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Time-varying Fiscal Multipliers Identified with Sign and Zero Restrictions: A Bayesian Approach to TVP-VAR-SV model

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Time-varying Fiscal Multipliers Identified with Sign and Zero Restrictions: A Bayesian Approach to TVP-VAR-SV model *

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Abstract

This study might be the first attempt applied zero and sign restrictions method proposed by Arias, Rubio-Ramirez and Waggoner (2014) to a time-varying-parameter VAR model, and identifies the structural model by imposing long-run zero restrictions and short-run sign restrictions for a fiscal policy shock to be orthogonal to monetary policy and business cycle shocks in the US economy between 1953:Q1-2013:Q4. Our estimation reports that time-varying fiscal multipliers could be negative during the Great Moderation, since private consumption was crowded out, and that they, however, changed positively after the Great Recession by increasing the consumption. And we also observed that the propagation effect of fiscal policy has had non-trivial time lag so that it has taken at least two years to work effectively after 1970's.

Keywords: Bayesian estimation, time-varying-parameter Structual VAR, Sign and Zero Restrictions,

JEL Classifications: C32, E32, E62

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1 Introduction

Since the Great Recession triggered by the Lehman crisis in 2008, the effect of expansionary fiscal stimulus has widely paid attention from many economist and policy analysts. Among them, it has often been argued that how much does fiscal policy impact on economic activity and contribute to real GDP growth during crises. And an elemental question that has happened is whether the impact of fiscal policy would have the same size over time even though an economic situation changes. In fact, to answer the question many empirical studies measuring fiscal multipliers using different econometric methods have appeared after the recession. Some of them estimate the shift of multipliers depending on business cycles. For example, Tagkalakis (2008), Auerbach and Gorodnichenko (2012), Bachmann and Sims (2012), Candelon and Lieb (2013) and Caggiano et al. (2015) report the multiplier is bigger in recessions than in booms. Another big factor influencing the multiplier is thought to be the volume of public debt level against GDP level, i.e., the multiplier is likely to become larger in the economies with low level of debt-to-GDP ratio, according to Favero et al. (2011), Corsetti et al. (2012) and Ilzetzki et al. (2013). This study might be located as an offspring of a series of these studies.

As many empirical studies suggested, instabilities in macroeconomic activities has been often observed and they are deeply associated with fluctuations of volatilities. It is plausible that a structural model is estimated by relaxing its parameters from time variation. Primiceri (2005) proposed a time-varying-parameter vector-autoregressive (TVP-VAR) model both with respect to the coefficients and the covariance matrix of innovations, in order to estimate the time variation in the effects of monetary policy on the rest of the economy. And his attempt has been widely and successfully accepted as one of the fundamental approaches to estimate temporal alteration of macroeconomies, in particular, facing the dramatic economic contraction related to the financial crisis. Recently, from the same framework by adding the sign and zero restrictions for short run, Baumeister, and Benati (2013) have analyzed the time variant effects of monetary policy including unconventional monetary policy and zero interest rate policy. However, there are a few studies of fiscal policy using TVP-VAR models.

On the other hand, an identification of structural VARs is another big issue for measuring the effects of economic policies. Different techniques of the identification such as zero restrictions and sign restriction (Uhlig, 2005) imposing impulse response functions (IRF) have been developed in the last decade. Peersmen (2009) discussed sign restrictions are superior to zero restrictions proposed by Blanchard and Quah (1985), while Kilian and Murphy (2012) mentioned just sign restrictions are not enough to identify underlined shocks (Kilian and Murphy (2014) used QR decomposition proposed by Rubio-Ramirez, Waggoner, Zha (2010) for sign restriction.)[1]. However, most of the existing studies have not discussed about the usage of both restrictions, except Baumeister, and Benati (2013) who adopted only contemporaneous restrictions but not both long-run and short-run restrictions. Arias, Rubio-Ramirez, Waggoner (2014) proposed an algorithm imposing both zero and sign restrictions by using QR decomposition. I think that both restrictions are needed to identify different structural shocks such as demand shocks imposed from both short and long runs, but one of them is not enough to identify them.

To our best knowledge, this study might be the first attempt applied a method imposing both zero and sign restrictions proposed by Arias, Rubio-Ramirez, Waggoner (2014) to a TVP-VAR model. The structural VAR is identified by imposing both of long-run zero restrictions and short-run sign restrictions for a fiscal policy shock orthogonalized with monetary policy and business cycle shocks in the US economy between 1953:Q1-2013:Q4. In our TVP-VAR approach, there are six endogenous variables, i.e., (1) government spending, (2) output (3) private consumption (4) public debt measured by the ratio against real GDP (5) price level (6) nominal interest rate. And, using the estimation results, we calculate time-varying fiscal multipliers as well as impulse responses to the three shocks. The main findings are as follows. Time-varying fiscal multipliers could be negative during the Great Moderation, since private consumption was crowded out. However, they changed positively after the Great Recession by increasing the consumption. And we also observed that the propagation effect of fiscal policy has had non-trivial time lag so that it has taken at least two years to work effectively after 1970's.

The remaining of this paper is organized as follows. Section 2 describes about the TVP-VARs as well as methods of the identification with sign and zero restrictions. And this section explains an estimation strategy such as a Bayesian method via MCMC simulation. Estimated results including time-varying impulse responses of structural shocks and time-varying fiscal multipliers are reported in Section 3. Section 4 concludes. Algorithms for MCMC simulation estimating TVP-VARs are described in two appendix sections.

[1]Also see a critical review of the sign restriction approach by Fry and Pagan (2011).

2 Empirical Methodology

In this section, we describe empirical methodology measuring time variations of fiscal multipliers. In the first subsection, a TVP VAR model incorporated with stochastic volatilities (SV) in its disturbance terms is introduced as our backbone model. The distinguished advantage of the model is to be designed to make coefficients and the covariance matrix of innovations time-vary in terms of all aspects from the viewpoint of 'agnostic'. The second subsection describes how to identify a fiscal policy shock using the zero restrictions and the sign sign restrictions for short-run and long-run. In the next subsection, we explain a Bayesian estimation method using Markov chain Monte Calro (MCMC) simulation. The final two subsections deals with calculating fiscal multipliers and data used for estimation.

2.1 TVP-VAR-SV

Consider the p-th lag length structual vector autoregression (SVAR(p)) model defined as

$$A_{0,t}Y_t = A_{1,t}Y_{t\text{-}1} + \cdots + A_{p,t}Y_{t-p} + \Sigma_t\varepsilon_t,\ \varepsilon_t \sim N(0, I), \tag{1}$$

where Y_t is a $k \times 1$ vector of observed variables, structural parameters $A_{i,t}$, $i = 1, ..., p$, are $k \times k$ matrices of time varying coefficients, and a contemporaneous matrix $A_{0,t}$ is invertible and decomposed into a orthogonal matrix Q_t , i.e., $Q_tQ_t' = I$, and a lower triangular matrix $A_{tr,t}$ such that $A_{0,t} = Q_t A_{tr,t}$, where

$$A_{tr,t} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ a_{21,t} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ a_{k1,t} & \cdots & a_{kk-1,t} & 1 \end{bmatrix}.$$

The disturbance ε_t is a $k \times 1$ vector of structural shocks and a time-varying covariance matrix Σ_t is a diagonal matrix that contains the stochastic volatilities which reflect the changes of the independent structural shocks $\sigma_{i,t}$ such as

$$\Sigma_t = \begin{bmatrix} \sigma_{1,t} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{k,t} \end{bmatrix}.$$

And the p-th lag length reduced VAR (p) model corresponding to above SVAR model is given by

$$Y_t = B_{1,t}Y_{t\text{-}1} + \cdots + B_{p,t}Y_{t-p} + u_t,\ u_t \sim N(0,\, \Omega_t\,), \tag{2}$$

where $B_{i,t}$ is a time varying reduced-form parameters given by $B_{i,t} = A_{0,t}^{-1}A_{i,t}$, and u_t is a one-period ahead forecasting error: $u_t = A_{0,t}^{-1}\Sigma_t\varepsilon_t$, because $A_{0,t}\Omega A_{0,t}' = \Sigma_t\Sigma_t'$. And also we can rewrite the one-period ahead forecasting errors as $u_t = A_{tr,t}^{-1}\, Q_t\, \Sigma_t\varepsilon_t$, using $A_{0,t}^{-1} = A_{tr,t}^{-1}\, Q_t'$. Notice that Q_t is a random matrix so that we can select its value to make structural shocks identified to satisfy zero and sign restrictions, as explained in the next subsection.

Letting β_t be a stacked $k^2p \times 1$ vector of the elements in the rows of the $k \times k$ matrices of the $B_{1,t},\cdots,B_{p,t}$, and a_t be the vector of non-zero and non-one elements of the lower triangular matrix $A_{tr,t}$. h_t is the logarithm of the diagonal elements of time varying volatilities matrix, $\ln\,\sigma_{j,t}^2$. The dynamics of the time varying parameters of the reduced form are following random walk process as below.

$$\beta_{t+1} = \beta_t + u_{\beta,t}, \tag{3}$$

$$a_{t+1} = a_t + u_{a,t}, \tag{4}$$

$$h_{t+1} = h_t + u_{h,t}, \tag{5}$$

where $\beta_t = (\beta_{11,t}, ..., \beta_{kk,t})$, $a_t = (a_{21,t}, ..., a_{kk-1,t})$ and $h_t = (h_{1,t}, ..., h_{k,t})$ with $h_{j,t} = \ln\sigma_{j,t}^2$ for $j = 1, ..., k$. And $u_{b,t}, u_{a,t}$, and $u_{h,t}$, are assumed to be normally distributed with a zero mean and diagonal covariance matrices, $\varSigma_\beta$, $\varSigma_a$, and $\varSigma_h$. The structural shocks are also assumed to independent with the time-varying parameters such as

$$\begin{bmatrix} \varepsilon_t \\ u_{\beta,t} \\ u_{a,t} \\ u_{h,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \Sigma_\beta^2 & 0 & 0 \\ 0 & 0 & \Sigma_a^2 & 0 \\ 0 & 0 & 0 & \Sigma_h^2 \end{bmatrix} \right). \tag{6}$$

2.2 Identification with Zero and Sign Restrictions

In our TVP-VAR approach, there are six endogenous variables, i.e., (1) government spending, (2) output (3) private consumption (4) public debt measured by the ratio against real GDP (5) price level (6) nominal interest rate. Since we assume that three 'demand' shocks such as fiscal policy shock, monetary policy shock and business cycle shock are independent to one another and impact only for short periods but not for long periods, the three shocks are imposed the orthogonality condition and the zero restrictions for the long run. To separate the three shocks from one another, we define them as follows. A positive government spending shock associates with contemporaneous positive impact on public debt and does not influence on output and private consumptions as well as the spending itself in the long run. A business cycles shock is just related to the private sector so that the positive shock increases the output and the private consumption only for the short periods. A discretionary monetary policy shock conducted by reducing the interest rate for the short run could increase the output and the consumption as well as the price level. Table 1 summarizes the zero and sign restrictions identifying the three structural shocks described above. In Table 1, signs '+' and '-' denote positive and negative sign restrictions, respectively. '0' is the zero restriction and '?' denotes unconstrained.

Table 1: Identification Restrictions of Fiscal Shock

Structural Shocks		Endogenous Variables					
		Gov	Output	Cons	Debt	Price Level	Interest Rate
Fiscal Policy Shock	Short Run	+	?	?	+	?	?
	Long Run	0	0	0	?	?	?
Business Cycles Shock	Short Run	?	+	+	?	?	?
	Long Run	?	0	0	?	?	?
Monetary Policy Shock	Short Run	?	+	+	?	+	-
	Long Run	?	0	0	?	0	0

Note: "+" and "-" denote postive and negative sign restrictions, respectively. "0" is zero restriction. And "?" denotes unconstrained.

As explained below, using the QR decomposition based on the algorithm proposed by Arias et al. (2014), this study identifies a fiscal policy shock to be orthogonal to monetary policy and business cycle shocks by imposing long-run zero restriction and short-run sign restrictions .

Impulse response functions (IRFs)

Firstly, we consider IRFs in a standard VAR with constant structural parameters:A_0, A_+, following Arias et al. (2014). Let $L_h(A_0, A_+)$ denote the IRF of the i-th variable to j-th structural shock at finite horizon h given by a $n \times n$ matrix as below.

$$\underbrace{IR_h(A_0, A_+}_{n\times n} = (A_0^{-1}J'F^hJ)'$$

where $A'_+ = [A'_1, \cdots, A'_p]$,

$$\underbrace{F}_{pn\times pn} = \begin{bmatrix} A_1 A_0^{-1} & I_n & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_{p-1}A_0^{-1} & 0 & \cdots & I_n \\ A_p A_0^{-1} & 0 & \cdots & 0 \end{bmatrix} \text{ and } \underbrace{J}_{pn\times n} = \begin{bmatrix} I_n \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

where I_n is a $n \times n$ identity matrix. Next, we apply them to the IRFs in the TVP-VARs. The IRFs: $L_h(A_0, A_+)$, can be rewritten as

$$\underbrace{IR_h(A_{t,0}, A_{t,+})}_{n\times n} = \left(A_{t,0}^{-1} J' \left(\prod_{i=t}^{t+h} F_i \right) J \right)^T, \tag{7}$$

where $A'_{t,+} = [A'_{t,1}, \cdots, A'_{t,p}]$,

$$\underbrace{F_t}_{pn\times pn} = \begin{bmatrix} A_{t,1} A_{t,0}^{-1} & I_n & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_{t,p-1}A_{t,0}^{-1} & 0 & \cdots & I_n \\ A_{t,p} A_{t,0}^{-1} & 0 & \cdots & 0 \end{bmatrix}.$$

Notice that the product of time-varying structural parameters: $A_{t,k}A_{t,0}^{-1}$ is equivalent to time-varying reduced-form parameters $B_{t,k}$ for $1 \leq k \leq p$.

Using the orthogonal matrix Q_t, the above IRF, $IR_h(A_0, A_+)$= $IR_h(A_{tr}Q, A_+)$, is transformed to $IR_h(A_{tr}, A_+Q')Q$, for $0 \leq h \leq \infty$. It indicates that the sets of structural parameters (A_0, A_+) and (A_{tr}, A_+Q') are observationally equivalent so that we can replace A_0 with A_{tr} in the IRF. Accordingly, instead of A_0, the lower triangular matrix A_{tr} derived from Cholesky decomposition is used together with the matrix Q to be convenient to calculate. Let $f(A_0, A_+)$ be the stacked IRF at horizon zero and long term: L, given by a $2n \times n$ matrix as below.

$$f(A_0, A_+) = \underbrace{\begin{bmatrix} IR_0(A_0, A_+) \\ IR_L(A_0, A_+) \end{bmatrix}}_{2n\times n}. \tag{8}$$

Using the above IRFs, we can identify the SVARs imposed from the zero and sign restrictions of the IRFs as explaining below.

Zero restrictions

We consider how to impose the IRFs from the zero restrictions, using the manner by Arias et al. (2014). Let Z_j denote a matrix in which the number of column is equal to the number of rows in $f(A_0, A_+)$ and j is the j-th structural shock imposing the zero restrictions. Using the orthogonal matrix Q_t, the product of the zero restrictions matrices and the IRF is transformed as below.

$$Z_j\, f(A_0 Q,\, A_+ Q)\, e_j = Z_j\, f(A_0,\, A_+)\, Q\, e_j = Z_j\, f(A_0,\, A_+)\, q_j,$$

where $q_j = Qe_j$. And then, the zero restrictions will hold if and only if

$$Z_j f(A_0, A_+) q_j = 0, \text{ for } 1 \leq j \leq n,$$

where Z_2, Z_3, and Z_4 are the zero restriction matrices for monetary policy shock, fiscal policy shock and business cycle shock, respectively. And their matrices are represented as

$$\underbrace{Z_2}_{4\times 2n} = \left[\begin{array}{cccccc:cccccc} 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&0&1 \end{array}\right], \text{ for the monetary policy shock,}$$

$$\underbrace{Z_3}_{3\times 2n} = \left[\begin{array}{cccccc:cccccc} 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0 \end{array}\right], \text{ for the fiscal policy shock}$$

$$\underbrace{Z_4}_{2\times 2n} = \left[\begin{array}{cccccc:cccccc} 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0 \end{array}\right], \text{ for the business cycle shock}$$

where elements with one correspond to the endogenous variables imposed zero restrictions, and those with zero do to the variables unconstrained. The first n columns of the zero restriction matrix correspond to the short run restriction; $LR_0(A_0, A_+)$, while the latter n columns of the matrix do to the long run restrictions: $LR_L(A_0, A_+)$. And the number of rows in the matrix Z_i is the number of the zero restrictions of the corresponding i-th shock shown in Table 1. Notice that the the number of the zero restrictions is equal to the number of endogenous variables: n, less the ordinal number

i of the i-th structual shock.

Sign restrictions

In the similar way to the above zero restrictions, sign restrictions can be implemented using a matrix expression. Let S_j be a matrix in which the number of column is equal to the number of rows in $f(A_0, A_+)$ and j is the j-th structural shock imposed the sign restrictions. Using the orthogonal matrix Q_t, the product of the sign restrictions matrices and the IRF is transformed as below.

$$S_j\, f(A_0 Q, A_+ Q)\, e_j = S_j f(A_0, A_+) Q e_j = S_j f(A_0, A_+) q_j,$$

And then, the sign restrictions will hold if and only if

$$S_j\, f(A_0, A_+)\, q_j > 0, \text{ for } 1 \leq j \leq n,$$

where S_2, S_3, and S_4 are the sign restriction matrices for monetary policy shock, fiscal policy shock and business cycle shock, respectively. And their matrices are represented as

$$\underbrace{S_2}_{4\times 2n} = \left[\begin{array}{cccccc:cccccc} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right], \text{ for the monetary policy shock,}$$

$$\underbrace{S_3}_{2\times 2n} = \left[\begin{array}{cccccc:cccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right], \text{ for the fiscal policy shock}$$

$$\underbrace{S_4}_{2\times 2n} = \left[\begin{array}{cccccc:cccccc} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right], \text{ for the business cycle shock}$$

where elements with one correspond to the endogenous variables imposed the sign restrictions, and those with zero do to the variables unconstrained. The first n columns of the sign restriction matrix correspond to the short run restriction; $LR_0(A_0, A_+)$, while the latter n columns of the matrix do to the long run restrictions: $LR_L(A_0, A_+)$. And the number of rows in the matrix S_i is the number of the sign restrictions of the corresponding i-th shock shown in Table 1.

QR decomposition

Let $X = QR$ be the QR decomposition of a $n \times n$ matrix X. The $n \times n$ random matrix Q has the uniform distribution, i.e., $QQ' = I$. and the $n \times n$ matrix R is a upper triangular matrix.

Let the matrix X be defined as

$$\underbrace{X_j(A_0, A_+)}_{n\times n} = \begin{bmatrix} Z_j f(A_0, A_+) \\ Q'_{j-1} \end{bmatrix}^T,$$

and the orthogonal matrix Q_j given from the QR decomposition of a $n \times n$ matrix $X_j(A_0, A_+)$ satisfies the zero restrictions, or $X_j(A_0, A_+)q_j = 0$ where $q_j = Q_j e_j$. By stacking them such as $Q = [q_1, \cdots, q_n]$, we obtain the rotation matrix Q to identify the SVAR model.

Algorithm for both restrictions

Finally, we show algorithm for both restrictions using the above QR decomposition. The sets of structural parameters are identified based on *Algorithm 4* by Arias et al. (2014) consisting of the following four steps.

1. Draw the sets of reduced-form parameters (B, Ω).

2. Using the QR decomposition mentioned above, draw an orthogonal matrix Q satisfies the zero restrictions, or $Z_j f(A_0, A_+)q_j = 0, \text{ for } 1 \leq j \leq n$.

3. Keep the draw if the sign restrictions are satisfied, or $S_j\, f(A_0, A_+)\, q_j > 0, \text{ for } 1 \leq j \leq n,$ otherwise discard the draw.

4. Return to step 1 until the required number of draws from the posterior distribution conditional on the sign and zero restrictions has been obtained.

Here, we remark as follows. In Step 2 and Step 3, the structural parameters A_0 are observationally equivalent to the lower triangular matrix A_{tr}. So instead of A_0, we use A_{tr} derived from the inverse of Cholesky decomposition of Ω. And A_+ is derived from BA_{tr}.

2.3 Estimation Methodology

State space model of TVP VARs

The TVP VARs are represented as state space models consisted of observation equations and state equations. In our model, the observation equation is Eq. (2) with observable variables y_t, and the state equations are Eq. (3), Eq.(4), and Eq.(5) with time-varying parameters, , regarded as state variables. And all parameters of the models are just three such as σ_β, σ_a and σ_h which determine covariances in Eq.(6).

Bayesian inference and MCMC Algorithm

Most of empirical studies dealing with TVP VARs have recently employed Bayesian inference via MCMC algorithm. Our study also follows them. There are four reasons to adopt the Bayesian estimation via the MCMC. First, its counterpart method: maximum likelihood estimation (MLE) method, is intractable to estimate because the state space model includes the nonlinear state equation (5) involved stochastic volatilities. Second, under the situation such as the uncertainty of parameters, the MCMC method is affordable to estimate simultaneously both of state variables and parameters. Third, the functions of both parameters and states variables such as the impulse response functions are also able to be sampled as the posterior distributions of the functions. Forth, all sampled parameters and state variables do not satisfied zero and sign restrictions. The impulse response functions just satisfied both restrictions are sampled as the products of the identified structural VAR.

In the state space model and the impulse response function involved the SVARs, draws generated iteratively from the following conditional posterior distributions of state variables and parameters must tend to convergence to the posterior joint distributions based on the property of Gibbs sampler. The MCMC algorithm estimating our model consists of the following nine steps.

1. Initialize parameters: Σ_β, Σ_a, Σ_h, and state variables: a_t, β_t, h_t.

2. Generate the state variables β_t given a_t, h_t, Σ_β, Y_t, from the conditional posterior distribution: $f(\beta_t|a_t, h_t, \Sigma_\beta, Y_t)$.

3. Generate the parametersΣ_β given β_t, from the conditional posterior distribution: $f(\Sigma_\beta|\beta_t)$.

4. Generate the state variables a_t given β_t, h_t, Σ_a, Y_t, from the conditional posterior distribution: $f(a_t|\beta_t, h_t, \Sigma_a, Y_t)$.

5. Generate the parametersΣ_a given a_t, from the conditional posterior distribution: $f(\Sigma_\alpha|\alpha_t)$.

6. Generate the state variables h_t given β_t, a_t, Σ_h, Y_t, from the conditional posterior distribution: $f(h_t|a_t, \beta_t, \Sigma_\beta, Y_t)$.

7. Generate the parameters Σ_h, given h_t, from the conditional posterior distribution: $f(\Sigma_h|h_t)$.

8. Generate the IRFs: $f(A_0, A_+)$, based on the structural parameters: A_0, A_+, identified with zero and sign restrictions, given a_t, β_t, h_t, Y_t.

9. Return to step 2 until the required number of draws from the posterior distribution

Here, we remark some points of the above MCMC simulation. In Step 8, the identification of SVARs and generation of IRFs are implemented from the way described of Section 2.2. In Steps 2 and 4, the simulation smoother of de Jong and Shephard (1995) is used for drawing β_t and a_t . In Step 7, a nonlinear filtering method based on block-sampling method is used for sampling stochastic volatility h_t, following Shephard and Pitt (1997), Watanabe and Omori (2004) and Nakajima et a. (2011). These parts explaining the MCMC procedure generating parameters in reduced-form TVP-VARs are described in Appendix A1in more detail.

The priors of the parameters are specified as:$(\Sigma_\beta)_i^2 \sim IG(20, 10^{-4})$, $(\Sigma_a)_i^2 \sim IG(20, 10^{-4})$, and $(\Sigma_h)_i^2 \sim IG(20, 10^{-4})$, where subscript i denotes the i-th diagonal elements of the covariance matrices and IG an inverse-Gamma distribution. The initial state variables are set as $\beta_0 \sim N(0, 10I)$, $a_0 \sim N(0, 10I)$, and $h_0 \sim N(0, 10I)$.

2.4 Measuring Fiscal Multipliers

In this study, we calculate an impact fiscal multiplier and a cumulative fiscal multiplier following Mountford and Uhlig (2009) and Arias et al. (2014). The impact fiscal multiplier at horizon i of structural shock s on endogenous variables y is defined as $IFM = \triangle y_{t+i}/\triangle g_t$, and is calculated from

$$IFM \equiv \frac{\triangle y_{t+i}}{\triangle g_t} = \frac{e_y' IR_i(A_0, A_+)q_s}{e_g' IR_0(A_0, A_+)q_s} \frac{1}{G/Y},$$

where g_t is the government spendings at period t, $IR_i(\bullet)$ is the IRF at horizon i as shown in Eq.(7), and G/Y denotes the average share of the government expenditure in GDP over the sample period. In the similar way, the cumulative fiscal multiplier at horizon i of structural shock s on endogenous variables y is given by $CFM = \sum_{i=0}^{H} \triangle y_{t+i} / \sum_{i=0}^{H} \triangle g_{t+i}$, and is calculated from

$$CFM \equiv \frac{\sum_{i=0}^{H} \triangle y_{t+i}}{\sum_{i=0}^{H} \triangle g_{t+i}} = \frac{\sum_{i=0}^{H} e_y' \, IR_i(A_0, A_+) \, q_s}{\sum_{i=0}^{H} e_g' \, IR_i(A_0, A_+) \, q_s} \frac{1}{(GOV/GDP)},$$

where H is the number of horizon to measure the impact of the policy shock for a specified interval. In our simulation, we calculate four cases characterized from different horizons, i.e., $H = 4,\ 8,\ 12,\ 20$.

2.5 Data

We use the quarterly data from the U.S. for the period between 1952:Q1 and 2013:Q4. Although our sample period contains the zero interest policy, we do not incorporate the zero lower bound constraint in light of the Nakagima (2011). The observed variables are composed from 6 variables: (1) government spending generated from sum of consumption expenditures and gross investment, (2) gross domestic product (GDP), (3) personal consumption expenditure, (4) dept-to-output ratio, (5) GDP deflator, and (6) nominal interest rate.

Because the level of public debt and the conduct of monetary policy are often suggested as candidates that affect the size of multipliers, we include debt-to-output ratio and monetary variables, such as price level and interest rate, in the TVP-VAR. Favero and Giavazzi (2012) and Chung and Leeper (2007) argue that inclusion of public debt in a VAR model is important to capture the effects of its dynamics on other variables. While they consider feedback between fiscal variables, we include debt-to-output ratio without imposing any restrictions as in Corsetti et al. (2013) because we choose to be agnostic towards the prevalence of either Ricardian or non-Ricardian fiscal regimes. For the very same reason, we include price level rather than inflation rate in our system following Uhlig (2005), Sims and Zha (2006) and Mountford and Uhlig (2009).

The first three variables are expressed in real per capita terms. We use the logarithm for all variables except nominal interest rate, debt-to-output-ratio. All variables except nominal interest rate are seasonally adjusted. All variables are detrended with a linear and quadratic trend. The lag length is set to p=4 following Blanchard and Perotti (2002).

3 Evidence on Time-Varying Fiscal Multiplier

3.1 MCMC Simulations

As described in the previous section, we adopt the Bayesian estimation with MCMC simulation to obtain the posterior estimates satisfied both of zero and sign restrictions showed in Table 1, based on the algorithm 4 proposed by Arias et al. (2014). We run MCMC simulations with 50,000 iterations, discarding the first 10,000 iterations to converge to the ergodic distribution, and sampling only draws satisfying the zero and sign restrictions out of the next 40,000 iterations. To calculate effects of the IRF for long run, we set $L = 20$ quarter (5 years) ahead in eq.(8). Figure 1 shows acceptance rates at individual period for the zero and sign restrictions out of the 40, 000 iterations. The size at each period is ranged between 1.2% and 3% as Figure 1. It indicates that around 400-1200 samples are recorded depending on the sample periods as the posterior estimates of the structural VAR imposed by the restrictions in Table 1, and used for making time-varing fiscal multipliers.

[Insert Figure 1 around here]

3.2 Time Variations of Impulse Responses

Figures 2 through 4 depict the time-varying impulse response of the six endogenous variables to the three shocks: government spending shocks, business cycle shock, and monetary policy shock, respectively. Panel (a) of these three figures show three-dimension format of the posterior means of the impulses with respect to both of the sample period and the horizon. Since the three-dimension figures, however, are not convenient to analyze, we also show the IRF of government spending, output, private consumption fixed by six time points: 60Q1, 70Q1, 90Q1, 2000Q1, 2010Q1, and bounded by 6 horizons: 1 Q ahead, 4 Q ahead, 8 Q ahead, 12 Q ahead, 16 Q ahead, 20 Q ahead, as panels (b)-(d). Blue solid lines represent the posterior means of the IRF fixed by periods, while light blue shaded areas represent 68% intervals in the former six graphs in the panel. Red solid lines and red shaded area in the latter six graphs are the posterior means and 68% band of the IRF fixed by horizons, respectively.

Firstly, let look into the responses to fiscal policy shock at Figure 2. As the zero restrictions shown Table 1, the impacts of government spending, output and consumption are set to zero for long run defined as 5 years after, since the fiscal policy shock belongs to demand shocks. As blue lines of the first six graphs in panel (b)-(d), all of the IRFs converge to zero at 20Q ahead. And we normalize the size of the posterior mean of positive fiscal policy shock at horizon 0 as a unity in panel (b), whereas the reactions of output and consumption are in panel (c) and (d), in which the unit of these impulses is dollar per one dollar increase in government spending.

Figure 3 shows the reponses to positive business cycle shock, in which the long-run restrictions are imposed to just real GDP and consumption set at zero as the corresponding blue lines at 20 horizons. Meanwhile, long run effect of government spending levels off positive at 20 horizons after 1960, since it is not imposed at all. This shock is normalized to make the output be one at 0 horizons. As blue solid line of the first six graphs of panel (c) and (d), the positive business cycle shock increases the output and consumption until the first 10 horizons, but changes them to zero or negative after 10 horizons. The positive shock changes not to increase the government spending after 1990 according to the red lines of $1-8$ Q ahead of the latter part of panel (b), while most of government spending change to be positive because of negative output and consumption after 10 Q ahead.

The responses to 1% increase interest rate shock are drawn in Figure 4. As shown in Table 1, the sign restriction imposes the monetary policy shock to decrease the output and consumption as well as to drop the price level, in which we can avoid to generate the price puzzle often observed in a standard SVAR approach. According to the result of the panel (b), the impact of government spending decreases for tightening monetary policy during 1960-1990. It indicates that fiscal policy must be taken to be consistent with the stance of monetary policy. However, fiscal policy is not likely to react against the change of monetary policy after 1990. In addition, we observed that fiscal restraint tends to be conducted for the long run, since red lines in 16 and 20 period ahead the later six graphs decline rapidly after 2000.

[Insert Figure 2 to 4 around here]

3.3 Time Variations of Fiscal Multipliers

Figures 5 and 6 show the estimation of impact fiscal multiplier and cumulative fiscal multiplier, respectively. The red solid line represents the posterior mean of time-varying fiscal multiplier, while blue dashed line represents the average of the means over the sample period. Panel (a), (b) and (c) of those figures show the impact of real

GDP, consumption, and debt-to-GDP ratio, respectively. As explained below, the results of time-varying multipliers in our estimations might be consistent with those of the existing empirical studies, some of which estimate how state dependent multipliers fluctuate. For example, the size of the multiplier depends on the states of business cycles, and it is larger in a recession than in an expansion, according to Auerbach and Gorodnichenko, (2012); Bachmann and Sims (2012); Batini et al. (2012); Candelon and Lieb (2013); Caggiano et al. (2015). Also, studies analyzing from another aspect found that the multiplier is larger in the economies with low debt-to-GDP ratio than with the high ratio (e.g., Favero et al. (2011); Corsetti et al. (2012); Ilzetzki et al. (2013)).

As the panel (a) and (b) in Figure 5, government spending impacts real GDP negatively for the first one year after the middle of 1960's, since it might make private consumption crowded out. However, the impact of output as well as consumption changes positively at two years later in the periods before 1984 and after 2008, say, except the Great Moderation most periods of which belong to boom. And the impact is positive over all of the sample period from three years later. On the other hand, the impact of debt-to-GDP ratio levels off at high level since two years later, after 1984 in which the Great Moderation starts, as shown red lines of in panel (c).

[Insert Figure 5 around here]

Each panel of Figure 6 shows the estimations of the cumulative fiscal multipliers from six intervals; 0-0.5year, 0-1 years, 0-2 years, 0-3 years, 0-4 years, 0-5 years. This further supports the findings of Figure 5. As shown the red lines in panel (a) and (b), cumulative multipliers for the output and consumption is negative during the Great Moderation (1984-2008) even a long-run effect such as the interval 0-5 years. Meanwhile, in the periods except the Great Moderation, the cumulative multipliers for the consumption is observed to change positively before that for the output does positively. It suggests that the reaction of the consumption on government spending is a key to decide the size of fiscal multipliers for the output. We turn to the blue dashed line representing the average over all periods. The averages of the output and consumption are negative for the short run; 0-2 years. And they finally reach to zero or positive after 3 years later, although they are even positive but almost are located around zero. The cumulative fiscal multipliers for the debt-to-GDP ratio gradually increase for the Great Moderation, as panel (c).

The above findings might be interpreted that the real GDP gradually increases because of the impact on increasing private consumption as well as private investment, but not on increasing spendings by the public sector during the Great Moderation. These good situation would have induced a reduction of government debt per real GDP by decreasing weight and influence of public sector on the US economy. However, this relationship between private and public sectors broke up at the beginning of the Great Recession caused by the Lehman Brother collapse in 2008, and it changed to expansion of government spending and stagnation of the output and private consumption.

[Insert Figure 6 around here]

4 Conclusion

This study might be the first attempt applied a method imposing both zero and sign restrictions proposed by Arias, Rubio-Ramirez, Waggoner (2014) to a TVP-VAR model. The structural VAR is identified by imposing both of long-run zero restrictions and short-run sign restrictions for a fiscal policy shock orthogonalized with monetary policy and business cycle shocks in the US economy between 1953:Q1-2013:Q4. In our TVP-VAR approach, there are six endogenous variables, i.e., (1) government spending, (2) output (3) private consumption (4) public debt measured by the ratio against real GDP (5) price level (6) nominal interest rate. And, using the estimation results, we calculate time-varying fiscal multipliers as well as impulse responses to the three shocks.

The main findings are as follows. Time-varying fiscal multipliers could be negative during the Great Moderation, since the private consumption was crowded out. However, they changed positively after the Great Recession by increasing the consumption. And we also observed that the propagation effect of fiscal policy has had non-trivial time lag so that it has taken at least two years to work effectively after 1970's.

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A Appendix

A.1 MCMC procedure for TVP-VARs

In Section 2.3, we describe the nine steps of the MCMC algorithm estimating our model. Here, we focus on the steps generating parameters in reduced-form TVP-VARs. This section is described based on Appendix of Nakajima (2011) and Nakajima et al. (2011).

A.1.1 Generate the state variables β_t given a_t, h_t, Σ_β, Y_t, from the conditional posterior distribution: $f(\beta_t|a_t, h_t, \Sigma_\beta, Y_t)$.

To generate β_t from the conditional posterior distribution: $f(\beta_t|a_t, h_t, \Sigma_\beta, Y_t)$, we introduce the simulation smoother by de Jong and Shephard (1995) and Durbin and Koopman (2002) using the state space model with respect to β_t given by

$$y_t = X_t\beta_t + A_t^{-1}\Sigma_t\varepsilon_t,\ t = s+1, \cdots, n, \tag{9}$$

$$\beta_{t+1} = \beta_t + u_\beta,\ t = s+1, \cdots, n-1,$$

where β_s is set as μ_{β_0}, and $u_{\beta_s} \sim N(0, \Sigma_{\beta_0})$.

A.1.2 Generate the state variables a_t given β_t, h_t, Σ_a, Y_t , from the conditional posterior distribution: $f(a_t|\beta_t, h_t, \Sigma_a, Y_t)$.

To generate a_t from the conditional posterior distribution: $f(a_t|\beta_t, h_t, \Sigma_a, Y_t)$, the simulation smoother is also adopted from the following state space model,

$$\hat{y}_t = \hat{X}_t a_t + \Sigma_t\varepsilon_t,\ t = s+1, \cdots, n,$$

$$a_{t+1} = a_t + u_{at},\ t = s, \cdots, n-1,$$

where $a_s = \mu_{a0}$, $u_{as} \sim N(0, \Sigma_{a0})$, $\hat{y}_t = y_t - X_t\beta_t$, and

$$\hat{X}_t = \begin{bmatrix} 0 & \cdots & & & & & & 0 \\ -\hat{y}_{1t} & 0 & 0 & \cdots & & & & \vdots \\ 0 & -\hat{y}_{1t} & -\hat{y}_{2t} & 0 & \cdots & & & \\ 0 & 0 & 0 & -\hat{y}_{1t} & \cdots & & & \\ \vdots & & & & \ddots & 0 & \cdots & 0 \\ 0 & \cdots & & & 0 & -\hat{y}_{1t} & \cdots & -\hat{y}_{k-1t} \end{bmatrix},$$

for $t = s+1, \cdots, n$.

A.1.3 Generate the state variables h_t given β_t, a_t, Σ_h, Y_t, from the conditional posterior distribution: $f(h_t|a_t, \beta_t, \Sigma_\beta, Y_t)$.

To generate the stochastic volatility h_t from the conditional posterior distribution: $f(h_t|a_t, \beta_t, \Sigma_\beta, Y_t)$, we conduct the inference for $h_{jt}{}_{t=s+1}^{n}$ separately for j, because it is assumed that Σ_h and Σ_{h0} are diagonal matrices. Let y_{it}^* denote the i-th element of $A_t y_t$. Then, we can write:

$$y_{it}^* = exp(h_{it}/2)\varepsilon_{it},\ t = s+1, \cdots, n,$$

$$h_{i,t+1} = h_{it} + \eta_{it},\ t = s, \cdots, n-1,$$

$$\begin{pmatrix} \varepsilon_{it} \\ \eta_{it} \end{pmatrix} \sim N\left(0, \begin{pmatrix} 1 & 0 \\ 0 & \nu_i^2 \end{pmatrix}\right),$$

where $\eta_{is} \sim N(0, \nu_{i0}^2)$, and ν_i^2 are the i-th diagonal elements of Σ_h and Σ_{h0}, respectively, and η_{it} is the i-th element of u_{ht}. We sample $h_t = (h_{i,s+1}, \cdots, h_{in})$ using the multi-move sampler developed by Shephard and Pitt (1997) and Watanabe and Omori (2004), the algorithm of which is described in the following subsection.

A.1.4 Generate the parameters Σ_α , Σ_β, andΣ_h.

To generate the parameter Σ_a given a_t, we draw the sample from the conditional posterior distribution: $\Sigma|a_t \sim IW(\hat{\nu}, \hat{\Omega}^{-1})$, where IW denotes the inverse-Wishart distribution, and $\hat{\nu} = \nu_0 + n - 1$, $\hat{\Omega} = \Omega_0 + \sum_{t=1}^{n-1}(a_{t+1} - a_t)(a_{t+1} - a_t)'$ in which the prior is set as $\Sigma \sim IW(\nu_0, \Omega_0^{-1})$. Sampling the diagonal elememts of Σ_β, Σ_h is also the same way to sample Σ_a.

A.2 Multi-Move Sampler of Stochastic Volatilities

This section is described based on Appendix of Nakajima (2011) and Nakajima et al. (2011). The algoritm of the multi-move sampler proposed by Shephard and Pitt (1997), Watanabe and Omori (2004) is adopted to generate draws of stochastic volatilities in the TVP-VARs from the conditional posterior distributions explainded in Appendix A2. We show the stochastic volatilities model again.

$$y_t^* = exp(h_t/2)\varepsilon_t,\ t = s+1, \cdots, n,$$

$$h_{t+1} = \phi h_t + \eta_t,\ t = s, \cdots, n-1,$$

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim N\left(0, \begin{pmatrix} 1 & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix}\right),$$

where y_t^* denote the i-th element of $A_t y_t$ shown in Eq.(9). For drawing a typical block such as$(h_r, \cdots, h_{r+d})$, we consider the draw of

$$\begin{aligned} (\eta_{r-1}, \cdots, \eta_{r+d-1}) &\sim \pi(\eta_{r-1}, \cdots, \eta_{r+d-1}|\omega) \\ &\propto \prod \frac{1}{e^{h_t/2}} \exp\left(\frac{y_t^{*2}}{2e^{h_t}}\right) \times \prod f(\eta_t) \times f(h_{r+d}) \end{aligned} \tag{10}$$

where

$$f(\eta_t) = \begin{cases} \exp\left\{-\frac{(1-\phi^2)\eta_0^2}{2\sigma_\eta^2}\right\} & (\text{if } t = 0), \\ \exp\left(-\frac{\eta_t^2}{2\sigma_\eta^2}\right) & (\text{if } t \geq 1), \end{cases}$$

$$f(h_{r+d}) = \begin{cases} \exp\left\{-\frac{(h_{r+d+1}-\phi h_{r+d})^2}{2\sigma_\eta^2}\right\} & (\text{if } r+d < n), \\ 1 & (\text{if } r+d = n), \end{cases}$$

and $\omega = (h_{r-1}, h_{r+d+1}, \beta, \gamma, \phi,)$. The posterior draw of $(h_r, \cdots, h_{r+d})$ can be obtained by running the state equation with the draw of $(\eta_{r-1}, \cdots, \eta_{r+d-1})$ given h_{r-1}.

We sample $(\eta_{r-1}, \cdots, \eta_{r+d-1})$ from the density (10) using the acceptance-rejection MH algorithm (Tierney, 1994; Chib and Greenberg, 1995) with the following proposal distribution constructed from the second-order Taylor expansion of

$$g(h_t) \equiv -\frac{h_t}{2} - \frac{y_t^{*2}}{2e^{h_t}},$$

around a certain point $\hat{h}_t$ which is given by

$$\begin{aligned} g(h_t) &\doteqdot g(h_t) + g'(\hat{h}_t)(h_t - \hat{h}_t) + \frac{1}{2}g''(\hat{h}_t)(h_t - \hat{h}_t)^2 \\ &\propto \frac{1}{2}g''(\hat{h}_t)\left\{h_t - \left(\hat{h}_t - \frac{g'(\hat{h}_t)}{g''(\hat{h}_t)}\right)\right\}^2, \end{aligned}$$

Here, the first and second derivatives are obtained such that

$$g'(\hat{h}_t) = -\frac{1}{2} + \frac{y_t^{*2}}{2e^{h_t}},\ g''(\hat{h}_t) = -\frac{y_t^{*2}}{2e^{h_t}},$$

And the proposal density of $\pi(\eta_{r-1}, \cdots, \eta_{r+d-1}|\omega)$ is given by

$$q(\eta_{r-1}, \cdots, \eta_{r+d-1}|\omega) \propto \prod \exp\left\{-\frac{(h_t^* - h_t)^2}{2\sigma_t^{*2}}\right\} \times \prod f(\eta_t),$$

where

$$\sigma_t^{*2} = -\frac{1}{g''(\hat{h}_t)},\ h_t^* = h_t + \sigma_t^{*2} g'(\hat{h}_t), \tag{11}$$

for $t = r, \cdots, r+d-1$, and $t = r+d$ in the case that $r+d = n$. Meanwhile, in the case that$r + d \leq n$,

$$\sigma_{r+d}^{*2} = \frac{1}{-g''(\hat{h}_{t+d}) + \phi^2/\sigma_\eta^2} \tag{12}$$

$$h_{r+d}^* = \sigma_{r+d}^{*2}\left\{g'(h_{r+d}) - g''(h_{r+d})h_{r+d} + h_{r+d}/\sigma_\eta^2\right\}, \tag{13}$$

for $t = r + d$. The proposal density of the AR-MH algorithm is derived from the following state space model,

$$h_t^* = h_t + \varsigma_t,\ t = s+1, \cdots, n,$$

$$h_{,t+1} = h_t + \eta_t,\ t = s, \cdots, n-1, \tag{14}$$

$$\begin{pmatrix} \varsigma_t \\ \eta_t \end{pmatrix} \sim N\left(0, \begin{pmatrix} \sigma_t^{*2} & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix}\right),$$

with $\eta_{r-1} \sim N(0, \sigma_\eta^2)$ when $r \geq 2$ and $\eta_s \sim N(0, \sigma_\eta^2/(1-\phi^2))$. Given ω, we draw candidate point of $(\eta_{r-1}, \cdots, \eta_{r+d-1})$ for AR-MH algorithm by running the simulation smoother over the state-space representation (14).

For realizing efficient drawings, we need to calculate the mode of the above posterior density for $(\hat{h}_r, \cdots, \hat{h}_{r+d})$. Numerically, we obtain the mode by iterating the following steps several times,

1. Initialize $(\hat{h}_r, \cdots, \hat{h}_{r+d})$.

2. Compute $(h^*_r, \cdots, h^*_{r+d})$, and $(\sigma^*_r, \cdots, \sigma^*_{r+d})$ by eq.(11) through eq.(13).

3. Run the simulation smoother for state space model eq.(14) with $(h^*_r, \cdots, h^*_{r+d})$, and $(\sigma^*_r, \cdots, \sigma^*_{r+d})$ as obervable variables. And Generate estimations $h^*_t = E(h_t|\omega)$ for $t = r, \cdots, r+d$.

4. Replace $(\hat{h}_r, \cdots, \hat{h}_{r+d})$ with $(h^*_r, \cdots, h^*_{r+d})$.

5. Return to Step 2.

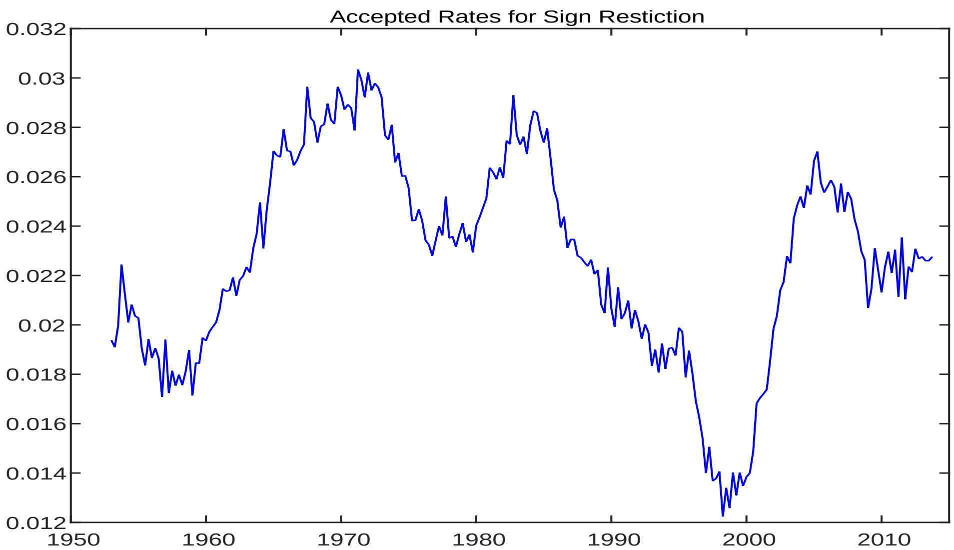
To implement a block sampling for h_t, they are devided into $K+1$ blocks, say, $(h_{k(i-1)}, \cdots, h_{k(i)})$ for $i = 1, \cdots, K+1$. Shephard and Pitt (1997) suggested to adopt stochastic knots for determining the positions of blocks: i, the rule of which is given by

$$k(i) = int\left[\frac{n(j+U_i)}{K+2}\right],$$

for $i = 1, \cdots, K$, where int is a function rounding to an integer value from the insight, and U_i is the random sample from the uniform distribution $U[0,1]$.

B Figures

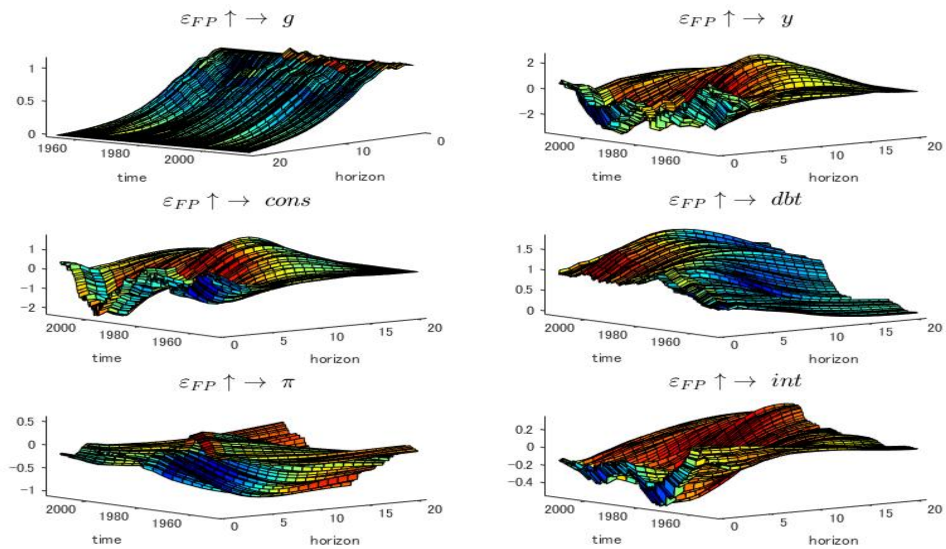
Figure 1: Acceptance rates of Zero and Sign Restrictions



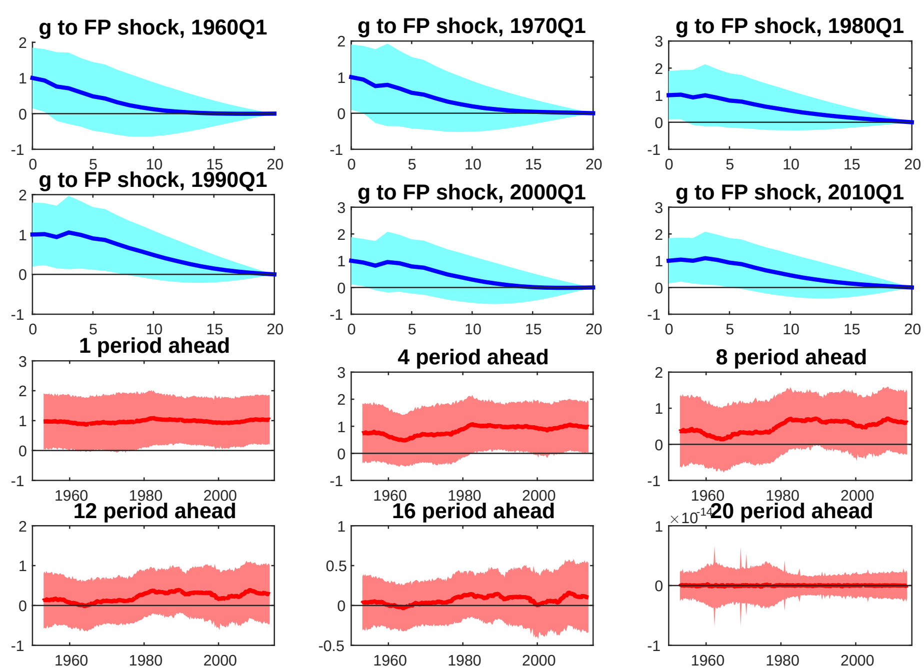
Notes: We run MCMC simulations with 50,000 iterations, discarding the first 10,000 iterations to converge to the ergodic distribution, and sampling only draws satisfying the zero and sign restrictions out of the next 40,000 iterations. Figure 1 shows acceptance rates fulfilling the zero restrictions and the sign restrictions out of the 40, 000 iterations at individual periods.

Figure 2: Impulse Response to Fiscal Policy Shock

(a) Three Dimensions of Impulse Response to Fiscal Policy shock

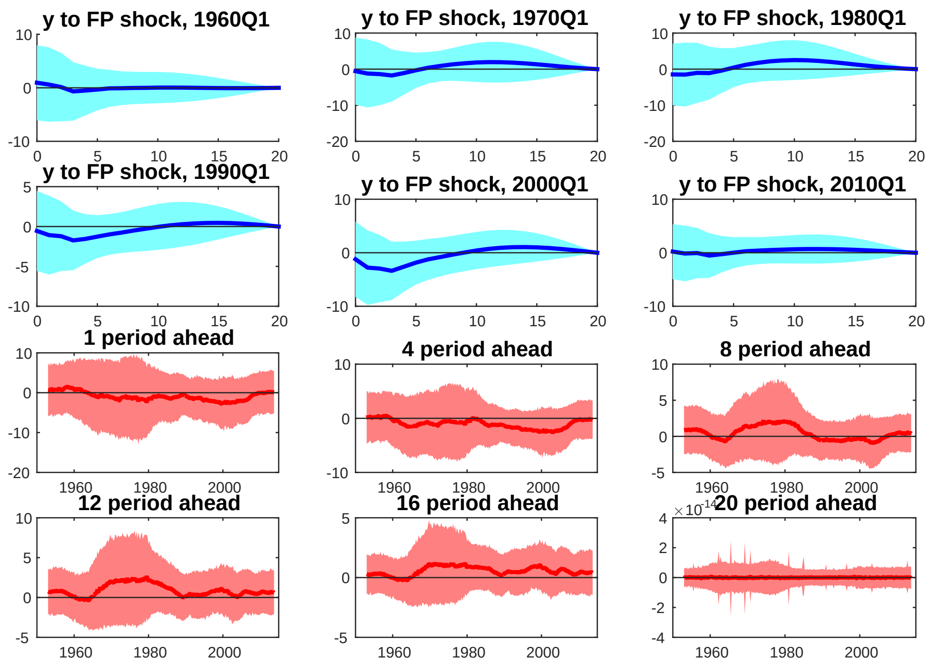


(b) Response of Govenment Spending to Fiscal Policy Shock

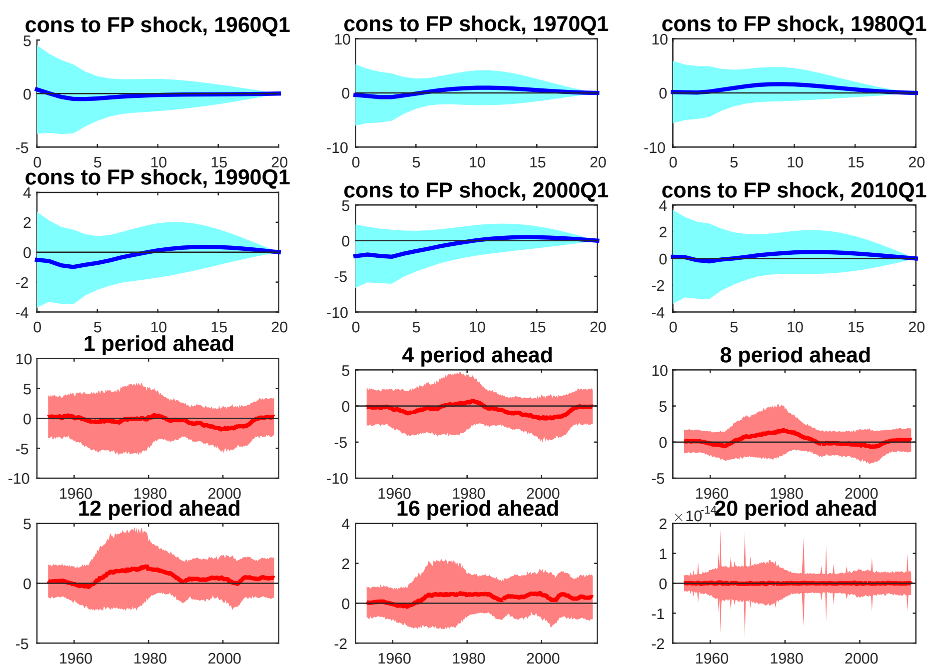


Notes: Blue and red solid lines represent the posterior means of the IRF in terms of specified periods and horizons, respectively. And light blue and red shaded areas represent 68% credible interval of the IRF.

(c) Response of real GDP to Fiscal Policy Shock



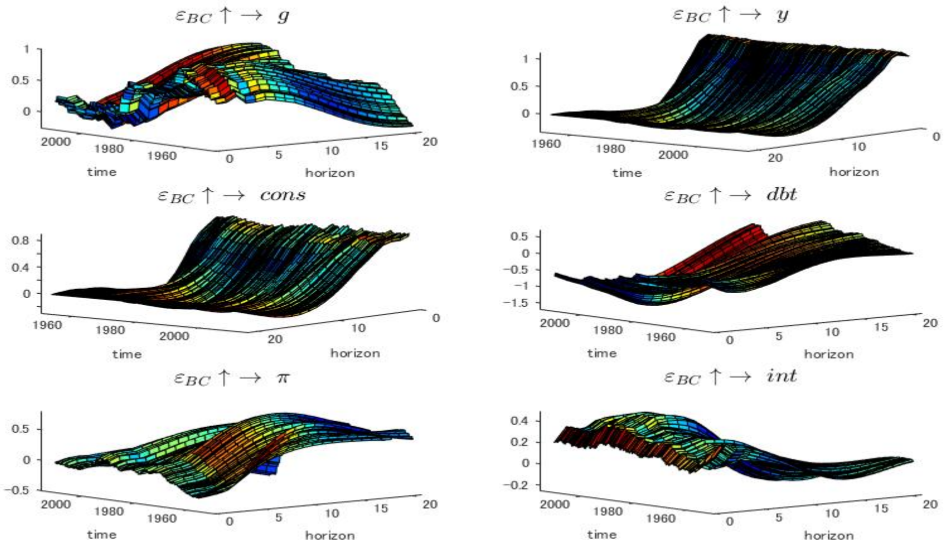
(d) Response of Private Consumption to Fiscal Policy Shock



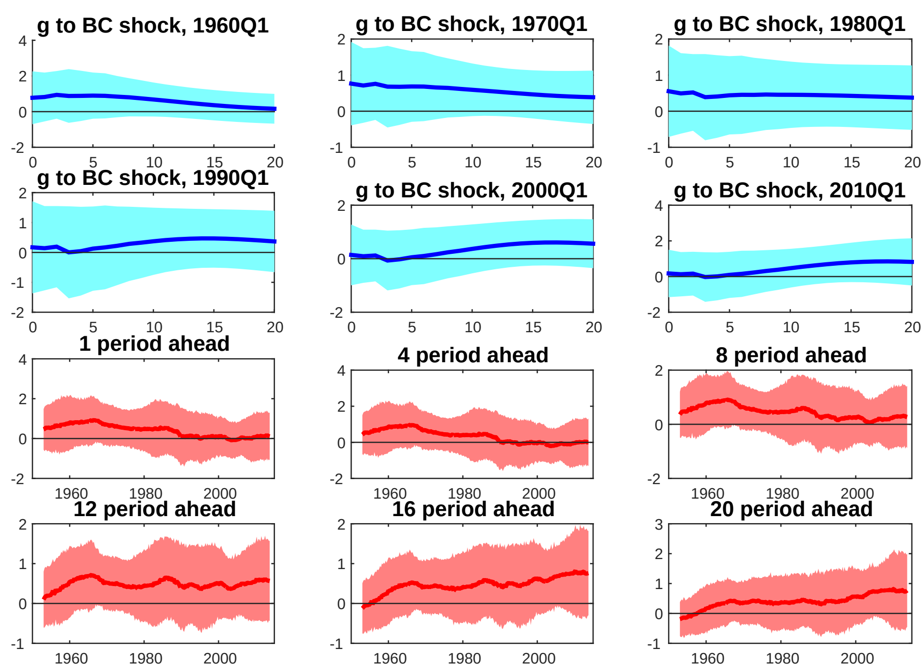
Notes: Blue and red solid lines represent the posterior means of the IRF in terms of specified periods and horizons, respectively. And light blue and red shaded areas represent 68% credible interval of the IRF.

Figure 3: Impulse Response to Business Cycles shock

(a) Three Dimensions of Impulse Response to Business Cycles shock

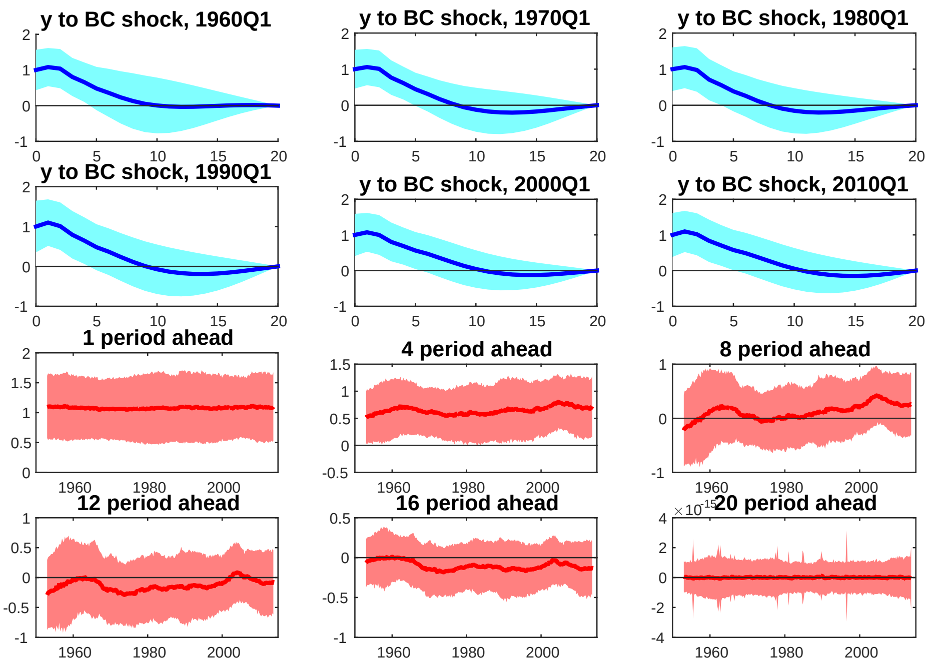


(b) Response of Govenment Spending to Business Cycles Shock

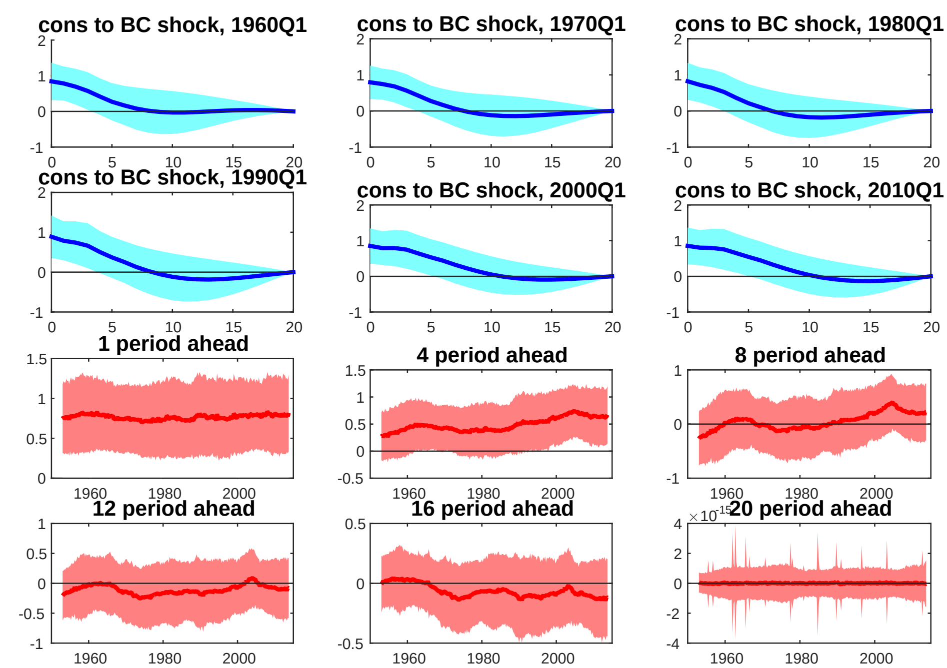


Notes: Blue and red solid lines represent the posterior means of the IRF in terms of specified periods and horizons, respectively. And light blue and red shaded areas represent 68% credible interval of the IRF.

(c) Response of real GDP to Business Cycles Shock



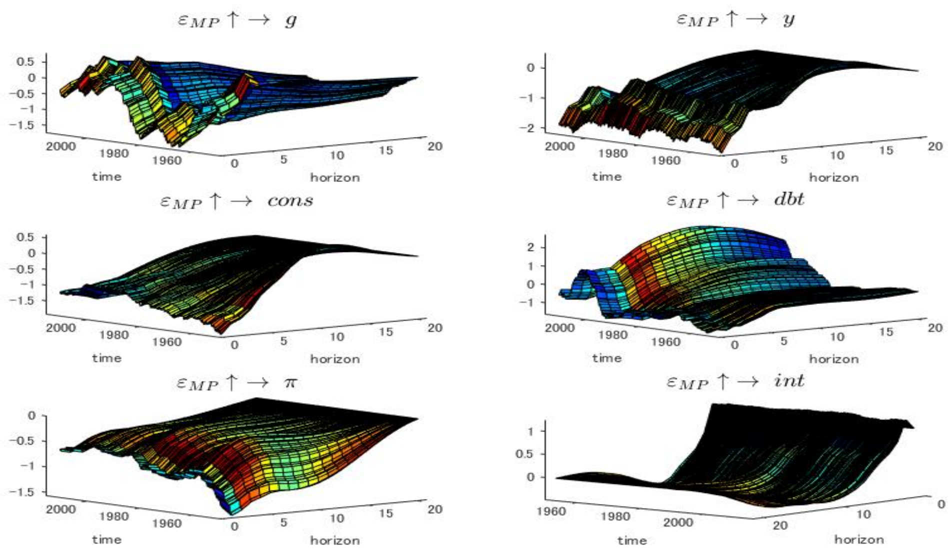
(d) Response of Private Consumption to Business Cycles Shock



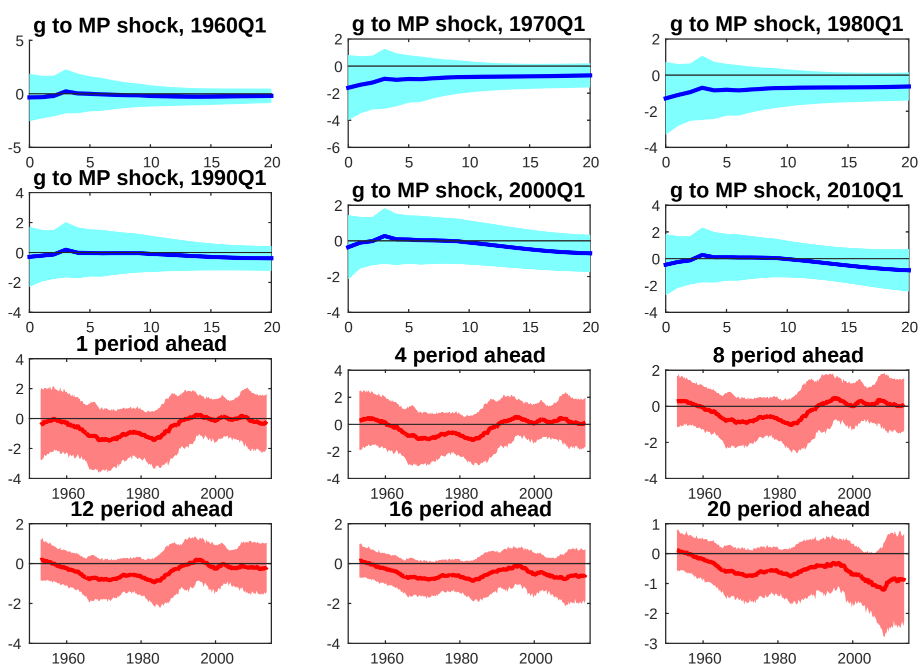
Notes: Blue and red solid lines represent the posterior means of the IRF in terms of specified periods and horizons, respectively. And light blue and red shaded areas represent 68% credible interval of the IRF.

Figure 4: Impulse Response to Monetary Policy Shock

(a) Three Dimensions of Impulse Response to Monetary Policy Shock

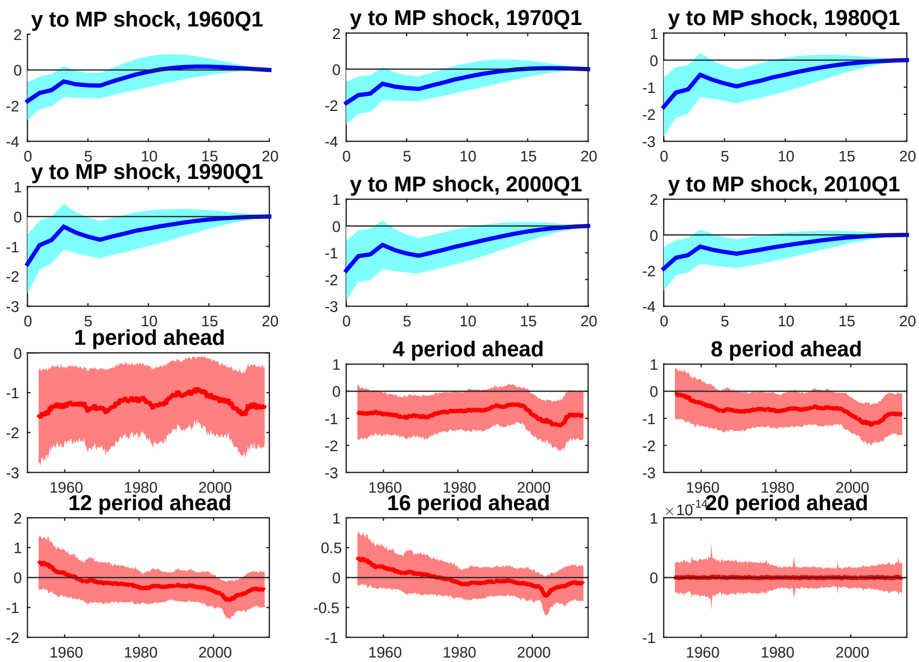


(b) Response of Gov to Monetary Policy Shock

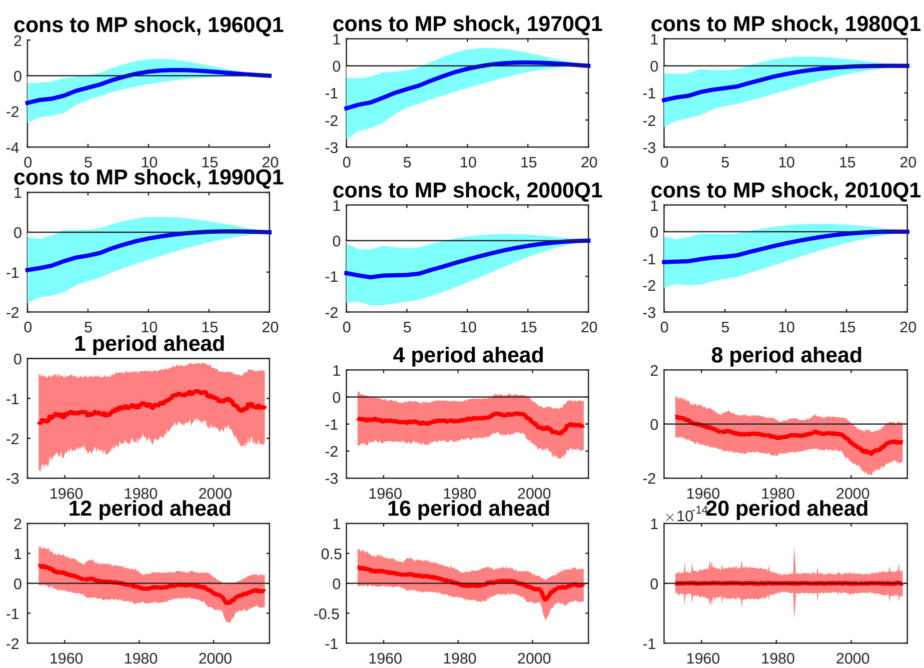


Notes: Blue and red solid lines represent the posterior means of the IRF in terms of specified periods and horizons, respectively. And light blue and red shaded areas represent 68% credible interval of the IRF.

(c) Response of real GDP to Monetary Policy Shock



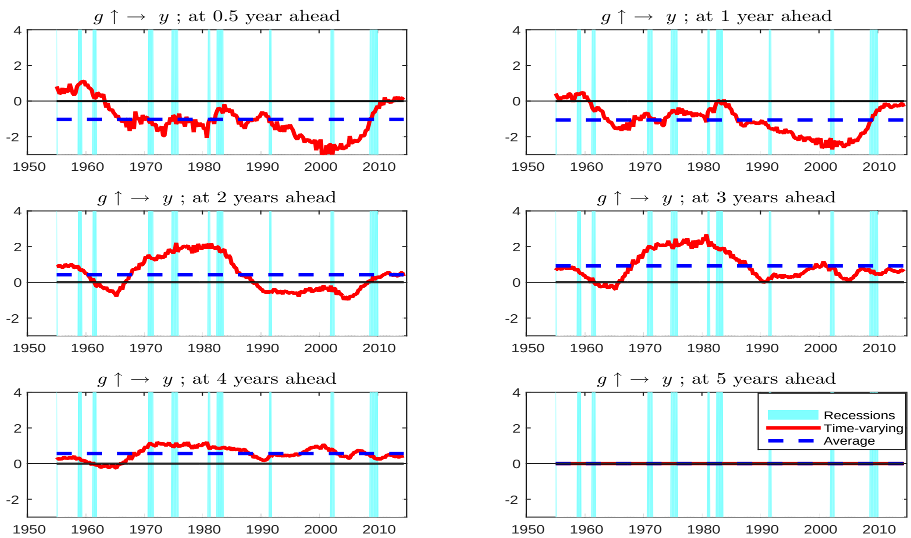
(d) Response of Private Consumption to Monetary Policy Shock



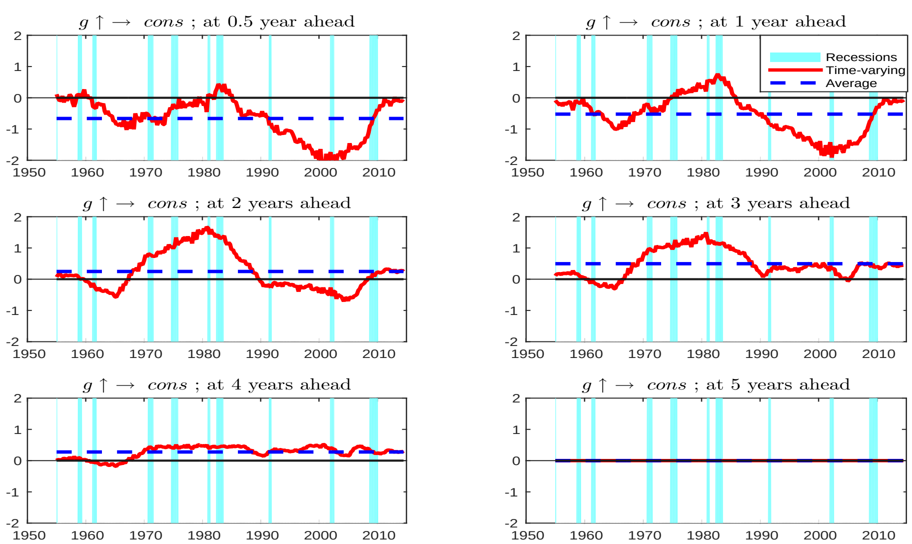
Notes: Blue and red solid lines represent the posterior means of the IRF in terms of specified periods and horizons, respectively. And light blue and red shaded areas represent 68% credible interval of the IRF.

Figure 5: Impact Multipliers of Government Spending

(a)Impact Multipliers for real GDP

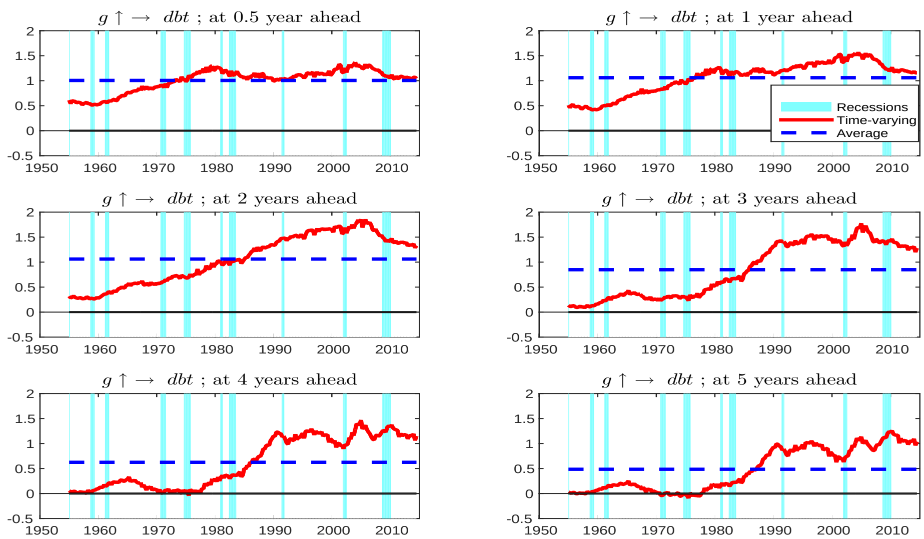


(b) Impact Multipliers for Private Consumption



Notes: Red solid and blue dashed lines represent the posterior means of the time-varying multipliers and the average of them over sample period, respectively.

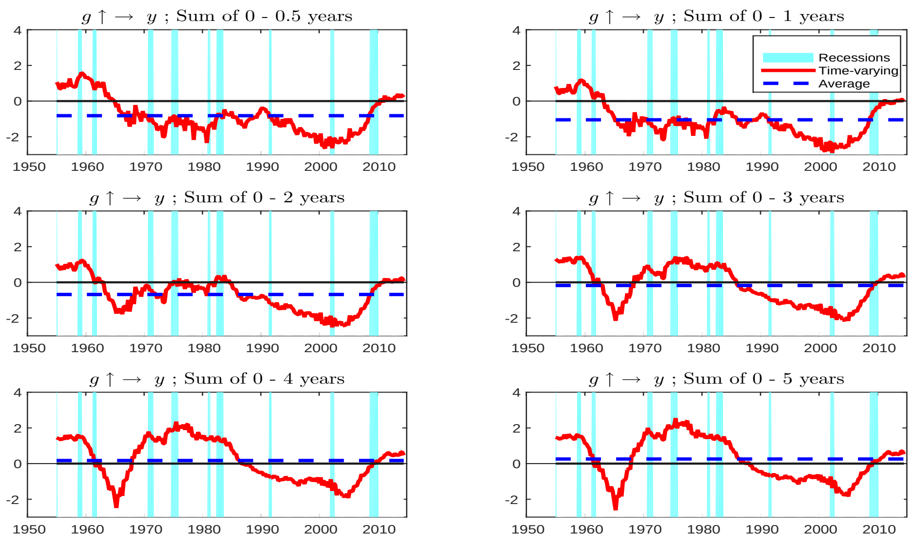
(c) Impact Multipliers for Debt-to-Output Ratio



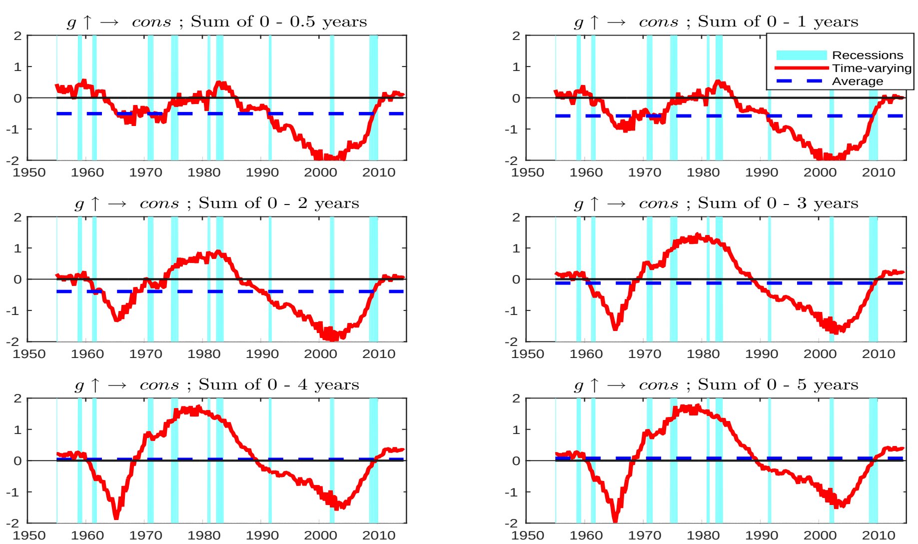
Notes: Red solid and blue dashed lines represent the posterior means of the time-varying multipliers and the average of them over sample period, respectively.

Figure 6: Cumulative Multipliers of Government Spending

(a)Cumulative Multipliers for real GDP

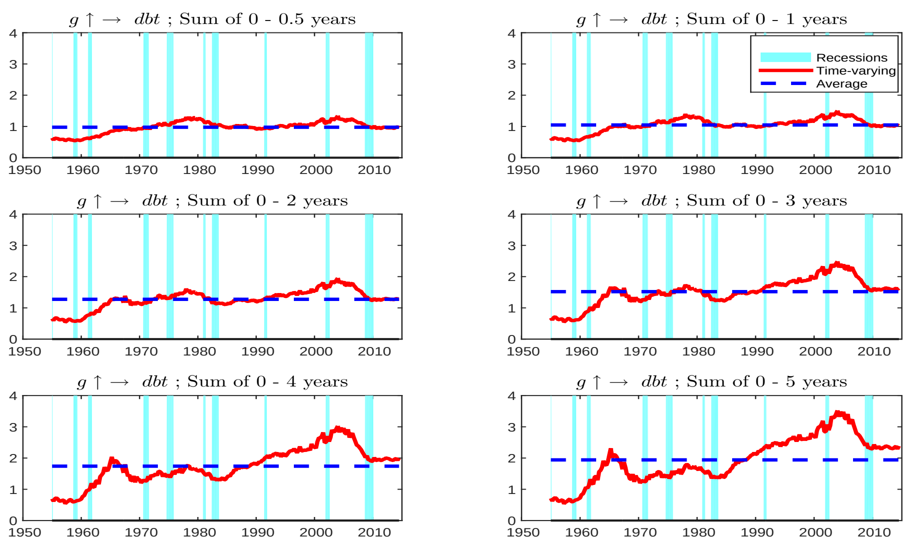


(b) Cumulative Multipliers for Private Consumption



Notes: Red solid and blue dashed lines represent the posterior means of the time-varying multipliers and the average of them over sample period, respectively.

(c) Cumulative Multipliers for Debt-to-Output Ratio



Notes: Red solid and blue dashed lines represent the posterior means of the time-varying multipliers and the average of them over sample period, respectively.