by the parameters satisfying $f_i^{\mathrm{n}} \geq 0$, for all $i$. In addition, we have that $f_1^{\mathrm{n}} \geq f_2^{\mathrm{n}}$, because $z_2 \geq z_1$. Here, we restrict our discussion of our model to the region defined by the parameters satisfying $f_2^{\mathrm{n}} \geq 0$. If $f_2^{\mathrm{n}} \geq 0$ is not satisfied, $(x_1^{\mathrm{n}}, x_2^{\mathrm{n}})$ is not derived as a unique solution.[9]

We define as $x^{\mathrm{n}} := x_1^{\mathrm{n}} + x_2^{\mathrm{n}}$ the *market noncooperative R&D*. Then, $x^{\mathrm{n}}$ is given by

$$x^{\mathrm{n}} = \frac{2\delta_1(a-c)+(\delta_2+\delta_3)(z_1+z_2)}{\delta_6-\delta_4-\delta_5}.$$

It is straightforward to obtain $\delta_2+\delta_3=-\delta_1\leq 0$ and $\delta_4+\delta_5=(1+\beta)\delta_1\geq 0$. Thus, we obtain the following result.

**Lemma 2.1.** *The market noncooperative R&D investment, denoted as $x^{\mathrm{n}}$, is given by*

$$x^{\mathrm{n}} = \frac{\delta_1}{\delta_6-(1+\beta)\delta_1}\big(2(a-c)-(z_1+z_2)\big). \tag{23}$$

*An increase in $z_1+z_2$ decreases $x^{\mathrm{n}}$.*

Note that $x^{\mathrm{n}}$ is a linear function of the term $2(a-c)-(z_1+z_2)$.

Finally, note that in the problem of noncooperative R&D, Firm $i$'s quantity, market quantity, market price, and consumer surplus are given by $q_i^{\mathrm{n}} := q_i(x_1^{\mathrm{n}}, x_2^{\mathrm{n}})$, $q^{\mathrm{n}} := q_1^{\mathrm{n}}+q_2^{\mathrm{n}}$, $p^{\mathrm{n}} := p(q_1^{\mathrm{n}}, q_2^{\mathrm{n}})$, and $cs^{\mathrm{n}} := (a-p^{\mathrm{n}})q^{\mathrm{n}}/2$, respectively ($i \in \{1,2\}$).

### 2.4. *R&D cooperation strategies*

This subsection describes the cooperative R&D investment strategy (i.e., the first stage is cooperative, whereas the second stage is noncooperative).[10]

The optimization problem is

$$\max_{x_1\geq 0, x_2\geq 0}\{f_1(x_1,x_2)+f_2(x_1,x_2)\}\geq 0. \tag{24}$$

Differentiating (24) with respect to $x_i$ yields

$$\frac{\partial(f_i(x_i,x_j)+f_j(x_i,x_j))}{\partial x_i} = \frac{1-\lambda}{(3-\lambda)^2(1+\lambda)}v_i(x_i,x_j)-\gamma x_i=0, \tag{25}$$

[9] Although several solutions may exist, many are difficult to derive. See Shibata (2014) for more information.
[10] These strategies are the same as those in d'Aspremont and Jacquemin (1988).

where

$$v_i(x_i,x_j) := h_{i1}(x_i,x_j)(2-\beta-\lambda) + h_{i2}(x_i,x_j)(2-\beta+\beta\lambda) + h_{j1}(x_i,x_j)(2\beta-1-\beta\lambda) + h_{j2}(x_i,x_j)(2\beta-1+\lambda), \tag{26}$$

for any $i$ $(i,j \in \{1,2\}, i \neq j)$.[11] As shown in the Appendix, the cooperative solution is given by $(x_1^{\mathrm{c}}, x_2^{\mathrm{c}})$, where

$$x_i^{\mathrm{c}} = \frac{1}{(\phi_6-\phi_4)^2-\phi_5^2} \times \Big((\phi_6-\phi_4+\phi_5)\phi_1(a-c) + [(\phi_6-\phi_4)\phi_2+\phi_5\phi_3]z_i + [(\phi_6-\phi_4)\phi_3+\phi_5\phi_2]z_j\Big), \tag{27}$$

where

$$\phi_1 := 2(1+\lambda)(1-\lambda)(1+\beta) \geq 0, \tag{28}$$
$$\phi_2 := 2\big(-5+4\beta+\beta\lambda^2+3\lambda(1-\beta)\big) \leq 0, \tag{29}$$
$$\phi_3 := 2\big(4-5\beta+\lambda^2-3\lambda(1-\beta)\big), \tag{30}$$
$$\phi_4 := 2\big((2-\beta-\lambda)(2-\beta+\beta\lambda)+(2\beta-1-\beta\lambda)(2\beta-1+\lambda)\big), \tag{31}$$
$$\phi_5 := 2\big((2-\beta-\lambda)(2\beta-1+\lambda)+(2\beta-1-\beta\lambda)(2-\beta+\beta\lambda)\big), \tag{32}$$
$$\phi_6 := \gamma\frac{(3-\lambda)^2(1+\lambda)}{1-\lambda} \geq 0. \tag{33}$$

As in (16), $(x_1^{\mathrm{c}}, x_2^{\mathrm{c}})$ is obtained as a unique solution for the region defined by the parameters satisfying $f_i^{\mathrm{c}} := f_i(x_1^{\mathrm{c}}, x_2^{\mathrm{c}}) \geq 0$, for all $i$.[12]

We define as $x^{\mathrm{c}} := x_1^{\mathrm{c}} + x_2^{\mathrm{c}}$ the *market cooperative R&D*. Then, $x^{\mathrm{c}}$ is given by

$$x^{\mathrm{c}} = \frac{2\phi_1(a-c)+(\phi_2+\phi_3)(z_1+z_2)}{\phi_6-\phi_4-\phi_5}.$$

Similarly to $\delta_2+\delta_3 = -\delta_1 \leq 0$ and $\delta_4+\delta_5 = (1+\beta)\delta_1 \geq 0$, we obtain $\phi_2+\phi_3 = -\phi_1$ and $\phi_4+\phi_5 = (1+\beta)\phi_1$, respectively. Thus, we have the following lemma.

**Lemma 2.2.** *The market cooperative R&D investment, denoted by $x^{\mathrm{c}}$, is given by*

$$x^{\mathrm{c}} = \frac{\phi_1}{\phi_6-(1+\beta)\phi_1}\big(2(a-c)-(z_1+z_2)\big). \tag{34}$$

*An increase in $z_1+z_2$ decreases $x^{\mathrm{c}}$.*

Note that $x^{\mathrm{c}}$ is a linear function of the term $2(a-c)-(z_1+z_2)$.

Finally, note that in the problem of cooperative R&D, Firm $i$'s quantity, market quantity, market price, and consumer surplus are given by $q_i^{\mathrm{c}} := q_i(x_1^{\mathrm{c}}, x_2^{\mathrm{c}})$, $q^{\mathrm{c}} := q_1^{\mathrm{c}}+q_2^{\mathrm{c}}$, $p^{\mathrm{c}} := p(q_1^{\mathrm{c}}, q_2^{\mathrm{c}})$, and $cs^{\mathrm{c}} := (a-p^{\mathrm{c}})q^{\mathrm{c}}/2$, respectively $(i \in \{1,2\})$.

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[11] The cooperative equilibrium $(x_1^{\mathrm{c}}, x_2^{\mathrm{c}})$ is obtained by solving the simultaneous equations, $\partial(f_1+f_2)/\partial x_1 = 0$ and $\partial(f_1+f_2)/\partial x_2 = 0$.

[12] Because $f_1^{\mathrm{c}} \geq f_2^{\mathrm{c}} \geq f_2^{\mathrm{n}}$ are always satisfied, it is sufficient to restrict the discussion to the region defined by the parameters satisfying $f_2^{\mathrm{n}} \geq 0$.

### 2.5. *Symmetric firms*

As a benchmark, we briefly review the case of symmetric firms, as in Shibata (2014).

In this subsection, we assume that the two firms are symmetric (denoted as $z_1 = z_2 = 0$).[13] We denote Firm $i$'s noncooperative and cooperative R&D investments by $x^{\mathrm{n}}_{\mathrm{S}i}$ and $x^{\mathrm{c}}_{\mathrm{S}i}$ ($i \in \{1, 2\}$), respectively, where the subscript "S" represents the case of "symmetric firms." Then, from Equations (16) and (27), we easily have $x^{\mathrm{n}}_{\mathrm{S}1} = x^{\mathrm{n}}_{\mathrm{S}2}$ and $x^{\mathrm{c}}_{\mathrm{S}1} = x^{\mathrm{c}}_{\mathrm{S}2}$. Recall that if $f_i(x^{\mathrm{n}}_{\mathrm{S}}, x^{\mathrm{n}}_{\mathrm{S}}) \geq 0$ is satisfied for any $i$ ($i \in \{1, 2\}$), $x^{\mathrm{n}}_{\mathrm{S}i}$ is obtained as a unique solution. Otherwise it is not.

We denote as $x^{\mathrm{n}}_{\mathrm{S}} := x^{\mathrm{n}}_{\mathrm{S}1} + x^{\mathrm{n}}_{\mathrm{S}2}$ and $x^{\mathrm{c}}_{\mathrm{S}} := x^{\mathrm{c}}_{\mathrm{S}1} + x^{\mathrm{c}}_{\mathrm{S}2}$ the market noncooperative and cooperative R&D investments, respectively. Substituting $z_1 = z_2 = 0$ into (23) and (34) yields

$$x^{\mathrm{n}}_{\mathrm{S}} = \frac{2\delta_1}{\delta_6 - (1+\beta)\delta_1}(a - c) \geq 0, \tag{35}$$

$$x^{\mathrm{c}}_{\mathrm{S}} = \frac{2\phi_1}{\phi_6 - (1+\beta)\phi_1}(a - c) \geq 0, \tag{36}$$

respectively. In addition, Firm $i$'s quantity, market quantity, market price, and consumer surplus are given by $q^k_{\mathrm{S}i} := q_i(x^k_{\mathrm{S}1}, x^k_{\mathrm{S}2})$, $q^k_{\mathrm{S}} := q^k_{\mathrm{S}1} + q^k_{\mathrm{S}2}$, $p^k_{\mathrm{S}} := p(q^k_{\mathrm{S}1}, q^k_{\mathrm{S}2})$, and $cs^k_{\mathrm{S}} := (a - p^k_{\mathrm{S}})q^k_{\mathrm{S}}/2$, respectively ($k \in \{\mathrm{n}, \mathrm{c}\}, i \in \{1, 2\}$). We use these symmetric solutions as a benchmark.[14]

### 2.6. *Properties of solutions*

Following Shy (1995), we define the social preference between cooperative and noncooperative R&D investments (i.e., between R&D cooperation and R&D competition strategies) as follows.

**Definition 2.3.** The R&D competition strategy is preferred if $x^{\mathrm{n}} > x^{\mathrm{c}}$, and the R&D cooperation strategy is preferred otherwise.

The social preference based on the magnitude of R&D quantity is appropriate,[15] because $f^{\mathrm{c}}_i \geq f^{\mathrm{n}}_i$ for any $i$, $\beta \in [0, 1]$, and $\lambda \in [0, 1]$ ($i, j \in \{1, 2\}, i \neq j$).

From Equations (23) and (34), noncooperative R&D investment is socially preferred if

$$\frac{\delta_1}{\delta_6 - (1+\beta)\delta_1} \geq \frac{\phi_1}{\phi_6 - (1+\beta)\phi_1} \tag{37}$$

is satisfied. Cooperative R&D investment is socially preferred otherwise. The most interesting part of this is that the inequality in (37) does not depend on $a$ (the magnitude of market demand), $c$ (the marginal cost of quantity), or $(z_1, z_2)$ (the two asymmetric cost parameters). The term in (37) consists of three parameters: $\beta$ (the spillover), $\lambda$ (the market competitiveness), and $\gamma$ (the cost parameter for R&D). We summarize the result in the following proposition.

[13] Here, we can assume $z_1 = z_1 = z > 0$ as the symmetric case. However, for ease of comparison between the symmetric and asymmetric cases, we assume $z_1 = z_2 = 0$ as the symmetric case.

[14] Here, $x^{\mathrm{n}}$ in (35) and $x^{\mathrm{c}}$ in (36) are the same as those in Shibata (2014).

[15] This idea is the same as that described in Shy (1995).

**Proposition 2.4.** *Suppose there are asymmetric firms (i.e., $z_1 \neq z_2$). Then, the social preference between noncooperative and cooperative R&D investments is independent of the firms' asymmetric cost parameters ($z_1$ and $z_2$).*

Proposition 2.4 is a very powerful statement. Proposition 2.4 indicates that any heterogeneity between firms has no influence on the social preference between R&D competition and R&D cooperation. Efficient low-cost (less efficient high-cost) firms with some cost advantage (disadvantage) can best approximate large/mature (small/young) firms. However, coexistence of large/mature and small/young firms in a market has no influence on the social preference for the R&D competition or cooperation strategies.

## 3. Model implications

This section considers the main implications of our solution. Because it is difficult to examine the properties of our solution analytically, we do so using numerical methods. In our model, the four important parameters are market competition intensity ($\lambda$), R&D investment spillover ($\beta$), and the two asymmetric-cost parameters ($z_1$, $z_2$). Subsection 3.1 defines the basic parameters and derives the optimal R&D investments numerically, as a benchmark. Subsection 3.2 examines the effects of cost asymmetry between firms. Subsection 3.3 investigates the effects of market competition on asymmetric firms. Subsection 3.4 examines how the difference between the R&D, production quantity, and profits of low-cost and high-cost firms varies with respect to market competition. Subsection 3.5 discusses the effects of production cost asymmetry between firms on each firm's R&D investment. Subsection 3.6 investigates the relationship between the optimal R&D investment and the resultant profit.

### 3.1. *Parameters and optimal R&D investment as a benchmark*

In this section, we defines the basic parameters and provides the optimal R&D investment numerically, as a benchmark.

The basic parameters are assumed to be

$$a = 100, \quad c = 50, \quad \gamma = 50, \quad z_i \in [-1, 1],$$

for any $i \in \{1, 2\}$. Under these parameters, $x^{\mathrm{n}} \geq 0$ and $x^{\mathrm{c}} \geq 0$ are stable, and $c_i(x_i^{\mathrm{n}}, x_j^{\mathrm{n}}) > 0$ and $c_i(x_i^{\mathrm{c}}, x_j^{\mathrm{c}}) > 0$ (i.e., $c + z_i > x_i^{\mathrm{n}} + \beta x_j^{\mathrm{n}}$ and $c + z_i > x_i^{\mathrm{c}} + \beta x_j^{\mathrm{c}}$, respectively), for any $i \in \{1, 2\}$, $\lambda \in [0, 1]$, and $\beta \in [0, 1]$. It is meaningful to compare $x_i^{\mathrm{n}} \geq 0$ and $x_i^{\mathrm{c}} \geq 0$ only when they are stable. Stability requires that the reaction functions cross correctly. See Shibata (2014) for further detail on the stability of the solutions. Thus, the basic parameters are reasonable.[16]

In Figure 1, we assume $z_1 = 0$ and $z_2 = 1$. Recall that the noncooperative R&D $x^{\mathrm{n}} \geq 0$ is satisfied when $f_i^{\mathrm{n}} \geq 0$, for any $i$ ($i \in \{1, 2\}$).[17] The line from $(\beta, \lambda) = (0, 0.7645)$ to $(\beta, \lambda) = (1, 0.7889)$ indicates the boundary of whether $f_2^{\mathrm{n}} = 0$ or $f_2^{\mathrm{n}} > 0$, because $f_i^{\mathrm{n}}$

[16]We assume $a = 100$ and $c = 50$, as in Shy (1995). In addition, we assume $z_i \in [-1, 1]$, for any $i \in \{1, 2\}$. We need to set $\gamma = 50$. Then, $x_i^{\mathrm{n}} \geq 0$ and $x_i^{\mathrm{c}} \geq 0$ are stable, and we have $c_i(x_i^{\mathrm{n}}, x_j^{\mathrm{n}}) > 0$ and $c_i(x_i^{\mathrm{c}}, x_j^{\mathrm{c}}) > 0$, for any $i \in \{1, 2\}$, $\lambda \in [0, 1]$, and $\beta \in [0, 1]$.

[17]Because we always obtain $f_1^{\mathrm{n}} \geq f_2^{\mathrm{n}}$, it is sufficient to consider $f_2^{\mathrm{n}} \geq 0$.

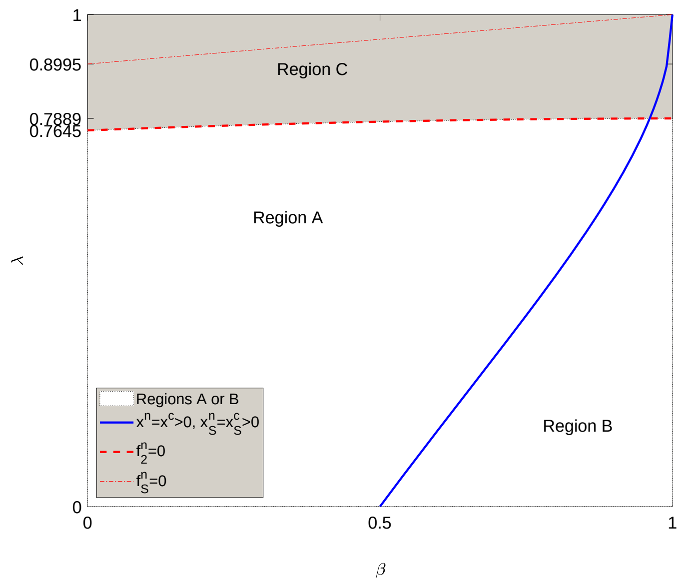


**Figure 1.** Regions A ($x^{\mathrm{n}} > x^{\mathrm{c}}$) and B ($x^{\mathrm{n}} \leq x^{\mathrm{c}}$) in space $(\beta, \lambda)$

is monotonically decreasing with $\lambda$.[18] For the lower regions of the boundary, we have $f_2^{\mathrm{n}} \geq 0$, and $x^{\mathrm{n}}$ is obtained as a unique solution. Thus, we define as Region C the upper region of the boundary. In contrast, we always obtain $f_i^{\mathrm{c}} \geq 0$, for any $i$ ($i \in \{1, 2\}$), as a unique solution for all regions of $\beta \in [0, 1]$ and $\lambda \in [0, 1]$. Consequently, henceforth, we compare only the magnitudes of $x^{\mathrm{n}} \geq 0$ and $x^{\mathrm{c}} \geq 0$ for any region other than Region C.

Figure 1 shows the regions of $x^{\mathrm{n}} \geq x^{\mathrm{c}} \geq 0$ in the $(\beta, \lambda)$-space. The line from $(\beta, \lambda) = (0.5, 0)$ to $(\beta, \lambda) = (1, 1)$ indicates the boundary of $x^{\mathrm{n}} = x^{\mathrm{c}} \geq 0$. Region A is defined by the upper-left region to the boundary, $x^{\mathrm{n}} > x^{\mathrm{c}} \geq 0$, where noncooperative R&D is preferred. Region B is defined by the lower-right region to the boundary, $x^{\mathrm{n}} \leq x^{\mathrm{c}}$, where cooperative R&D is preferred. As shown in Proposition 2.4, the important result is that the boundary of $x^{\mathrm{n}} = x^{\mathrm{c}} \geq 0$ is the same as that of $x_{\mathrm{S}}^{\mathrm{n}} = x_{\mathrm{S}}^{\mathrm{c}} \geq 0$. This confirms the result of Proposition 2.4 using numerical calculations.

### 3.2. *Effects of cost asymmetry on each firm*

This subsection examines the effects of cost asymmetry on each firm's R&D. The properties of $x_i^{\mathrm{n}}$ and $x_i^{\mathrm{c}}$ for $\lambda = 0$ are the same as those for any $\lambda \in (0, 1]$, on condition that $x_i^{\mathrm{n}}$ is obtained as a unique solution, for any $i$ ($i \in \{1, 2\}$). In this subsection, without loss of generality, we assume $\lambda = 0$.

In the two top, middle, and bottom panels of Figure 2, we assume that the asymmetric cost parameters are $(z_1, z_2) = (-0.5, 0.5)$, $(z_1, z_2) = (0, 1)$, and $(z_1, z_2) = (-1, 0)$, respectively. Under these three cases, we have $z_1 + z_2 = 0$, $z_1 + z_2 = 1$, and $z_1 + z_2 = -1$, respectively. The three left panels depict $x_i^{\mathrm{n}}$ and $x_i^{\mathrm{c}}$ with $\beta$ for the three cases. We find

[18] It is straightforward to obtain $\mathrm{d}f_i^{\mathrm{n}}/\mathrm{d}\lambda \leq 0$, for any $i$ ($i \in \{1, 2\}$). In the proof, differentiating $f_i^{\mathrm{n}}$ with respect to $\lambda$ yields

$$\frac{\mathrm{d}f_i^{\mathrm{n}}}{\mathrm{d}\lambda} = \frac{\partial f_i}{\partial x_i^{\mathrm{n}}}\frac{\partial x_i^{\mathrm{n}}}{\partial \lambda} + \frac{\partial f_i}{\partial \lambda} = \frac{\partial f_i}{\partial \lambda} = -\pi_i(x_i^{\mathrm{n}}, \pi_j^{\mathrm{n}}) \leq 0, \tag{38}$$

where we use the envelope theorem (i.e., $\partial f_i/\partial x_i^{\mathrm{n}} = 0$).

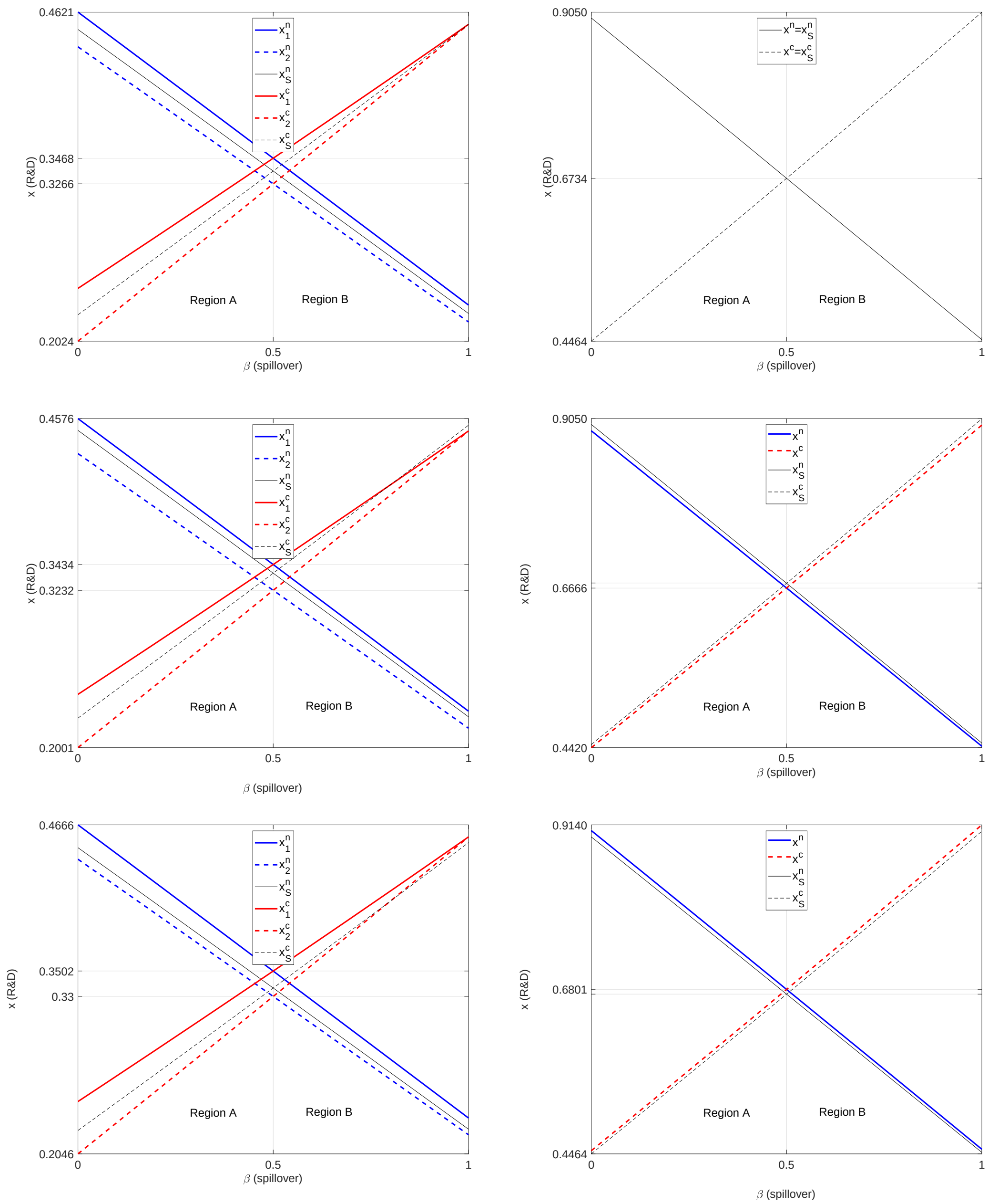


**Figure 2.** Effects of asymmetry between firms with $\beta$

We assume $\lambda = 0$. For symmetric cases, we assume $z_1 = z_2 = 0$. For asymmetric cases, we assume three cases: $(z_1, z_2) = (-0.5, 0.5)$, $(z_1, z_2) = (0, 1)$, and $(z_1, z_2) = (-1, 0)$, corresponding to the two top, two middle, and two bottom panels, respectively.

that $x_i^{\rm n}$ and $x_i^{\rm c}$ are decreasing and increasing with $\beta$, respectively, and we always have $x_1^k \geq x_2^k$ ($k \in \{{\rm n},{\rm c}\}$). The numerical calculation suggests the following observation.

**Observation 1.** The cooperative and noncooperative R&D investments of a low-cost firm are always larger than those of a high-cost firm.

As noted previously, efficient low-cost (less efficient high-cost) firms with some cost advantage (disadvantage) can best approximate large/mature (small/young) firms. Thus, our results fit well with those of the empirical studies by Shefer and Frenkel (2005) and Manez et al. (2009) that the investment in R&D is larger in large firms than it is in medium-sized and small firms.

In addition, in the three left panels, we obtain an interesting result. For any Firm $i$ ($i \in \{1,2\}$), we obtain $x_i^{\rm n} > x_i^{\rm c}$ for $\beta \in [0, 1/2)$, whereas $x_i^{\rm n} \leq x_i^{\rm c}$ for $\beta \in [1/2, 1]$. For any Firm $i$, regardless of whether $z_1 + z_2 = 0$, $z_1 + z_2 = 1$, or $z_1 + z_2 = -1$, we have $x_i^{\rm n} = x_i^{\rm c}$, for $\beta = 1/2$. This property always holds for any $\lambda \in (0, 1)$, subject to $f_2^{\rm n} \geq 0$.[19] Thus, our numerical calculation suggests the following observation.

**Observation 2.** The fact that low-cost firm's noncooperative R&D is larger than its cooperative R&D is equivalent to the fact that the high-cost firm's noncooperative R&D is larger than its cooperative R&D.

Observation 2 implies Proposition 2.4. To confirm the results in Proposition 2.4, we display $x^{\rm n}$ and $x^{\rm c}$ (market R&D investments) in the three right-hand panels of Figure 2.

Recall that we assume $(z_1, z_2) = (0, 0)$ as the symmetric case, and $(z_1, z_2) = (-0.5, 0.5)$, $(z_1, z_2) = (0, 1)$, and $(z_1, z_2) = (-1, 0)$ as asymmetric cases. Here, $z_1 + z_2 < 0$ ($z_1 + z_2 > 0$) implies an increase (decrease) in the cost efficiency of the asymmetric oligopoly market. From Equations (23), (34), (35), and (36), we have

$$\begin{cases} x^k > x_{\rm S}^k, & \text{if } z_1 + z_2 < 0, \\ x^k = x_{\rm S}^k, & \text{if } z_1 + z_2 = 0, \\ x^k < x_{\rm S}^k, & \text{if } z_1 + z_2 > 0. \end{cases}$$

Thus, we obtain the following observation.

**Observation 3.** As the production cost efficiency increases (i.e., $z_1 + z_2 < 0$), market noncooperative and cooperative R&D investments increase. In contrast, as the production cost efficiency decreases (i.e., $z_1 + z_2 > 0$), market noncooperative and cooperative R&D investments decrease.

### 3.3. *Effects of market competition on each firm*

This subsection considers the effects of market competition intensity on noncooperative R&D investment and on each firm's profit.

The two top panels of Figure 3 depict $f_i^{\rm n} := f_i(x_1^{\rm n}, x_2^{\rm n})$ with $\lambda$ ($i \in \{1,2\}$), for $\beta \in \{0, 1/2\}$. We see that $f_i^{\rm n}$ is monotonically decreasing with $\lambda$, for any $i$ ($i \in \{1,2\}$), and that $f_1^{\rm n} \geq f_2^{\rm n}$. For $\beta = 0$ and $\beta = 1/2$, the critical values of $\lambda$ satisfying $f_2(x_1^{\rm n}, x_2^{\rm n}) = 0$ are obtained as $\lambda = 0.7645$ and $\lambda = 0.7889$, respectively. The critical value of $\lambda$ is increasing with $\beta$.

[19] It is difficult to obtain such a property analytically, although it is likely possible. To the extent that we have solved the solutions numerically, we always obtain such properties.

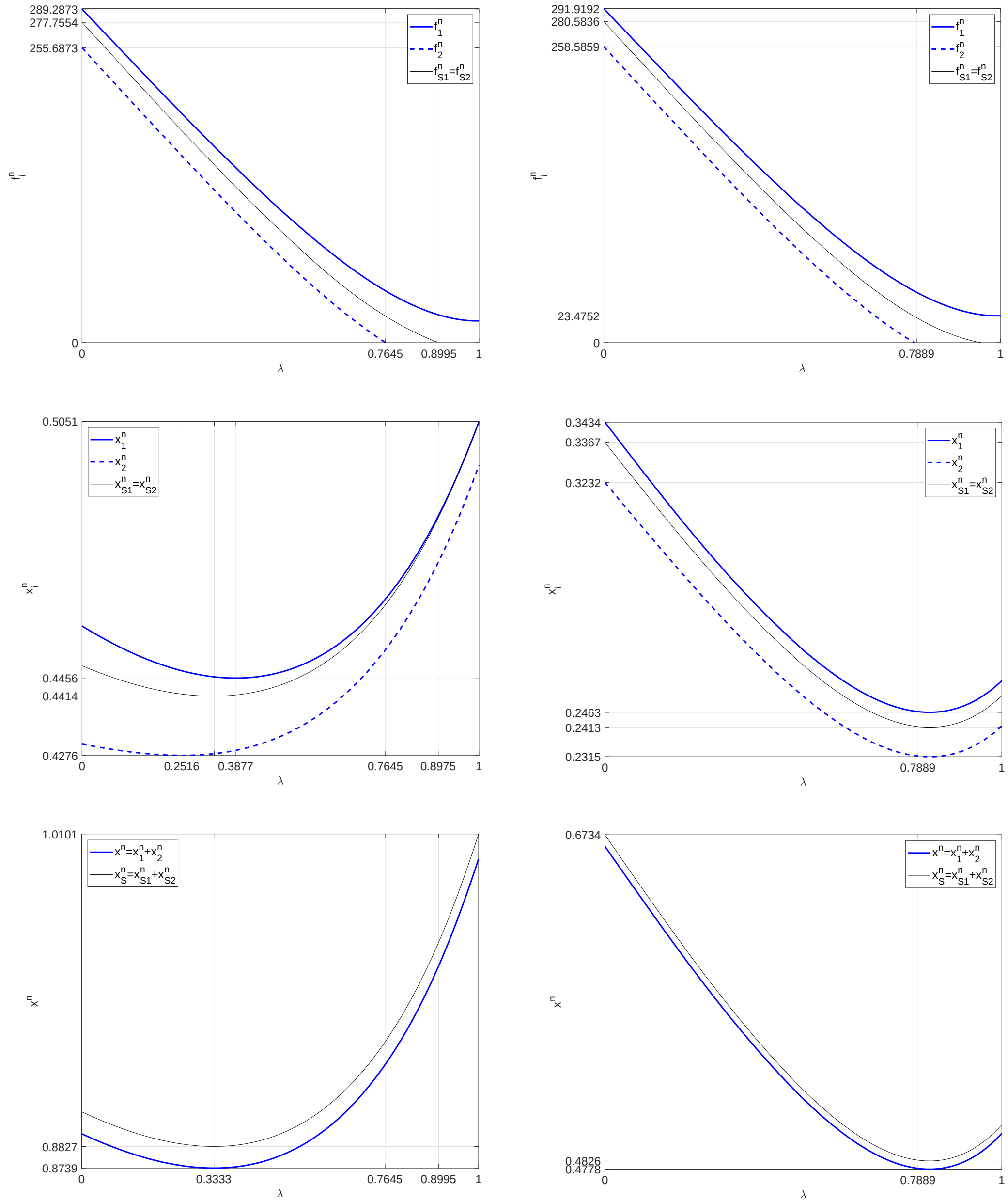


**Figure 3.** Effects of $\lambda$

The three left and three right panels represent $\beta = 0$ and $\beta = 0.5$, respectively.

In the middle-left panel of Figure 3, $x_i^{\mathrm{n}}$ has a U-shaped curve with $\lambda$, for any $i$ ($i \in \{1, 2\}$). This nonlinearity property fits with the findings of the empirical studies of Aghion et al. (2005) and Sacco and Schmutzler (2011) as well as with those of the theoretical studies of Vives (2008), Matsumura et al. (2013), and Shibata (2014). We can see that $x_1^{\mathrm{n}}$ is decreasing with $\lambda$ for $\lambda < 0.3877$, and is increasing with $\lambda$ for $\lambda > 0.3877$. Thus, $x_1^{\mathrm{n}}$ has a minimum value at $\lambda = 0.3877$. Similarly, $x_2^{\mathrm{n}}$ is decreasing with $\lambda$ for $\lambda < 0.2516$, and is increasing with $\lambda$ for $\lambda > 0.2516$. Thus, $x_2^{\mathrm{n}}$ has a minimum value at $\lambda = 0.2516$. These results yield the following interesting observation. For $\lambda \in [0.2516, 0.3877]$, $x_1^{\mathrm{n}}$ is decreasing with $\lambda$, while $x_2^{\mathrm{n}}$ is increasing with $\lambda$.[20] We summarize the result as follows.

**Observation 4.** A low-cost firm's noncooperative R&D investment may be decreasing with $\lambda$, but a high-cost firm's noncooperative R&D investment may be increasing with $\lambda$.

The results of Observation 4 contrast with those of Ishida et al. (2011), who show that an increase in the number of high-cost firms can increase (decrease) low-cost (high-cost) firms' R&D. The empirical studies by Shefer and Frenkel (2005) and Manez et al. (2009) show that R&D investments are higher for firms in high-tech industries than they are for firms in low- and medium-tech industries. Consider the hypothesis that the market in a high-tech industry is relatively less competitive than those in the low- and medium-tech industries are. Then, as the competitiveness intensifies, the market is regarded as being the same as those in the low- and medium-tech industries. Thus, if the hypothesis is correct, our result is consistent with those of Shefer and Frenkel (2005) and Manez et al. (2009).

In the bottom-left panel, we see that both $x^{\mathrm{n}} := x_1^{\mathrm{n}} + x_2^{\mathrm{n}}$ and $x_{\mathrm{S}}^{\mathrm{n}} := x_{\mathrm{S}1}^{\mathrm{n}} + x_{\mathrm{S}2}^{\mathrm{n}}$ have a minimum value at $\lambda = 1/3$, which is the same result established in Matsumura et al. (2013) and Shibata (2014).

In the middle-right and bottom-right panels, we assume $\beta = 1/2$. Then, recall that $x_1^{\mathrm{n}}$ and $x_2^{\mathrm{n}}$ are obtained as a unique solution for $\lambda \in [0, 0.7889)$. We see that $x_i^{\mathrm{n}}$ is monotonically decreasing with $\lambda$. Thus, a nonlinearity property between $x_i^{\mathrm{n}}$ and $\lambda$ is not obtained for $\beta = 1/2$.

Figure 4 displays the regions of the U-shaped curves between $x_i^{\mathrm{n}}$ and $\lambda$ ($i \in \{1, 2\}$). The line from $(\beta, \lambda) = (0, 0.3877)$ to $(\beta, \lambda) = (1, 1)$ represents the boundary of $\mathrm{d}x_1^{\mathrm{n}}/\lambda = 0$. The line from $(\beta, \lambda) = (0, 0.2516)$ to $(\beta, \lambda) = (1, 1)$ represents the boundary of $\mathrm{d}x_2^{\mathrm{n}}/\lambda = 0$. Then, the upper-left regions to the boundaries represent $\mathrm{d}x_i^{\mathrm{n}}/\mathrm{d}\lambda > 0$, and the lower-right regions represent $\mathrm{d}x_i^{\mathrm{n}}/\mathrm{d}\lambda < 0$. We see that the lines of $\mathrm{d}x_1^{\mathrm{n}}/\mathrm{d}\lambda = 0$ for $\beta \geq 0.4305$, $\mathrm{d}x_{\mathrm{S}}^{\mathrm{n}}/\mathrm{d}\lambda = 0$ for $\beta \geq 0.4340$, and $\mathrm{d}x_2^{\mathrm{n}}/\mathrm{d}\lambda = 0$ for $\beta \geq 0.4355$ are in Region C. Thus, $x_1^{\mathrm{n}}$, $x_{\mathrm{S}}^{\mathrm{n}}$, and $x_2^{\mathrm{n}}$ have a U-shaped curve with $\lambda$ for $\beta \in [0.0.4305)$, $\beta \in [0.0.4340)$, and $\beta \in [0, 0.4355)$, respectively.

### 3.4. *Low-cost and high-cost firms' profits and the degree of market competition*

This section investigates how the difference between the profits of low-cost and high-cost firms changes with the degree of market competition.

Figure 5 depicts $x_1^k - x_2^k \geq 0$, $q_1^k - q_2^k \geq 0$, and $f_1^k - f_2^k \geq 0$, with $\lambda$ ($k \in \{\mathrm{n}, \mathrm{c}\}$). The

[20] These results are obtained for $\beta < 0.4305$. See Figure 4 for details.

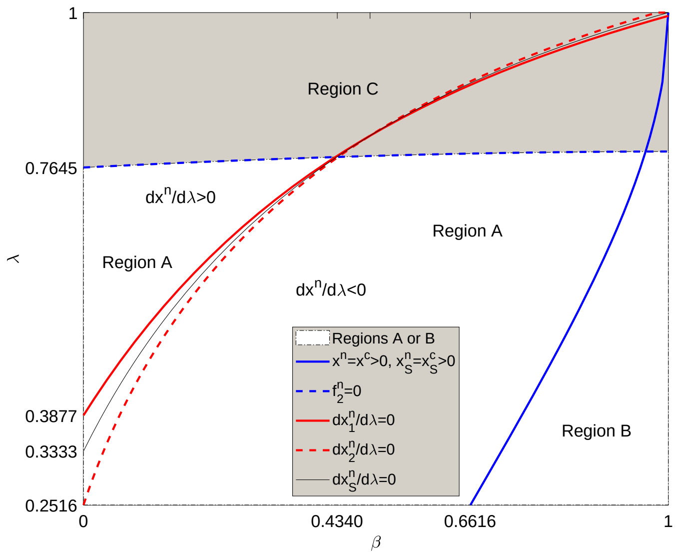


**Figure 4.** Regions of nonmonotonicity for the R&D competition strategy in space $(\beta, \lambda)$

This figure depicts the line of $\mathrm{d}x_i^{\mathrm{n}}/\mathrm{d}\lambda = 0$ in space $(\beta, \lambda)$. Under the basic parameters, $x_1^{\mathrm{n}}$, $x_{\mathrm{S}}^{\mathrm{n}}$, and $x_2^{\mathrm{n}}$ have a U-shaped curve with $\lambda$ for $\beta \in [0, 0.4305)$, $\beta \in [0, 0.4330)$, and $\beta \in [0, 0.4355)$, respectively.

three left, middle, and right panels represent $\beta = 0$, $\beta = 1/2$, and $\beta = 1$, respectively.

In the three left-hand panels, because $x_1^{\mathrm{n}} - x_2^{\mathrm{n}}$, $q_1^{\mathrm{n}} - q_2^{\mathrm{n}}$, and $f_1^{\mathrm{n}} - f_2^{\mathrm{n}}$ are defined for $\lambda \in [0, 0.7645)$, we compare them for $\lambda \in [0, 0.7645)$ rather than for $\lambda \in [0.7645, 1]$. Thus, we have

$$\begin{cases} x_1^{\mathrm{n}} - x_2^{\mathrm{n}} < x_1^{\mathrm{c}} - x_2^{\mathrm{c}}, & \text{for } \lambda \in [0, 0.4250), \\ x_1^{\mathrm{n}} - x_2^{\mathrm{n}} \geq x_1^{\mathrm{c}} - x_2^{\mathrm{c}}, & \text{for } \lambda \in [0.4250, 0.7645). \end{cases}$$

Her, we find that $x_1^k - x_2^k$ and $q_1^k - q_2^k$ are decreasing with $\lambda$, whereas $f_1^k - f_2^k$ is increasing with $\lambda$.

In the three middle panels, $x_1^{\mathrm{n}} - x_2^{\mathrm{n}}$, $q_1^{\mathrm{n}} - q_2^{\mathrm{n}}$, and $f_1^{\mathrm{n}} - f_2^{\mathrm{n}}$ are defined for $\lambda \in [0, 0.7889)$. Here, we find that $x_1^k - x_2^k$ and $q_1^k - q_2^k$ are decreasing with $\lambda$, whereas $f_1^k - f_2^k$ is increasing with $\lambda$.

The six panels on the left yield the following interesting results.

**Observation 5.** Suppose that the spillover is small. Then, as the market becomes more competitive, $f_1^k - f_2^k$ increases, whereas $x_1^k - x_2^k$ and $q_1^k - q_2^k$ decrease.

In the three right-hand panels, $x_1^k - x_2^k$, $q_1^k - q_2^k$, and $f_1^k - f_2^k$ are defined for $\lambda \in [0, 1]$ ($k \in \{\mathrm{n}, \mathrm{c}\}$). Here, $x_1^{\mathrm{n}} - x_2^{\mathrm{n}}$ is increasing with $\lambda$, and $x_1^{\mathrm{c}} - x_2^{\mathrm{c}}$ has a $\Lambda$-shaped curve with $\lambda$. These results are contrary to those in the top-left and top-middle panels. Furthermore, $q_1^{\mathrm{n}} - q_2^{\mathrm{n}} = q_1^{\mathrm{c}} - q_2^{\mathrm{c}}$ in the middle-right panel, and $f_1^k - f_2^k$ is increasing with $\lambda$ in the bottom-right panel.

In the three bottom panels, we see that $f_1^k - f_2^k$ is increasing with $\lambda$. As already shown in the two top-left and top-right panels of Figure 3, we obtain $\mathrm{d}f_i^k/\mathrm{d}\lambda < 0$; that is, $f_i^k$ is decreasing with $\lambda$ for any $i$ and $k$ ($i \in \{1, 2\}$, $k \in \{\mathrm{n}, \mathrm{c}\}$). We summarize these results as follows.

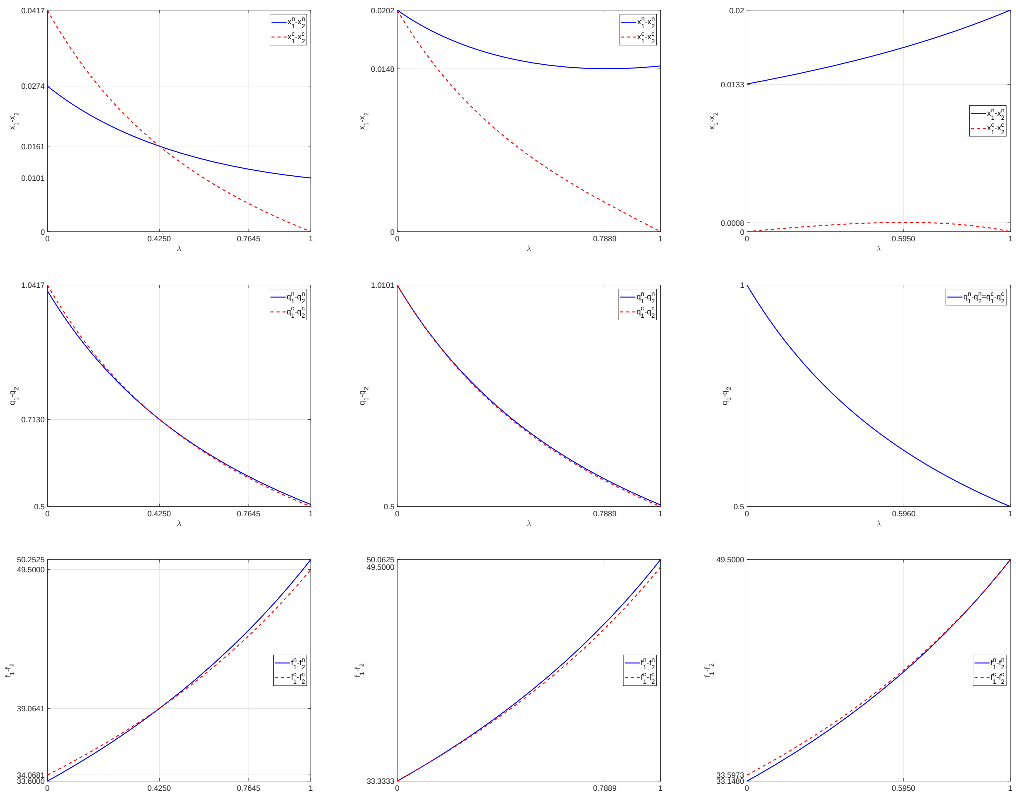

**Figure 5.** Difference of $x_1 - x_2$ with $\beta$

In the left, middle, and right panels, we assume $\lambda = 0$, $\lambda = 0.25$, and $\lambda = 0.5$, respectively.. The other parameters are $a = 100$, $c = 50$, $\gamma = 50$, $z_1 = 0$, and $z_2 = 1$.

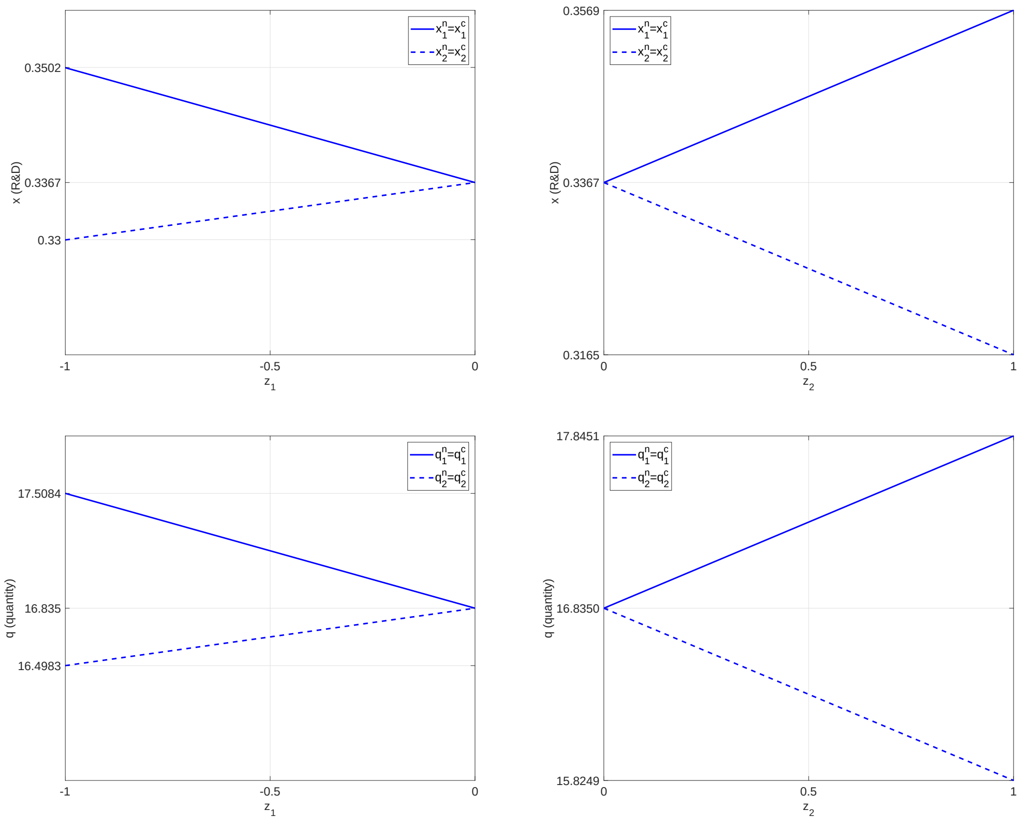


**Figure 6.** Effects of production cost asymmetry ($z_2$)

The parameters are $a = 100$, $c = 50$, $\gamma = 50$, $\lambda = 0$, and $\beta = 1/2$. In the two left and two right panels, we assume $z_2 = 0$ and $z_1 = 0$, respectively.

**Observation 6.** When a market becomes more competitive, the difference in profit between low-cost and high-cost firms increases, although the actual profits of both firms decrease.

That an increase in the number of firms reduces each firm's profit is a standard result. However, Observation 6 indicates that an increase in the number of firms increases the difference in profit between low-cost and high-cost firms. This is a new prediction corresponding to an interesting and testable implication.[21]

### 3.5. *Effects of production cost asymmetry*

This subsection examines the effects of production cost asymmetry between firms on each firm's R&D investments. As in Subsection 3.2, the properties of $\lambda = 0$ and $\beta = 0.5$ are the same as those for any $\lambda \in (0, 1]$ and $\beta \in [0, 1]\backslash\{0.5\}$. In this subsection, without loss of generality, we assume $\lambda = 0$ and $\beta = 0.5$, which leads to $x_i^{\rm n} = x_i^{\rm c}$ $(i \in \{1, 2\})$.[22]

The two upper panels of Figure 6 depict $x_i^{\rm n} = x_i^{\rm c}$ with $z_1$ and $z_2$, respectively $(i \in \{1, 2\})$. In the upper-left panel, we consider the effect of $z_1 \in [-1, 0]$ for a fixed

[21]To the best of our knowledge, there are no empirical studies on Observation 5.

[22]In the proof, we obtain $\delta_j = \phi_j$, for any $j$ $(j \in \{1, 2, 3, 4, 5, 6\})$, by substituting $\lambda = 0$ and $\beta = 1/2$ into (17)–(22) and (28)–(33).

$z_2 = 0$. An increase in $z_1$ decreases $x_1^k$, but increases $x_2^k$ ($k \in \{\text{n}, \text{c}\}$). In the upper-right panel, we examine the effect of $z_2 \in [0, 1]$ for a fixed $z_1 = 0$. Here, an increase in $z_2$ increases $x_1^k$, but decreases $x_2^k$. Thus, importantly, an increase in $z_i$ affects both $x_i^k$ and $x_j^k$ ($i, j \in \{1, 2\}, i \neq j$). Equations (16) and (27) makes it difficult to prove these results analytically. However, we show numerically that the denominators of $x_i^\text{n}$ and $x_i^\text{c}$ in (16) and (27), respectively, are positive; that is,

$$(\delta_6 - \delta_4)^2 - \delta_5^2 > 0, \quad (\phi_6 - \phi_4)^2 - \phi_5^2 > 0.$$

Using the results from the two upper panels, we obtain the following numerically:

$$\begin{aligned} &(\delta_6 - \delta_4)\delta_3 + \delta_5\delta_2 > 0, \quad &(\delta_6 - \delta_4)\delta_2 + \delta_5\delta_3 < 0, \\ &(\phi_6 - \phi_4)\phi_3 + \phi_5\phi_2 > 0, \quad &(\phi_6 - \phi_4)\phi_2 + \phi_5\phi_3 < 0. \end{aligned}$$

These results are summarized as the following observation.

**Observation 7.** As the production cost of a firm increases, the firm decreases its R&D investment, while the rival firm increases its R&D investment.

The two lower panels of Figure 6 depict $q_i^\text{n} = q_i^\text{c}$ with $z_1$ and $z_2$, respectively ($i \in \{1, 2\}$). In the lower-left panel, we examine the effect of $z_1 \in [-1, 0]$ for a fixed $z_2 = 0$. An increase in $z_1$ decreases $q_1^k$, but increases $q_2^k$ ($k \in \{\text{n}, \text{c}\}$). In the lower-right panel, we consider the effect of $z_2 \in [0, 1]$ for a fixed $z_1 = 0$. An increase in $z_2$ increases $q_1^k$, but decreases $q_2^k$. From the results of the two lower panels, an increase in $z_i$ decreases $q_i^k$, but increases $q_j^k$ ($i, j \in \{1, 2\}, i \neq j$). Thus, we have the following observation.

**Observation 8.** As the production cost of a firm increases, the firm decreases its production quantity, while the rival firm increases its production quantity.

The results of Observation 8 follow from those of Observation 7. In addition, the results of Observations 7 and 8 provide a new and testable implication, because few empirical studies have examined how a firm's production cost affects the R&D investments of its rival firms.

### 3.6. *The relationship between each firm's R&D and its profit*

This subsection numerically examines the relationship between each firm's optimal R&D expenditure and its profit. The parameters for asymmetric production costs are $z_1 = -0.5$ and $z_2 = 0.5$. The other parameters are defined as before.

Figure 7 depicts $f_i^k$ (Firm $i$'s profit) with respect to $x_i^k$ (the firm's R&D), for $i \in \{1, 2\}, k \in \{\text{n}, \text{c}\}$. Here, we derive and plot the pairs $(x_i^k, f_i^k)$, for $\beta \in [0, 1]$. Recall that for $\beta \in [0, 1]$ under $(z_1, z_2) = (-0.5, 0.5)$, we obtained $x_1^\text{n} = x_1^\text{c} = 0.3468$ and $x_2^\text{n} = x_2^\text{c} = 0.3266$ for $\beta = 1/2$ (see the top-left panel of Figure 3). From these results, we obtain $f_1^\text{n} = f_1^\text{c} = 297.6712$ for $x_1^\text{n} = x_1^\text{c} = 0.3468$, and $f_2^\text{n} = f_2^\text{c} = 290.4012/(1.1) = 264.0011$ for $x_2^\text{n} = x_2^\text{c} = 0.3266$. Most interestingly, we find that $f_i^\text{n}$ decreases with $x_i^\text{n}$, while $f_i^\text{c}$ increases with $x_i^\text{c}$ ($i \in \{1, 2\}$). These results are summarized in the following observation.

**Observation 9.** An increase in expenditure on R&D competition decreases a firm's profit, whereas an increase in expenditure on R&D cooperation increases the firm's profit.

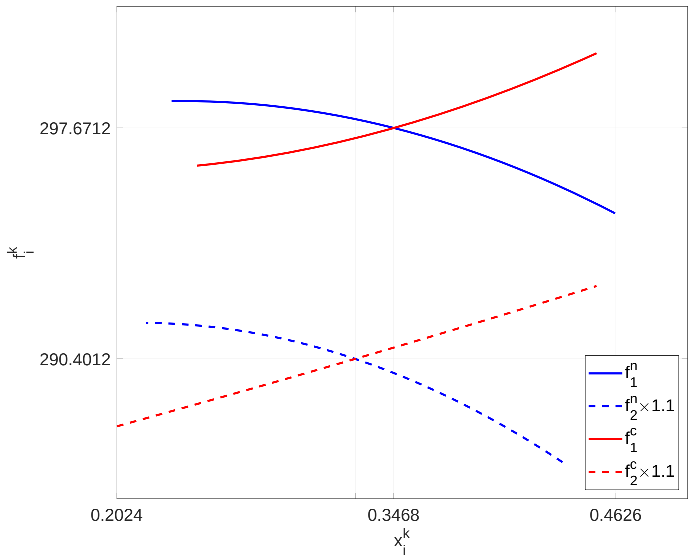


**Figure 7.** Relationship between each firm's R&D ($x_i^k$) and its profit ($f_i^k$)
The parameters are $a = 100$, $c = 50$, $\gamma = 50$, $\lambda = 0$, $\beta = 1/2$, $z_1 = -0.5$, and $z_2 = 0.5$.

The second part of the result described in Observation 9 fits well with the results of empirical studies of Aschhoff and Schmidt (2008) and Bogliacino and Pianta (2012), which show that an increase in a firm's R&D cooperation increases its profit. However, few empirical studies have examined how an increase in R&D competition affects a firm profit (see the first part of Observation 9). Thus, the results of Observation 9 also provide a new and testable implication on whether an increase in R&D competition decreases a firm's profit.

## 4. Concluding remarks

This study extends the work of Shibata (2014) by incorporating asymmetry in firms' production costs. The contribution of this work is to examine the effects of this asymmetry on firms' R&D investments and profits, where we obtain five novel results.

Several extensions to this study are possible. First, it would be interesting to consider asymmetries other than those in firms' production costs, for example, those in firms' spillovers or R&D costs. Suppose, for example, $\beta_1 = 0.3$ and $\beta_2 = 0.5$, where $\beta_i \in [0, 1]$ represents the R&D spillovers from Firm $i$ to Firm $j$ ($i, j \in \{1, 2\}, i \neq j$). Additionally, assume $\gamma_1 = 50$ and $\gamma_2 = 70$, where $\gamma_i > 0$ represents Firm $i$'s R&D cost parameter ($i \in \{1, 2\}$). Here, it would be interesting to consider how these asymmetries affect R&D investment.

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## Appendix

### *Derivation of $x_i^{\mathrm{n}}$*

The noncooperative Nash solution is the solution to the following simultaneous equations:

$$\delta_1(a-c)+\delta_2 z_1+\delta_3 z_2+\delta_4 x_1+\delta_5 x_2-\delta_6 x_1=0, \tag{A.1}$$
$$\delta_1(a-c)+\delta_2 z_2+\delta_3 z_1+\delta_4 x_2+\delta_5 x_1-\delta_6 x_2=0, \tag{A.2}$$

where $\delta_1$, $\delta_2$, $\delta_3$, $\delta_4$, $\delta_5$, and $\delta_6$ are given by

$$\begin{aligned}\delta_1 :=&(1+\lambda)(2-\beta-\lambda)+(1-\lambda)(2-\beta+\beta\lambda)\\ &-\lambda\big((1+\lambda)(2\beta-1-\beta\lambda)+(1-\lambda)(2\beta-1+\lambda)\big)\geq 0,\end{aligned} \tag{A.3}$$
$$\begin{aligned}\delta_2 :=&-2(2-\beta-\lambda)-(2-\lambda)(2-\beta+\beta\lambda)\\ &-\lambda\big((1-\lambda)(2\beta-1-\beta\lambda)+(2\beta-1+\lambda)\big)\leq 0,\end{aligned} \tag{A.4}$$
$$\begin{aligned}\delta_3 :=&(1-\lambda)(2-\beta-\lambda)+(2-\beta+\beta\lambda)\\ &-\lambda\big(-2(2\beta-1-\beta\lambda)-(2-\lambda)(2\beta-1+\lambda)\big)\geq 0,\end{aligned} \tag{A.5}$$
$$\delta_4 :=2\big((2-\beta+\beta\lambda)(2-\beta-\lambda)-\lambda(2\beta-1+\lambda)(2\beta-1-\beta\lambda)\big), \tag{A.6}$$
$$\delta_5 :=(1-\lambda)\big((2\beta-1+\lambda)(2-\beta-\lambda)+(2\beta-1-\beta\lambda)(2-\beta+\beta\lambda)\big), \tag{A.7}$$
$$\delta_6 :=\gamma(3-\lambda)^2(1+\lambda)\geq 0, \tag{A.8}$$

respectively. Rearranging (A.3)–(A.8) yields (17)–(22). Equations (A.1) and (A.2) are rewritten as

$$\begin{pmatrix}\delta_6-\delta_4 & -\delta_5\\ -\delta_5 & \delta_6-\delta_4\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}\delta_1(a-c)+\delta_2 z_1+\delta_3 z_2\\ \delta_1(a-c)+\delta_2 z_2+\delta_3 z_1\end{pmatrix}. \tag{A.9}$$

The solution $x_i$ is

$$\begin{aligned}x_i^{\mathrm{n}} =&\frac{1}{(\delta_6-\delta_4)^2-\delta_5^2}\\ &\times\big((\delta_6-\delta_4)(\delta_1(a-c)+\delta_2 z_i+\delta_3 z_j)+\delta_5(\delta_1(a-c)+\delta_2 z_j+\delta_3 z_i)\big),\end{aligned} \tag{A.10}$$

for any $i$ $(i,j\in\{1,2\}, i\neq j)$. Rearranging (A.10) yields (16).

### *Derivation of* $x_i^{\mathrm{c}}$

The cooperative solution is obtained by solving the following simultaneous equations:

$$\phi_1(a-c)+\phi_2 z_1+\phi_3 z_2+\phi_4 x_1+\phi_5 x_2-\phi_6 x_1=0, \tag{A.11}$$
$$\phi_1(a-c)+\phi_2 z_2+\phi_3 z_1+\phi_4 x_2+\phi_5 x_1-\phi_6 x_2=0, \tag{A.12}$$

where $\phi_1$, $\phi_2$, $\phi_3$, $\phi_4$, $\phi_5$, and $\phi_6$ are given by

$$\begin{aligned}\phi_1 :=&(1+\lambda)\big((2-\beta-\lambda)+(2\beta-1-\beta\lambda)\big)\\ &+(1-\lambda)\big((2-\beta+\beta\lambda)+(2\beta-1+\lambda)\big)\geq 0,\end{aligned} \tag{A.13}$$
$$\begin{aligned}\phi_2 :=&-2(2-\beta-\lambda)-(2-\lambda)(2-\beta+\beta\lambda)\\ &+(1-\lambda)(2\beta-1-\beta\lambda)+(2\beta-1+\lambda)\leq 0,\end{aligned} \tag{A.14}$$
$$\begin{aligned}\phi_3 :=&(1-\lambda)(2-\beta-\lambda)+(2-\beta+\beta\lambda)\\ &-2(2\beta-1-\beta\lambda)-(2-\lambda)(2\beta-1+\lambda)\geq 0,\end{aligned} \tag{A.15}$$
$$\phi_4 :=2\big((2-\beta+\beta\lambda)(2-\beta-\lambda)+(2\beta-1+\lambda)(2\beta-1-\beta\lambda)\big), \tag{A.16}$$
$$\phi_5 :=2\big((2\beta-1+\lambda)(2-\beta-\lambda)+(2\beta-1-\beta\lambda)(2-\beta+\beta\lambda)\big), \tag{A.17}$$
$$\phi_6 :=\gamma\frac{(3-\lambda)^2(1+\lambda)}{1-\lambda}\geq 0, \tag{A.18}$$

respectively. Rearranging (A.13)–(A.18) yields (28)–(33). Equations (A.11) and (A.12) are rewritten as

$$\begin{pmatrix}\phi_6-\phi_4 & -\phi_5\\ -\phi_5 & \phi_6-\phi_4\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}\phi_1(a-c)+\phi_2 z_1+\phi_3 z_2\\ \phi_1(a-c)+\phi_2 z_2+\phi_3 z_1\end{pmatrix}. \tag{A.19}$$

The solution $x_i$ is

$$\begin{aligned}x_i^{\mathrm{c}} =&\frac{1}{(\phi_6-\phi_4)^2-\phi_5^2}\\ &\times\big((\phi_6-\phi_4)(\phi_1(a-c)+\phi_2 z_i+\phi_3 z_j)+\phi_5(\phi_1(a-c)+\phi_2 z_j+\phi_3 z_i)\big),\end{aligned} \tag{A.20}$$

for any $i$ $(i,j\in\{1,2\}, i\neq j)$. Rearranging (A.20) yields (27).

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