

Research Paper Series

No. 185

**Zero interest rate policy and asymmetric price  
adjustment in Japan: an empirical analysis  
of a nonlinear DSGE model**

Mototsugu Shintani<sup>†</sup>      Hirokuni Iiboshi<sup>‡</sup>

July, 2017

---

<sup>†</sup> University of Tokyo, Research Center for Advanced Science and Technology

<sup>‡</sup> Graduate School of Social Sciences, Tokyo Metropolitan University

# Zero interest rate policy and asymmetric price adjustment in Japan: an empirical analysis of a nonlinear DSGE model\*

Hirokuni Iiboshi<sup>†</sup> and Mototsugu Shintani<sup>‡</sup>

August 2016

## Abstract

We investigate the effect of incorporating zero lower bound (ZLB) in monetary policy rule and asymmetric adjustment costs (AAC) in firm's price setting mechanism in a standard New Keynesian DSGE model on explaining the unique experience of Japanese economy over the last three decades. To improve the accuracy of evaluating the nonlinear feature of the model, the projection method is employed in solving the model. We estimate the model using the Bayesian method combined with a particle filter and show that the estimated model with both ZLB and AAC outperforms the benchmark model in term of explaining the data. The adjustment cost in reducing prices is estimated to be 24 to 32 percent higher than raising prices. The presence of this downward price rigidity is likely to play a role in preventing further deflation by mitigating the deflationary pressure from the reduction of productivity.

*Keywords*, New Keynesian model, zero lower bound, Rotemberg price setting, policy function iterations, projection method, particle filter, Metropolis-Hastings algorithm

*JEL Classifications*, C11, E32, E52

---

\*We thank Gary Anderson, Yasuo Hirose, Jinill Kim, Junior Maih, Marco Ratto, Kozo Ueda and seminar and conference participants at the University of Tokyo, the 22nd International Conference of Computing in Economics and Finance in Bordeaux for helpful comments and discussions. Shintani gratefully acknowledges the financial support of Grant-in-aid for Scientific Research. The views expressed here are those of the authors and do not reflect the official views of the organizations the authors belongs to.

<sup>†</sup>Tokyo Metropolitan University, Japan. Email: iiboshi@tmu.ac.jp

<sup>‡</sup>the University of Tokyo, Japan and Vanderbilt University, USA. Email: moto.shintani@gmail.com

# Introduction

Over the past two decades, nominal interest rate has been set at near zero in Japan. Such a prolonged spell of zero interest rates makes it difficult to ignore the presence of this strong nonlinearity in monetary policy function when estimating a macroeconomic model using Japanese data. Another unique feature of Japanese experience during the same time period is deflation and near-zero inflation. The zero lower bound (ZLB) constraint on nominal interest rates implies that government cannot reduce interest rate below zero even if the inflation rate is below target and the output gap is negative. One possibility of explaining the unique movement of inflation rate below and near zero is to consider the asymmetric price setting behavior by firms who raise prices and who reduce prices.

In this paper, we use Japanese data and estimate a small-scale New Keynesian model which emphasizes two types of nonlinear dynamic features in the economy. In particular, we introduce the ZLB constraint on nominal interest rates and the asymmetric cost of price adjustment in a canonical DSGE model of An and Schorfheide (2007). We show that the estimated model with those features outperforms the benchmark model in term of explaining the data.

In the literature of the ZLB constraint, it is well-known that log-linearization and higher-order perturbation method are not reliable techniques to solve the model. Instead, recent studies, including Aruoba, Cuba-Borda and Schorfheide (2013), Fernandez Villaverde, Gordon, Guerron-Quintana and Rubio-Ramirez (2015), Gust, Lopez-Salido, and Smith (2013), Gavin, Keen, Richter and Throckmorton (2015), to name a few, typically utilize some form of global solution methods in solving the fully nonlinear model. In this paper, we also solve a fully nonlinear DSGE model by a global projection method followed by the Bayesian estimation based on full-information likelihood evaluated by the particle filter. The use of particle filters was first introduced in the DSGE model estimation by Fernandez-Villaverde and Rubio-Ramirez (2005, 2007)

and was employed by Gust, Lopez-Salido, and Smith (2012) in their estimation of the model with ZLB constraint.

Using the data from 1981:Q3 to 2015:Q1, our estimates suggest that the adjustment costs in deflation are about 24-32% higher than those in inflation. This is in line with results obtained by Kim and Ruge-Murcia (2009) and Aruoba et al. (2013) who found the significant effect of downward rigidity in the wage adjustment in the U.S. rather than the price adjustment.

Our estimated model also implies that expected duration of the zero interest rate is 4.2-5.5 quarters which is longer than the U.S. evidence obtained by Gust, Lopez-Salido, and Smith (2012) who found that the average duration for a lower bound spell is just over three quarters and the median duration is two quarters.

Our paper is organized as follows. Japanese experience over the past three decades is first described in Section 1. The model is provided in section 2. The estimation procedure described and results are reported in section 3. A concluding remark is made in section 4.

## 1 Overview of the data

Figure 1 shows the output growth ( $YGR_t$ ), inflation ( $INFL_t$ ), and nominal interest rate rates ( $INT_t$ ) from 1981:Q3 through 2015:Q1 in Japan. The output growth series computed as the log difference of real GDP from the Cabinet Office's National Accounts. We used official 2005 constant price series that cover the period 1994:Q1-2015:Q1 and merged it with the 2000 constant price series which is available for earlier years. The inflation series is year on year log growth rate of consumption price index excluding foods (core CPI) from Statistics Bureau. The nominal interest rate series is quarterly averages of monthly uncollateralized call rate obtained from the Bank of Japan. Two unique features stand out from the figure. First, nominal interest rate has been decreasing over the first half of the sample, and set at near zero in

the second half of the sample. Second, frequencies of observing negative inflation rate seems to be increased in the second half of the sample compared the first half of the sample.

[ Insert Figure 1 ]

## 2 Model

We modify a canonical small-scale New Keynesian DSGE model of An and Schorfheide (2007) and Herbst and Schorfheide (2016) by introducing ZLB constraint on monetary policy function and AAC of firm' price setting behavior<sup>1</sup>. Since ZLB and AAC are only two features that differ from the benchmark model, we first explain each nonlinearity one by one.

The other issue we want to emphasize is that we numerically solve a rational expectations equilibrium (REE) directly from a nonlinear equations instead of relying on a log-linearized version of the model. To this end, we adapt the technique suggested in Richtcher et al. (2014). This method will be briefly explained in this section (additional details are described in Technical Appendix).

### 2.1 Monetary policy under zero lower bound (ZLB)

Monetary policy rule is constrained by ZLB and written as

$$R_t = \max(1, R_t^{*1-\rho_R} R_{t-1}^{\rho_R} e^{\epsilon_{R,t}}), \quad (1)$$

where  $R_t$  is the gross nominal interest rate,  $R_t^*$  is nominal target rate and  $\epsilon_{R,t}$  is monetary policy shock. The nominal target rate,  $R_t^*$ , is given as

$$R_t^* = r\pi^* \left(\frac{\pi_t}{\pi^*}\right)^{\psi_1} \left(\frac{Y_t}{Y_t^*}\right)^{\psi_2}, \quad (2)$$

---

<sup>1</sup>Aruoba et al. (2014) and Aruoba and Schorfheide (2015) also used the model and extent it to regime switching model under ZLB.

where  $\pi^*$  and  $Y_t^*$  is inflation target (or steady state of inflation) and output target, respectively.

## 2.2 Firms with asymmetric adjustment cost (AAC)

In what follows, Rotemberg type adjustment costs of price change are extended to respond asymmetrically depending on whether current price is below or above the steady state of inflation  $\pi$ .

Monopolistically competitive intermediate goods producing firms maximize the present value of future profits:

$$\Pi = E_t \left[ \sum_{s=0}^{\infty} \beta^s Q_{t+s|t} \left( \frac{P_{t+s}(j)}{P_t} Y_{t+s}(j) - W_{t+s} N_{t+s}(j) - AC_{t+s}(j) \right) \right],$$

where  $\beta$  is discount factor and  $AC_t(j)$  is asymmetric adjustment costs (AAC) of price change given by

$$AC_t(j) = \frac{\psi(\pi_t)}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - \pi \right)^2 Y_t(j), \quad (3)$$

$$\psi(\pi_t) = \begin{cases} \phi, & \text{if } \frac{P_t(j)}{P_{t-1}(j)} (= \pi_t) \geq \pi \\ e^\delta \phi & \text{if } \frac{P_t(j)}{P_{t-1}(j)} (= \pi_t) < \pi \end{cases},$$

where  $\delta \in (-\infty, \infty)$  is a parameter which controls the degree of asymmetric adjustment which reduces to the standard symmetric adjustment cost when  $\delta = 0$ .

The function implies downward price rigidity if  $\delta > 0$ , while it implies the upward price rigidity if  $\delta < 0$ . Note that, for a small value of  $\delta$ , we have  $e^\delta \approx (1 + \delta)$ , so that  $e^\delta$  can be approximated by  $(1 + \delta)\phi$ . For a positive (small) value of  $\delta$ , we can interpret that reducing prices is  $100 \times \delta$  percent more costly than increasing prices. Similarly, for a negative (small) value of  $\delta$ , we can interpret that reducing prices is  $100 \times \delta$  percent less costly than increasing the price.

Although a similar idea has already adapted using an alternative functional form by Kim and Ruge-Murcia (2009) and Aruoba et al. (2014), our functional form seems

to be simple and intuitive.<sup>2</sup>

This firm producing the intermediate good,  $j$ , faces demand given by

$$Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-1/\nu} Y_t,$$

where  $\nu$  is inverse elasticity of demand for goods  $j$ . The linear production technology of the firm is given by

$$Y_t(j) = A_t N_t(j),$$

where  $A_t$  and  $N_t(j)$  are exogenous common productivity process and the labor input of firm  $j$ , respectively.

Aggregate productivity  $A_t$  follows a nonstationary process

$$A_t = \gamma A_{t-1} z_t, \quad z_t = z_{t-1}^{\rho_z} e^{\epsilon_{z,t}} (> 0), \quad (4)$$

where  $\epsilon_{z,t}$  is the productivity shock.

## 2.3 Closing the model

The remaining part of the model consists of household and government sectors.

The households maximize utilities

$$U = E_t \left[ \sum_{s=0}^{\infty} \beta^s \left( \frac{(C_{t+s}/A_{t+s})^{1-\tau} - 1}{1-\tau} + \chi_M \ln \left( \frac{M_{t+s}}{P_{t+s}} \right) - \chi_H H_{t+s} \right) \right],$$

where  $\tau$  is inverse elasticity of intertemporal substitution of consumption, subject to budget constraint

$$P_t C_t + B_t + M_t + T_t = P_t W_t H_t + R_{t-1} B_{t-1} + M_{t-1} + P_t D_t + P_t S C_t,$$

The government's budget is given by

---

<sup>2</sup>Kim and Ruge-Marcia (2009), Arouba et al. (2013) employed a linex function of AAC given by

$$AC_t(j) = \phi \left( \frac{\exp(-\psi(P(j)_t/P(j)_{t-1} - 1)) + \psi(P(j)_t/P(j)_{t-1} - 1) - 1}{\psi^2} \right), |$$

where  $\phi > 0$ . A restriction  $\psi > 0$  implies downward rigidity, while  $\psi < 0$  implies the upward rigidity.

$$P_t G_t + R_{t-1} B_{t-1} + M_{t-1} = T_t + B_t + M_t,$$

where the government's expenditure

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t. \quad (5)$$

is determined exogenously from

$$g_t/g = (g_{t-1}/g)^{\rho_g} e^{\epsilon_{g,t}} (> 0), \quad (6)$$

where  $\epsilon_{g,t}$  is the government shock which can be also interpreted as the aggregate demand shock.

The steady state is given by

$$\begin{aligned} \pi &= \pi^*, \quad r = \frac{\gamma}{\beta}, \quad R = r \pi^*, \\ C/A &= (1 - \nu)^{1/\tau}, \quad Y/A = g C/A = Y^*/A. \end{aligned}$$

The potential aggregate output (or target level of output in the monetary policy rule) in case of the no price adjustment cost is given by

$$Y_t^* = (1 - \nu)^{1/\tau} A_t g_t, \quad (7)$$

The market clearing conditions are given by

$$Y_t = C_t + G_t + AC_t, \quad N_t = H_t, \quad (8)$$

The optimality conditions of households (or the consumption Euler equation) and firms are respectively given by

$$1 = \beta E_t \left[ \left( \frac{C_{t+1}/A_{t+1}}{C_t/A_t} \right)^{-\tau} \frac{A_t}{A_{t+1}} \frac{R_t}{\pi_{t+1}} \right] \quad (9)$$

and

$$\begin{aligned} 1 &= \phi(\pi_t) (\pi_t - \pi) \left[ \left(1 - \frac{1}{2\nu}\right) \pi_t + \frac{\pi}{2\nu} \right] \\ &- \beta E_t \left[ \phi(\pi_{t+1}) \left( \frac{C_{t+1}/A_{t+1}}{C_t/A_t} \right)^{-\tau} \frac{Y_{t+1}/A_{t+1}}{Y_t/A_t} (\pi_{t+1} - \pi) \pi_{t+1} \right] \\ &+ \frac{1}{\nu} \left[ 1 - \left( \frac{C_t}{A_t} \right)^\tau \right]. \end{aligned} \quad (10)$$



Notice that because of our specification of AAC, the sizes of  $\phi(\pi_t)$  and  $\phi(\pi_{t+1})$  in the first and the second terms of the RHS depend on the values of current inflation,  $\pi_t$ , and expected inflation,  $E_t(\pi_{t+1})$ , respectively.

## 2.4 Solving the model

To solve our nonlinear models, we use above ten equations (1) through (10) which involve ten endogenous variables:  $Y_t, Y_t^*, C_t, G_t, AC_t, \pi_t, R_t, R_t^*, A_t, g_t$ , and three exogenous structural shocks:  $\varepsilon_{R,t}, \varepsilon_{z,t}, \varepsilon_{g,t}$ . There are 15 parameters including as  $\beta, \tau, \nu, \phi_2, \delta, \psi_1, \psi_2, \gamma, \pi^*, \rho_r, \rho_g, \rho_z, \sigma_r, \sigma_g, \sigma_z$ , all of which are to be estimated. The definitions of the parameters are summarized in Table 1.

The model is solved using a *time iteration method with linear interpolation* (TL) within the class of projection methods (or policy function iteration methods). Richter et al. (2014) reported TL provides the best balance between speed and accuracy. In addition, TL outperforms time iteration with Chebyshev polynomial, which is the popular method in the class of policy function iteration, when the ZLB constraint is embedded.

The policy functions of the model (or decision rules) can be written as

$$(Y_t/A_t, \pi_t, R_t) = \mathcal{P}(R_{t-1}, g_t, z_t, \varepsilon_{R,t}), \quad (11)$$

where  $Y_t/A_t, \pi_t$ , and  $R_t$  in the LHS are control variables in the decision rules and  $R_{t-1}, g_t, z_t$ , and  $\varepsilon_{R,t}$  in the RHS are state variables of the function. We specify seven grid points on each continuous state variables and five grid points on an exogenous shock, which implies 1715 ( $= 7 \times 7 \times 7 \times 5$ ) nodes. See the Technical Appendix for more detail.

As in the case of Aruoba et al. (2014) and Aruoba and Schorfheide (2015), we use the perfect foresight as rational expectations when we calculate expectations in the TL. This approach has an advantage in obtaining stable solution with faster speed and fewer iterations, when we sample from wide range of many parameters in a MH

algorithm.

Using this solution method, let us first investigate the effect of introducing AAC on inflation dynamics under ZLB.

To be specific, we consider the effect of the parameter  $\delta$  on the policy function of inflation. Figure 2 shows predicted policy functions of three cases of price rigidities, namely, symmetric price adjustment ( $\delta = 0$ ), upward price rigidity ( $\delta = -0.5$ ) and downward price rigidity ( $\delta = 0.5$ ). All other values for remaining parameters are taken from An and Schorfheide (2007).

[ Insert Figure 2 ]

Figure 2 implies two interesting features of our model. First, the presence of ZLB causes kinked policy function of inflation with steeper slope within the negative TFP range. Second, if there is upward rigidity in the price adjustment, deflationary pressure from negative TFP seems to be strengthened, while the presence of downward rigidity mitigate the such a deflationary pressure.

### 3 Estimation strategy

We estimate the following four variants of the model depending on the presence and absence of ZLB and AAC.

- Model 1: no ZLB and no AAC
- Model 2: ZLB but no AAC
- Model 3: AAC but no ZLB
- Model 4: ZLB and AAC

In Model 1 and Model 3 where the ZLB is absent, we incorporate the standard monetary policy rule,  $R_t = R_t^{*1-\rho_R} R_{t-1}^{\rho_R} e^{\epsilon_{R,t}}$  instead of using (1). In Model 1 and Model 2 where the AAC is absent, we impose  $\delta = 0$ .

Following Gust et al. (2012), we estimate the four nonlinear model using Bayesian methods with particle filter. Particle filter Metropolis-Hastings algorithm (PFMH), or Particle Markov Chain Monte Carlo (PMCMC), was established in Andrieu et al. (2010). The algorithm of Herbst and Schorfheide (2016, ch 8 and ch 9) which we employ is described in the Technical Appendix. After solving for the decision rule,  $\mathcal{P}(R_{t-1}, g_t, z_t, \varepsilon_{R,t})$ , our economic environment can be represented as a nonlinear state space model which consists of (12) and (13) below.

To keep acceptance rate between 25 to 35% during PFMH, we use a random-block MH algorithm proposed by Chib and Ramamurthy (2010) which is also explained in Herbst and Schorfheide (2016, p83).

### **State equations**

The policy function (11) combined with (1), (5) and (6), can be rewritten as

$$s_t = \Phi(s_{t-1}, \varepsilon_t, \theta), \quad (12)$$

where  $s_t$  is endogenous variables:  $s_t = (Y_t/A_t, \pi_t, R_t, g_t, z_t)$ , and  $\varepsilon_t = (\varepsilon_{R,t}, \varepsilon_{g,t}, \varepsilon_{z,t})$ .  $\theta$  is the parameter.

### **Measurement equations**

A measurement equation represents connection between endogenous variables and observed variables as

$$y_t = \psi(s_t, \theta) + \sigma_u u_t, \text{ for } u_t \sim \text{i.i.d. } N(0, I)$$

where  $y_t$  is observed variables,  $\sigma_u$  and  $u_t$  are standard deviation and disturbance term of the measurement error. In our case, we use

$$\begin{bmatrix} YGR_t(\%) \\ INFL_t(\%) \\ INT_t(\%) \end{bmatrix} = \begin{bmatrix} 100 \times (\ln(Y_t/A_t) - \ln(Y_{t-1}/A_{t-1}) + \ln z_t + \ln \gamma) \\ 400 \times \ln \pi_t \\ 400 \times \ln R_t \end{bmatrix} + \begin{bmatrix} \sigma_{\Delta y} u_{y,t} \\ \sigma_{\pi} u_{\pi,t} \\ \sigma_r u_{r,t} \end{bmatrix}. \quad (13)$$

where three observed variables on the LHS are the ones described in Section 1.

## 4 Empirical results

### 4.1 Prior and posterior

The left half of Table 1 shows the prior distributions of the structural parameters which are assumed to be mutually independent. In estimation, we calculate likelihood approximation using particle filter with 10,000 particles, and choose the mode of posterior density out of parameters sampled from prior distributions. By setting the mode as initial values of MH, we obtain 30,000 draws of MCMC samplings after discarding the first 10,000 burn-in draws.

[ Insert Table 1 ]

The right half of Table 1 contains log marginal likelihoods and posterior model probabilities of the four models as well as posterior means of 15 parameters. The result clearly shows that Models 2 and 4 which incorporate ZLB performs much better than Models 1 and 3 without the ZLB. In addition, since the posterior model probability of Model 4 is as large as 99.2 percent, it seems to be fair to say that the model with both AAC and ZLB seems to be the most appropriate specification among the four models.

For the degree of asymmetric adjustment parameter  $\delta$ , the sign of posterior means differ between Model 3 and 4. However, given that Model 4 with ZLB is a better specification, the adjustment costs in reducing prices are likely to be about 60.8 percent ( $\exp(0.45)=1.608$ ) higher than those in increasing prices.

This result combined with the calibration exercise in the previous section suggest that the presence of the downward price rigidity is likely to play a role in preventing further deflation by mitigating the deflationary pressure from the reduction of productivity.

## 4.2 Estimated policy functions

Using posterior means of the parameters in four models, calculated policy functions of output, inflation, and interest rate are respectively shown in Figures 3 to 5. For example, Figure 3 shows the reaction of output in response to the TFP and monetary policy shocks:  $z_t$  and  $\epsilon_{R,t}$ . Panels (a), (b), (c) and (d) of Figure 3 correspond to policy functions of output predicted by Models 1 to 4, respectively.

Let us first focus on the effect of TFP in some detail.

Figure 6 shows the estimated policy function of output, inflation, and interest rate in response to change in  $z_t$  when the three state variables are fixed at some values. Note that since the policy functions of the model with only AAC (Model 3) did not differ much from the benchmark model (Model 1), we only compare the policy functions among three models excluding Model 3.

It should be noted that shapes of the policy functions differ not only because the models are different but also estimated parameter values differ. Most importantly, responses of output and inflation in Models 2 and 4 are larger when  $z_t$  is below one than when  $z_t$  is above one. This kinked policy functions is caused by the interest rate hitting at the ZLB in the two models.

On the other hand, the benchmark model without ZLB and AAC predict that inflation respond more to positive TFP and that output respond less to negative TFP.

Figure 7 shows estimated policy function of output, inflation, and interest rate in response to monetary policy shock  $\epsilon_{R,t}$ . Panel (a) of the figure shows the case of low level of TFP, when the interest rate is around the ZLB. Panel (b), shows the case of

high level of TFP, when the interest rate is far from the ZLB. In the low TFP case, response of inflation and output to expansionary monetary policy shock is modest in the model with both ZLB (Models 2 and 4) compared to Benchmark model. In contrast, output response are almost the same between the model with both ZLB and AAC (Model 4) and Benchmark model.

[ Insert Figures 3 to 7 ]

### **4.3 Estimated duration of zero interest rate policy**

Finally, we calculate frequency distribution of the duration of lower bound spell, i.e., the number of consecutive periods hitting the ZLB, of the two models imposing the ZLB constraint by generating artificial data for 1,000,000 periods from the estimated policy function, along the line of Gust et al. (2012) who conducted similar analysis in their Figure 6. Panel (a) of Figure 8 shows CDF of duration of spell, and panel (b) represents right tail of the spell after 15 quarters. Red and blue line shows the prediction of Models 2 and 4, respectively. As can be seen from the figure, the model with only ZLB is skewed toward longer duration in the distribution than the model which also considers AAC. In fact, we obtain  $\text{Prob}(\text{Duration} > 12) = 0.089$  and  $\text{Prob}(\text{Duration} < 4) = 0.45$  in Model 2. In contrast, corresponding values for Model 4 are 0.055 and 0.549, respectively. This suggests that positive AAC reduces the predicted duration for same shocks and that it helps in stabilizing the economy with higher probability. Averages of the spells predicted by Models 2 and 4 are 5.5 and 4.2 quarters, respectively, and longer than the prediction in the U.S. case obtained by Gust et al. (2012) which is over three. Although Japanese long stagnation has brought over 28 consecutive periods of the zero interest rate policy between 2009:Q1 and 2016:Q1, the probability of this situation is no more than 0.03 percent based on the prediction of Model 4. Therefore, it is still challenging to fully simulate the recent Japanese experience by a simple nonlinear DSGE model considered here.

[ Insert Figure 8 ]

## 5 Conclusion

We incorporated asymmetric adjustment costs (AAC) in Rotemberg price setting mechanism and zero lower bound (ZLB) in monetary policy rule and solve nonlinear New Keynesian DSGE models using projection method. Using the Bayesian method combined with a particle filter, we estimate the model and show that the estimated model with both ZLB and AAC outperforms the benchmark model in term of explaining the Japanese data from 1981:Q3 to 2015:Q1

The adjustment cost in reducing prices is estimated to be 24 to 32 percent higher than raising prices. The presence of this downward price rigidity is likely to play a role in preventing further deflation by mitigating the deflationary pressure from the reduction of productivity.

## References

- [1] An S., and F. Schorfheide (2007) "Bayesian Analysis of DSGE Models," *Econometric Reviews*, 26(2-4), 113-172.
- [2] Andrieu, C., A. Doucet, and R. Holenstein (2010) "Particle Markov Chain Monte Carlo Methods," *Journal of the Royal Statistical Society Series B*, 72(3), 269-342.
- [3] Aruoba, S.B., P. Cuba-Borda, and F. Schorfheide (2014) "Macroeconomic Dynamics Near the ZLB: A Tale of Two Countries," NBER Working Paper, 19248.
- [4] Aruoba, S.B., L. Bocola, and F. Schorfheide (2013) "Assessing DSGE Model Non-linearities," NBER Working Paper, 19693.
- [5] Aruoba, S.B., and F. Schorfheide (2015) "Inflation during and after the Zero Lower Bound," mimeo.

- [6] Chib, S., and S. Ramamurthy (2010) "Tailored Randomized Block MCMC Methods with Application to DSGE Models," *Journal of Econometrics*, 155(1), 19-38.
- [7] Fernandez-Villaverde, J., J. G. Gordon, P. Guerron-Quintana and J.F. Rubio-Ramirez (2015) "Nonlinear Adventures at the Zero Lower Bound," *Journal of Economic and Dynamic Control*, 57, 182-204.
- [8] Fernandez-Villaverde, J., and J.F. Rubio-Ramirez (2005) "Estimating Dynamic Equilibrium Economies: Linear versus Nonlinear Likelihood," *Journal of Applied Econometrics*, 20, 891-910.
- [9] Gust, C., D. López-Salido, and M.E. Smith (2012) "The Empirical Implications of the Interest-Rate Lower Bound," mimeo, Federal Reserve Board.
- [10] Herbst, E.P., and F. Schorfheide (2016) *Bayesian Estimation of DSGE models*, Princeton University Press.
- [11] Keen, B.D., Richter, A. W., and N.A. Throckmorton, (2015) "Forward Guidance and the State of Economy," mimeo.
- [12] Kim, J. and F. Ruge-Murcia (2009) "How Much Inflation is Necessary to Grease the Wheels," *Journal of Monetary Economics*, 56, 365-377.
- [13] Koop, G., M.H. Pesaran, and S. M. Potter (1996) "Impulse Response Analysis in Nonlinear Multivariate Models," *Journal of Econometrics*, 74, 119–147.
- [14] Richter, A., and N. Throckmorton (2015) "The Zero Lower Bound: Frequency, Duration, and Numerical Convergence," *The B.E. J. Macroecon.* 15(1), 157–182.
- [15] Richter, A., N. Throckmorton, and T. B. Walker (2014) "Accuracy, Speed and Robustness of Policy Function Iteration," *Computational Economics*, 44, 445–476.
- [16] Rotemberg, J.J. (1982) "Sticky Prices in the United States," *Journal of Political Economy*, 90, 1187–1211.



Table 1: Prior and Posterior Estimations

parameters	Prior Setting			Posterior			
	distribution	param 1	param 2	Model 1	Model 2	Model 3	Model 4
ZLB				No	Yes	No	Yes
AAC				No	No	Yes	Yes
$\beta$	beta	0.99	0.01	0.994	0.995	0.995	0.995
$\tau$	gamma	2.0	0.1	1.755	1.566	1.819	1.617
$\nu$	gamma	0.3	0.1	0.423	0.562	0.418	0.494
$\phi_1$	gamma	20.0	10.0	16.85	16.74	16.91	16.97
$\delta$	normal	0.00	0.25	-	-	<b>-0.058</b>	<b>0.475</b>
$\psi_1$	normal	1.8	0.5	1.60	1.67	1.61	1.66
$\psi_2$	normal	0.5	0.5	0.61	0.63	0.60	0.58
$\gamma$	normal	1.01	0.01	1.0048	1.0005	1.0056	1.0003
$\pi$	normal	1.0	0.01	1.0017	1.0034	1.0024	1.0017
$\rho_r$	beta	0.75	0.1	0.657	0.635	0.667	0.689
$\rho_g$	beta	0.75	0.1	0.639	0.837	0.644	0.838
$\rho_z$	beta	0.75	0.1	0.851	0.768	0.857	0.791
$\sigma_r$	inv. gamma	0.5	5	1.538	0.921	1.478	0.872
$\sigma_g$	inv. gamma	0.5	5	0.731	1.765	0.681	1.744
$\sigma_z$	inv. gamma	0.5	5	1.565	0.768	1.506	0.781
$\sigma_{\Delta y}$	inv. gamma	0.5	5	0.552	0.576	0.573	0.571
$\sigma_\pi$	inv. gamma	0.9	5	0.882	0.861	0.874	0.851
$\sigma_R$	inv. gamma	1.25	5	1.241	1.242	1.259	1.241
Accept.Rate.	-	-	-	0.142	0.328	0.167	0.400
Log Mgrl.Lik	-	-	-	-966.75	<b>-789.14</b>	-957.11	<b>-781.38</b>
Posterior Prob	-	-	-	0.000	<b>0.0004</b>	0.000	<b>0.9996</b>

Notes: In the first column, ZLB and AAC stand for zero lower bound and asymmetric adjustment cost, respectively. Accept. Rate, Log Mgrl Lik and Posterior Prob denotes acceptance rate of MH algorithm, marginal likelihood and posterior model probabilities, respectively. In the third and fourth column, parameter 1 and 2 represent mean and standard deviation in Beta and Normal distributions, respectively. In the fifth through eighth column, the values of parameters represent their posterior means.

Figure 1: Output, Inflation and Interest Rate

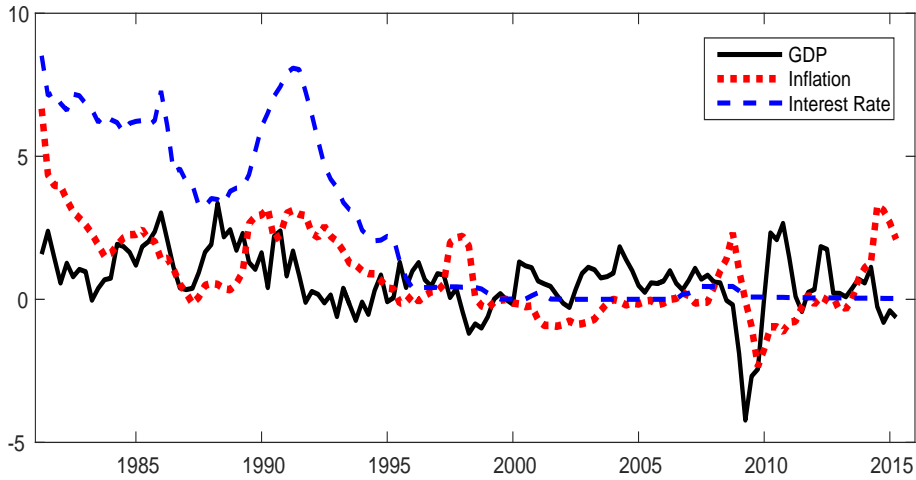
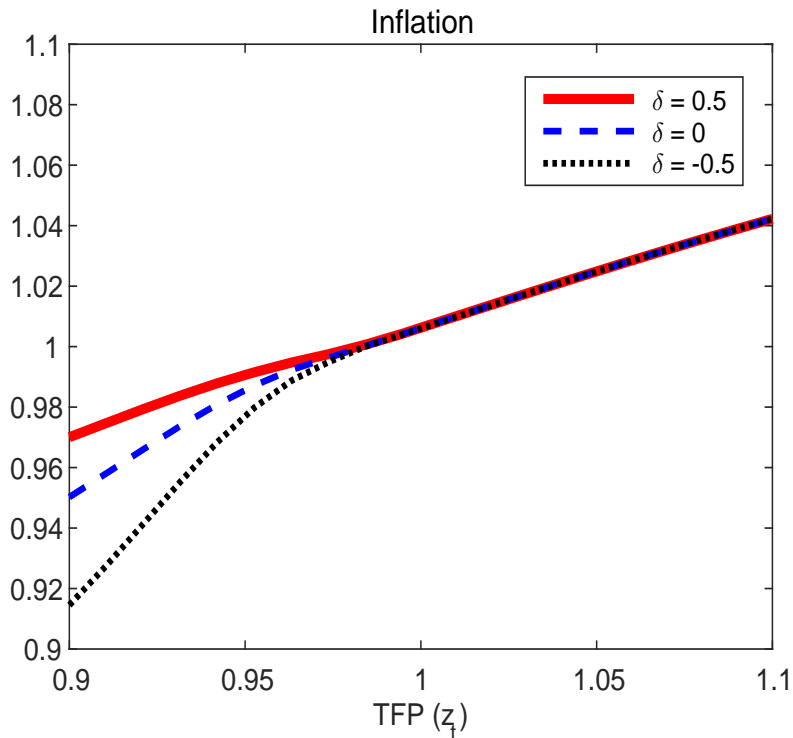


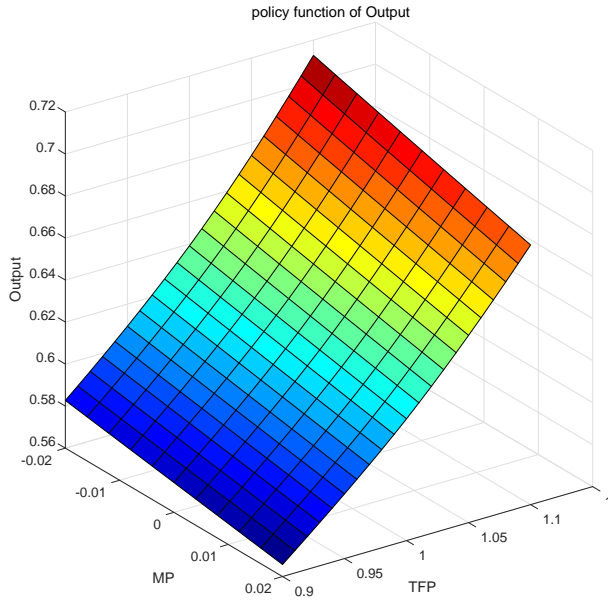
Figure 2: Policy Functions with Asymmetric Price Adjustment



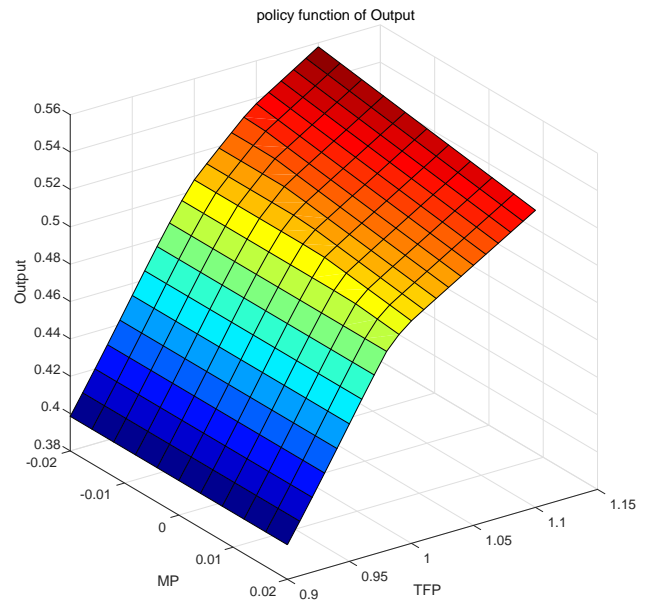
Notes: The response of inflation to TFP shock when adjustment cost parameters are  $\delta = -0.5, 0, 0.5$ . Other parameter values are same as the ones in An and Schorfheide (2007). The policy function is calculated from the time iteration method with linear interpolation described in Section 2.

Figure 3: Estimated Policy Function of Output

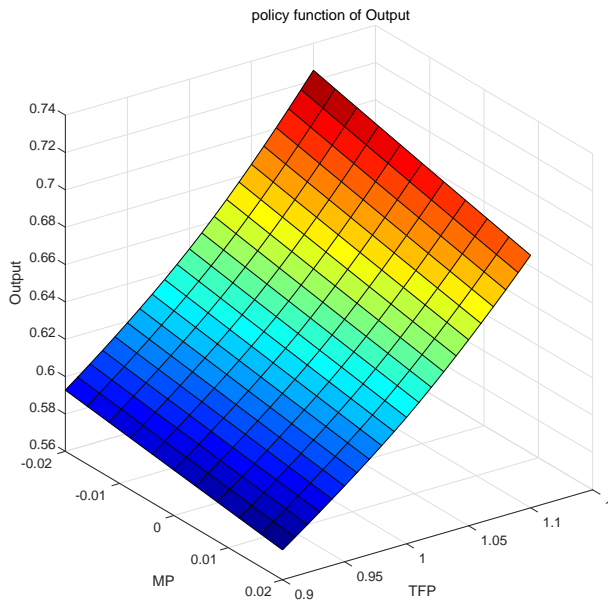
(a) Model 1 (Benchmark)



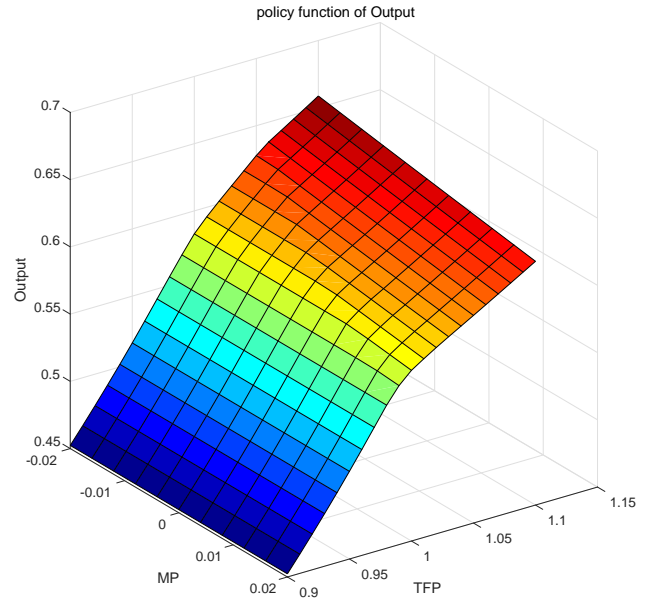
(b) Model 2 (ZLB only)



(c) Model 3 (AAC only)



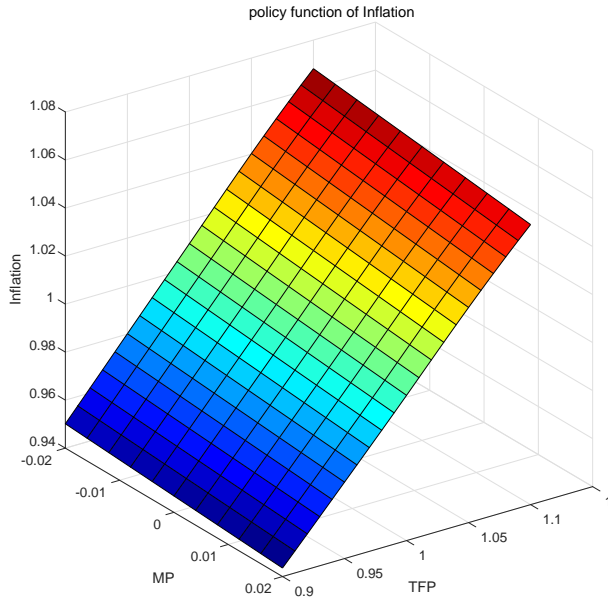
(d) Model 4 (ZLB/AAC)



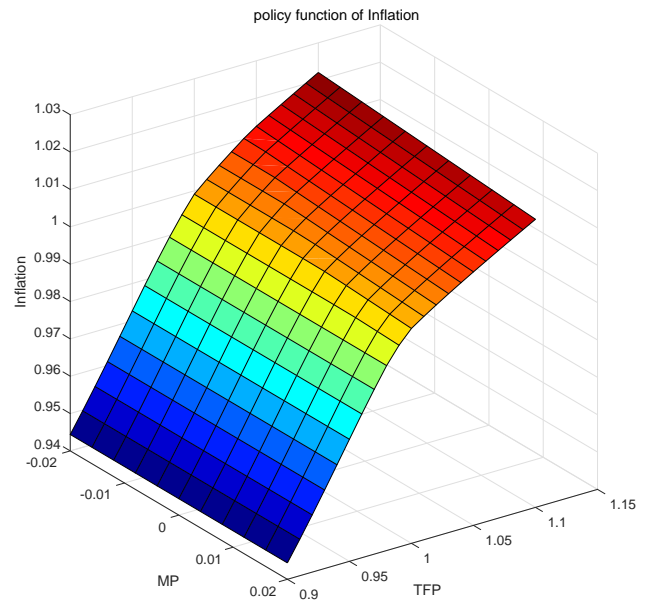
Notes: Calculated from posterior means of parameters, using time iteration method with linear interpolation described in Section 2. And the reaction of a control variable are represented in terms of TFP and monetary policy shock by setting other state variables as constant. ZLB and AAC stand for zero lower bound and asymmetric adjustment cost, respectively.

Figure 4: Estimated Policy Function of Inflation

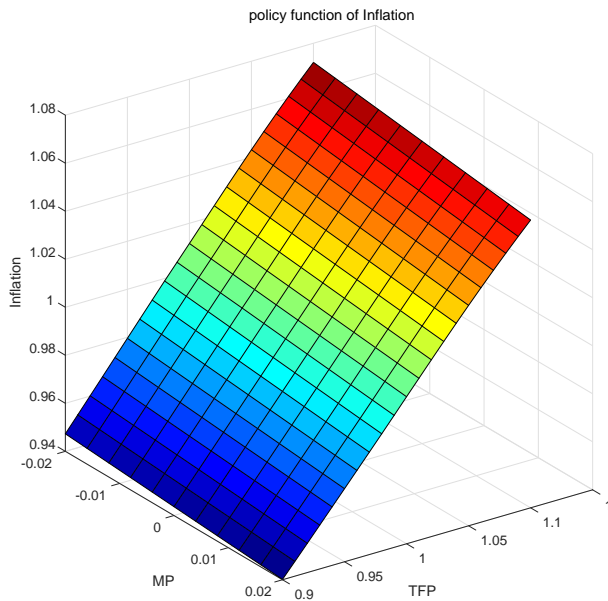
(a) Model 1 (Benchmark)



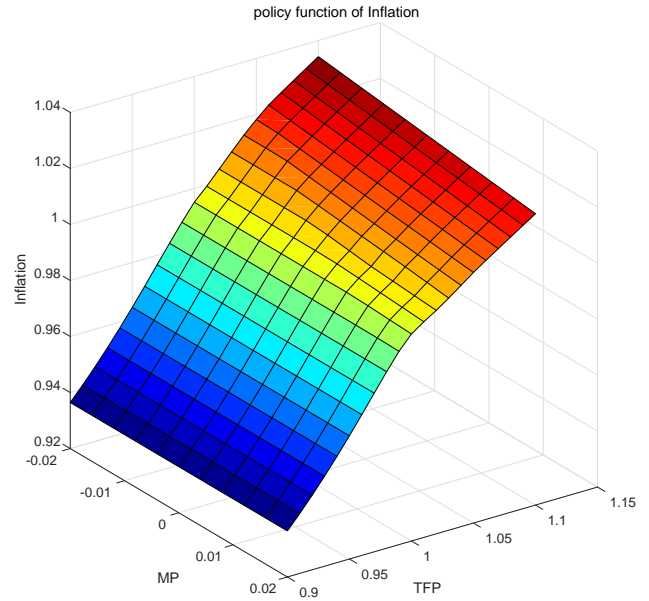
(b) Model 2 (ZLB only)



(c) Model 3 (AAC only)



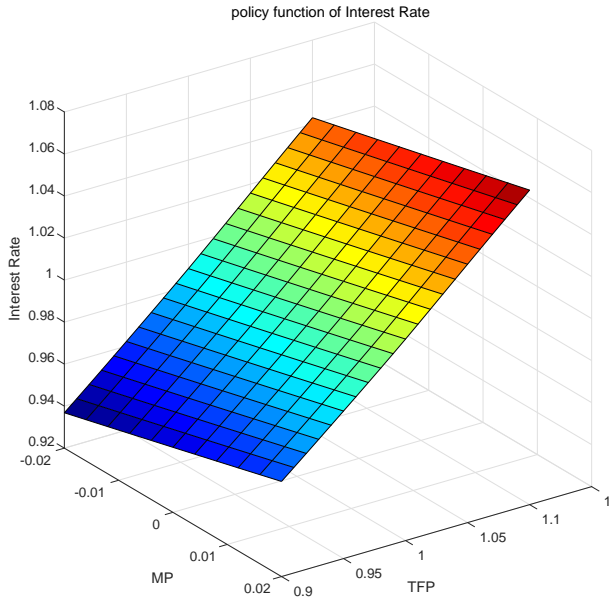
(d) Model 4 (ZLB/AAC)



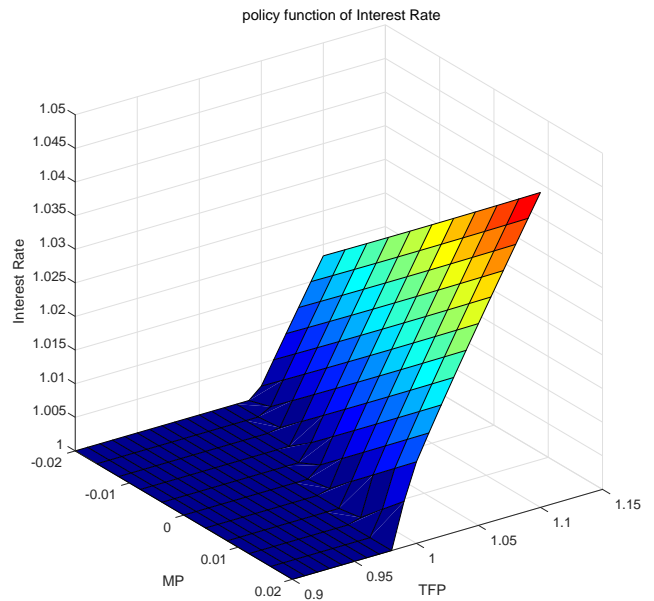
Notes: Calculated from posterior means of parameters, using time iteration method with linear interpolation described in Section 2. And the reaction of a control variable are represented in terms of TFP and monetary policy shock by setting other state variables as constant. ZLB and AAC stand for zero lower bound and asymmetric adjustment cost, respectively.

Figure 5: Estimated Policy Function of Interest Rate

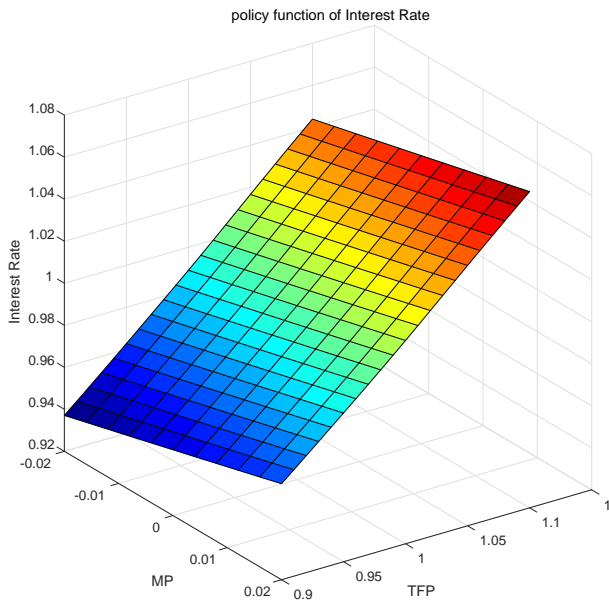
(a) Model 1 (Benchmark)



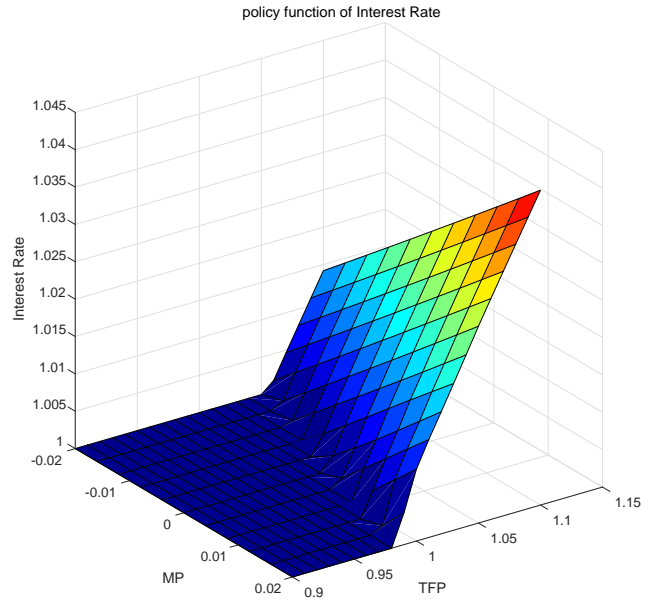
(b) Model 2 (ZLB only)



(c) Model 3 (AAC only)

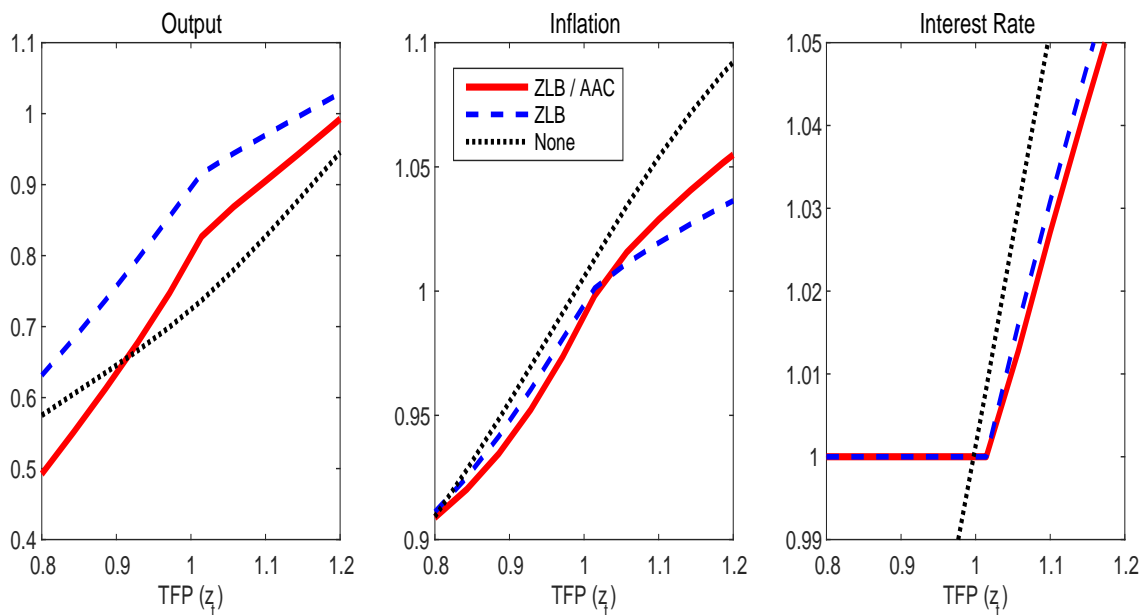


(d) Model 4 (ZLB/AAC)



Notes: Calculated from posterior means of parameters, using time iteration method with linear interpolation described in Section 2. And the reaction of a control variable are represented in terms of TFP and monetary policy shock by setting other state variables as constant. ZLB and AAC stand for zero lower bound and asymmetric adjustment cost, respectively.

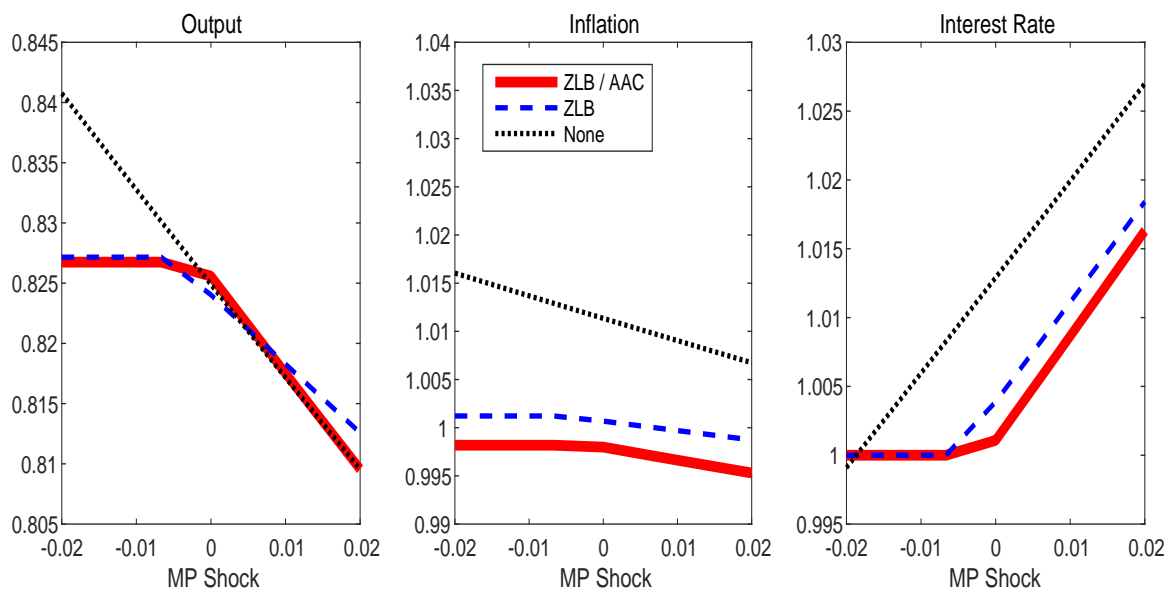
Figure 6: Estimated Policy Functions



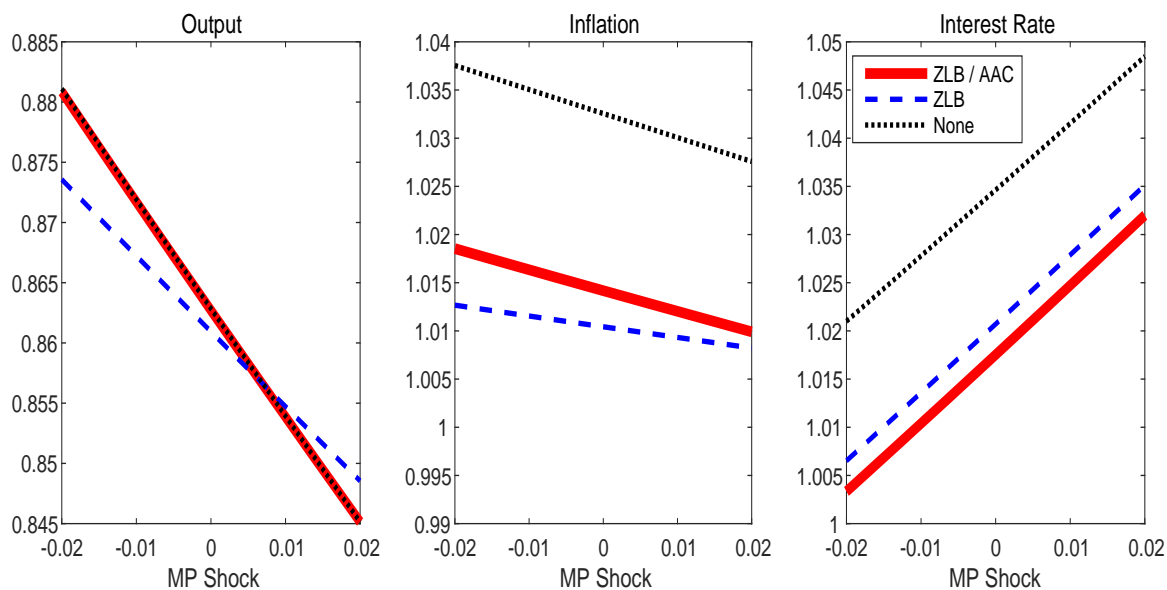
Notes: Calculated from posterior means of parameters. ZLB and AAC stand for zero lower bound and asymmetric adjustment cost, respectively. We set as  $g_t = 1.2$ ,  $R_{t-1} = 1.006$ , and  $\varepsilon_t^R = 0$ .

Figure 7: Estimated Policy Functions

(a) Near ZLB (Case of low TFP)

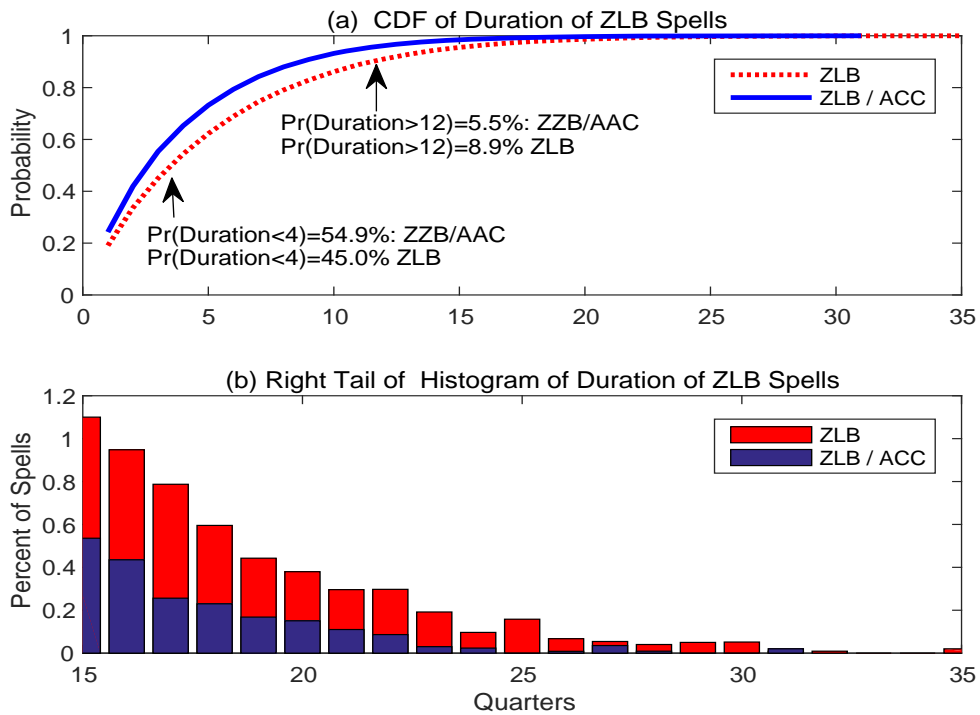


(b) Far away from ZLB (Case of high TFP)



Notes: Calculated from posterior means of parameters. ZLB and AAC stand for zero lower bound and asymmetric adjustment cost, respectively. For panel (a) we set  $z_t = 1.01$ ,  $g_t = 1.2$ , and  $R_{t-1} = 1.006$ , whereas we set  $z_t = 1.05$ ,  $g_t = 1.2$ , and  $R_{t-1} = 1.006$  for Panel (b).

Figure 8: Estimated Duration of a ZLB Spell



Notes: Generating artificial data for 1,000,000 periods from the estimated policy function, we calculate frequency distribution of the duration of zero interest rate policy. Panel (a) shows CDF of spell derived from number of periods that belong to the corresponding duration of ZLB, and panel (b) represents right tail of histogram of the spell after 15 quarters.