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Abstract: This paper examines the optimal investment timing decision problem of a firm that is constrained by a debt issuance limit determined by collateral value. Our model provides five important results. First, collateral-based financing constraints do not always delay corporate investment, compared with that without upper limit. Second, collateral-based financing constraints are likely to change bankruptcy strategies during financial distress via a change of capital structure. Third, collateral-based financing constraints create a low-risk and low-return scenario for debt holders. Fourth, the agency costs of financing constraints are not monotonic with cash-flow volatility. Finally, debt financing with an upper limit does not always accelerate investment, compared with all-equity financing, even when debt financing with an upper limit is preferred to all-equity financing.

Keywords: Real options; Financing constraints; Credit spreads; Default probabilities.

JEL classification: G32; G33, G21.

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1 Introduction

Modigliani and Miller (1958) argue that in a frictionless market, financing and investment decisions are completely separable. Once capital market imperfections are introduced, however, financing and investment decisions are not necessarily independent. Since their seminal work, corporate finance literature has highlighted the role of financial frictions between financing and investment decisions. For example, following Rauh (2006) and Almeida and Campello (2007), the issue of how financing frictions affect financing and investment decisions is a central, unsettled issue in corporate finance.

Brennan and Schwartz (1984), Mauer and Triantis (1994), Hackbarth et al. (2007), and Sundaresan and Wang (2007) examine the interaction between financing and investment decisions in contingent claim (real option) model. These models have a limitation. There are no financial frictions. In other words, the authors assume a frictionless market.

Several studies incorporate financial frictions in a contingent claim model. Boyle and Guthrie (2003), Hirth and Uhrig-Homburg (2010), and Nishihara and Shibata (2013) investigate the interactions between financing and investment decisions under internal financing constraints in a contingent claim (real option) model. Shibata and Nishihara (2012), Shibata and Nishihara (2015a), and Shibata and Nishihara (2015b) examine the interactions between financing and investment decisions under external financing constraints in a contingent claim model. An interesting result in these papers is that investment strategies are non-monotonic with respect to financial frictions. These models do not, however, consider the role of collateral during the times of financial distress.[1] Mella-Barral and Perraudin (1997) and Fan and Sundaresan (2000) derive bankruptcy strategies that account for the role of collateral. However, these models do not examine the interactions between financing and investment decisions.

In this paper, we investigate the interactions between financing and investment decisions that account for the role of collateral in a contingent claim model. Specifically, we consider how collateral constraints influence the interactions between financing and investment decisions. Our contribution is to provide a theoretical examination of how collateral constraints affect investment timing, default timing, liquidation timing, leverage, credit spreads, default probabilities, and so forth. To the best of our knowledge, this is the first

[1] Collateral plays an important role in bank lending in most developed countries.

paper to incorporate collateral constraints in a contingent claim (real options) model.[2] Despite the strong belief in non-stochastic dynamic models (e.g., macroeconomics) and, in practice, in the importance of collateral constraints, there has not been a contingent claim model that identifies and quantifies the impact of collateral constraints on firm's investment timing, default timing, liquidation timing, credit spreads, default probabilities, and so forth. This paper represents an effort to fill this gap.

Our model builds primarily on two papers: Mella-Barral and Perraudin (1997) and Shibata and Nishihara (2012). Our paper presents the optimal bankruptcy decision model after considering financing and investing (i.e., consistent with Mella-Barral and Perraudin (1997)). In our model we incorporate collateral constraints where the upper limit of debt issuance is restricted by the value of collateral when a firm is financed by debt financing. These constraints are similar to those used by Kiyotaki and Moore (1997). Our model is consistent with Shibata and Nishihara (2012) if the upper limit of debt issuance is restricted by the amount of investment expenditure instead of collateral value. Thus, our model can be regarded as a natural extension of these two papers, and it provides a useful framework for addressing many questions in the contingent claim (real option) literature. For example, how do collateral constraints affect financing and investment decisions? How do collateral constraints influence corporate leverage, credit spreads, and default probabilities? How do collateral constraints change corporate management (e.g. bankruptcy) strategies after investment? Answering these questions is the purpose of our model.

We provide five important results. The first result is that collateral-based financing constraints do not always delay corporate investment. This is because the investment thresholds of a constrained levered firm may be smaller than those of an unconstrained levered firm, implying that the investment thresholds of a constrained levered firm are not necessarily in between those of an unconstrained levered firm and an unlevered firm. In the literature, once the state variable, starting at a sufficiently low level, reaches the investment threshold, the firm makes the investment. Following existing studies, we assume that a smaller (larger) threshold implies earlier (later) investment. We intuit that

[2] A partial list of non-stochastic dynamic models that incorporate collateral constraints is Kiyotaki and Moore (1997), Bianchi (2010), Jeanne and Korinek (2010), Rampini and Viswanathan (2010), and Gottardi and Kubler (2015).

collateral-based financing constraints always delay investment. However, our theoretical result is contrary to our intuition. The coupon payment is the other control variable at the time of investment in our model. The coupon payments of a constrained levered firm are in between those of an unconstrained levered firm and an unlevered firm. Thus, if a firm is financially constrained with a collateral-based upper limit, it is less costly to distinguish the constrained investment thresholds from the unconstrained ones than to distinguish the constrained coupon payments from the unconstrained ones.

The second result is that financing constraints may change the default ("operating concern bankruptcy") and shutdown ("liquidation bankruptcy") strategies from sequential to simultaneous. Suppose that, as a benchmark, a firm exercises sequential bankruptcy strategies at the equilibrium when there are no financing constraints. The firm may exercise simultaneous bankruptcy strategies at the equilibrium because the firm is financially constrained when financing constraints exist. The reason is as follows. Because the coupon payment is reduced by the upper limit of debt issuance at the time of investment, the face value of the debt is reduced. The reduced face value of debt leads to a change in bankruptcy strategies from sequential to simultaneous. This result is similar to that in Nishihara and Shibata (2016b), where asymmetric information leads to change the bankruptcy strategy.

The third result is that upper limits of debt financing create low return and low risk for debt holders. In our model, we show that financing constraints decrease credit spreads and default probabilities. The return and risk for debt holders can be measured by credit spreads and default probabilities, respectively. Thus, we shed light on the determinants of upper limits for debt holders. In addition, financing constraints decrease corporate leverages. In the absence of financing constraints, leverages are almost flat with collateral, credit spreads decrease with collateral, and default probabilities are almost flat with collateral. If financing constraints exist, leverages, credit spreads, and default probabilities increase with collateral.

The fourth result is that the agency costs of financing constraints are not monotonic with cash-flow volatility. The reason is as follows. The firm is financially constrained in low-volatility or high-volatility regions, while it is not in an intermediate-volatility region. These results imply that agency costs exist in low-volatility or high-volatility regions, while they do not exist in an intermediate-volatility region. The agency costs decrease with volatility for the low-volatility region, they are zero for the intermediate-volatility region, and they increase with volatility for the high-volatility region. Thus, agency costs have a discontinuous U-shaped curve with volatility.

The fifth result is that debt financing with an upper limit does not always accelerate investment, compared with all-equity financing, even when debt financing with an upper limit is preferred to all-equity financing. Suppose that, as a benchmark, there are no financing constraints. Under these conditions, investment thresholds for a levered firm are always smaller than those for an unlevered firm, implying that debt financing always accelerates investment. The equity option values for a levered firm are always larger than those for an unlevered firm. Thus, debt financing always decreases the investment threshold and increases the equity option value before investment. We call this property the symmetric relationship between investment threshold and equity option value. This relationship has already been shown by Sundaresan and Wang (2007) and Shibata and Nishihara (2010). By contrast, suppose that financing constraints exist. In that case, the investment thresholds for a constrained levered firm may be larger than those for an unlevered firm, implying that debt financing with an upper limit does not always accelerate investment, even though the equity option values for a constrained levered firm are larger than those for an unlevered firm. As a result, financing constraints distort the symmetric relationship that is always obtained in a frictionless market. This result is similar to that in the Modigliani and Miller (1958) theorem, where financial frictions distort the independence between investment and capital structure that is obtained in a frictionless market. This outcome is a new result. In previous models about financing constraints, debt financing with an upper limit always accelerates investment, because the investment thresholds for a constrained levered firm are always smaller than those for an unlevered firm. See Shibata and Nishihara (2012), Shibata and Nishihara (2015a), and Shibata and Nishihara (2015b) for detail.

The reminder of the paper is organized as follows. Section 2 describes the model setup and derivation of the value functions. Section 3 provides the solution of our model. Section 4 examines the model's implications. Section 5 concludes.

2 Model

In this section, we begin with a description of the model. We then provide the value functions after investment and derive the optimal exit (default and shutdown) strategies. Finally, we formulate the financing and investment decision problem.

2.1 Setup

A firm possesses an option to invest in a single project at any time. If the investment option is exercised at time T^{i} where superscript "i" represents the investment strategy, the firm pays a fixed cost $I > 0$ at time T^{i} and receives an instantaneous cash inflow $X(t)$ after time T^{i}. Here, $X(t)$ follows the geometric Brownian motion:

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}z(t), \quad X(0) = x > 0, \tag{1}$$

where $\mu > 0$ and $\sigma > 0$ are positive constants and $z(t)$ denotes a standard Brownian motion. For convergence, we assume that $r > \mu$ where $r > 0$ is the risk-neutral interest rate. We assume that the current value $X(0) = x > 0$ is sufficiently low that equity holders do not undertake the investment option immediately.

In this paper, we assume that the firm issues a debt at the time of investment in order to finance the investment cost of $I > 0$. For analytical convenience, we limit the condition that the debt is perpetual. If the firm issues the debt, the firm obtains $X(t) - c$, where $c > 0$ is the coupon payment. When $X(t)$ is decreased after investment, it is difficult for the firm to pay $c > 0$. In that case, the firm files bankruptcy. At the same time, debt holders decide whether to operate or liquidate the corporation, depending on the collateral as an exogenous liquidation value $\theta \geq 0$. Following Leland (1994), we assume that there is a bankruptcy cost during financial distress. Thus, if the corporation is operated by debt holders (as new equity holders) who gain ownership of the corporation after default, the new owners obtain the cash inflow $(1 - \alpha)X(t)$ at time t where $\alpha X(t)$ represents a bankruptcy cost ($\alpha \in (0, 1)$). Alternatively, when the firm is liquidated at the time of default, debt holders obtain $\min\{c/r, (1 - \alpha)\theta\}$.[3] To summarize, the default ("operating concern bankruptcy") is defined by the transfer of management rights from equity holders

[3]Under these conditions, when $\theta \geq 0$ goes to zero, value functions converge to those in Sundaresan and Wang (2007) and Shibata and Nishihara (2012).

to debt holders. The shutdown ("liquidation bankruptcy") is defined by the cessation of management.

Let T^{i}, T^{d}, and T^{s} denote the investment, default ("operating concern bankruptcy"), and shutdown ("liquidation bankruptcy") times, where the superscripts "i," "d," and "s" stand for the respective investment, default, and shutdown strategies. Mathematically, these times are defined as $T^{\mathrm{i}} := \inf\{t \geq 0 | X(t) \geq x^{\mathrm{i}}\}$, $T^{\mathrm{d}} := \inf\{t \geq T^{\mathrm{i}} | X(t) \leq x^{\mathrm{d}}\}$, and $T^{\mathrm{s}} := \inf\{t \geq T^{\mathrm{d}} | X(t) \leq x^{\mathrm{s}}\}$. Here, x^{i}, x^{d}, and x^{s} denote the respective investment, default, and shutdown thresholds. Note that $0 \leq T^{\mathrm{i}} \leq T^{\mathrm{d}} \leq T^{\mathrm{s}}$ a.s., implying that the default is defined only after the investment is exercised, and that the liquidation is defined only after the firm is bankrupted.[4]

2.2 Value function after investment for an unlevered firm

This subsection provides the value functions after investment for an unlevered (all-equity financed) firm. In this case, coupon payment is zero, i.e., $c = 0$.

Now we consider any time t after the investment is exercised ($t > T_0^{\mathrm{i}}$). Here, the subscript "0" represents the unlevered firm. We denote by $E_0^{\mathrm{a}}(X(t))$ the equity value after investment for the unlevered firm, where the superscript "a" represents the value function after investment. The equity value after investment, $E_0^{\mathrm{a}}(X(t))$, is defined as

$$E_0^{\mathrm{a}}(X(t)) := \sup_{T_0^{\mathrm{s}}(\geq t)} \mathbb{E}^{X(t)} \Big[\int_t^{T_0^{\mathrm{s}}} \mathrm{e}^{-r(u-t)}(1-\tau)X(u)\mathrm{d}u + \mathrm{e}^{-r(T_0^{\mathrm{s}}-t)}(1-\alpha)\theta \Big], \tag{2}$$

where $\mathbb{E}^{X(t)}$ denotes the expectation operator conditional on $X(t)$, $\tau > 0$ represents the tax rate, and $\alpha \in (0, 1)$ represents the proportional bankruptcy cost parameter to the firm value during financial distress. Using standard arguments, $E_0^{\mathrm{a}}(X(t))$ is given by

$$E_0^{\mathrm{a}}(X(t)) = \max_{x_0^{\mathrm{s}}(\geq 0)} \Big\{ vX(t) + \Big((1-\alpha)\theta - vx_0^{\mathrm{s}}\Big) \Big(\frac{X(t)}{x_0^{\mathrm{s}}}\Big)^{\gamma} \Big\}, \tag{3}$$

where $v := (1-\tau)/(r-\mu) > 0$ and $\gamma := 1/2 - \mu/\sigma^2 - ((\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2)^{1/2} < 0$. Then, the optimal threshold for shutdown ("liquidation bankruptcy") is obtained by

$$x_0^{\mathrm{s}} = (1-\alpha)\lambda\theta \geq 0, \tag{4}$$

[4] "a.s." stands for "almost surely."

where $\lambda := \gamma/((\gamma-1)v) > 0$. Note that x_0^{s} is a linear function of θ with $\lim_{\theta\downarrow 0} x_0^{\mathrm{s}} = 0$. Clearly we have $\lim_{\theta\downarrow 0} E_0^{\mathrm{a}}(X(t)) = vX(t)$, which is the value of discounted cash inflow without shutdown.

2.3 Value functions after investment for a levered firm

In this subsection, we derive the value functions after investment for a levered (debt-equity financed) firm. In this case, coupon payment is strictly positive, i.e., $c > 0$.[5]

Because we assume a perpetual debt, $c/r > 0$ is the value for the principal of debt (i.e., face value of debt). Thus, if the following inequality

$$F^{\mathrm{a}}(c) := \frac{c}{r} \leq (1-\alpha)\theta, \tag{5}$$

is satisfied for a fixed $c > 0$, debt holders never take risks. Here, we denote the face value of debt as $F^{\mathrm{a}}(c)$ for a fixed $c > 0$. We define $\theta_2(c) > 0$ as

$$\theta_2(c) := \inf\left\{\theta \geq 0 \mid \theta \geq \frac{c}{r(1-\alpha)}\right\}, \tag{6}$$

for a fixed $c > 0$. Thus, we define risky debt and riskless debt as follows.

Definition 1 *Debt is risky if $\theta \in [0, \theta_2(c))$ for a fixed coupon payment $c > 0$. Debt is riskless otherwise.*

If $\theta \in [0, \theta_2(c))$ for a fixed $c > 0$, debt holders take a risk at the time of liquidation. Otherwise ($\theta \in [\theta_2(c), +\infty)$ for a fixed $c > 0$), and debt holders do not have any risks because there will always be sufficient collateral to meet the firm's liabilities (i.e., the value for the principal of perpetual debt). Let $g(X(t), c)$ denote the value for the levered firm after investment as follows:

$$g(X(t), c) = \begin{cases} g_1(X(t), c), & \theta \in [0, \theta_2(c)), \\ g_2(X(t), c), & \theta \in [\theta_2(c), +\infty), \end{cases}$$

for a fixed $c > 0$ where $g_k \in \{D_k^{\mathrm{a}}, E_k^{\mathrm{a}}, V_k^{\mathrm{a}}\}$ for any k ($k \in \{1, 2\}$). Here, D_k^{a}, E_k^{a}, and $V_k^{\mathrm{a}} (:= D_k^{\mathrm{a}} + E_k^{\mathrm{a}})$ denote the value functions of debt, equity, and total firm. In addition, subscripts "1" and "2" represent the value functions for "risky debt" and "riskless debt," respectively.

[5]In our model, $c = 0$ is equivalent to the fact that the debt value is zero.

2.3.1 Value functions when debt is risky

Suppose that debt is risky (i.e., $\theta \in [0, \theta_2(c))$ for a fixed $c > 0$). Consider any time $t(> T^{\mathrm{i}})$ after investment. For a fixed $c > 0$, the equity value after investment, $E_1^{\mathrm{a}}(X(t), c)$, is defined by

$$\begin{aligned} E_1^{\mathrm{a}}(X(t), c) &:= \sup_{T_1^{\mathrm{d}}(\geq t)} \mathbb{E}^{X(t)} \Big[\int_t^{T_1^{\mathrm{d}}} \mathrm{e}^{-r(u-t)} (1-\tau)(X(u) - c) \mathrm{d}u \Big] \\ &= vX(t) - \frac{1-\tau}{r} c - \Big(v x_1^{\mathrm{d}}(c) - \frac{1-\tau}{r} c \Big) \Big(\frac{X(t)}{x_1^{\mathrm{d}}(c)} \Big)^{\gamma}, \end{aligned} \tag{7}$$

where the optimal default threshold, $x_1^{\mathrm{d}}(c)$, is obtained by

$$x_1^{\mathrm{d}}(c) = \kappa^{-1} c > 0, \tag{8}$$

where $\kappa := r/(\lambda(1-\tau)) > 0$. Note that $E_1^{\mathrm{a}}(X(t), c)$ and $x_1^{\mathrm{d}}(c)$ are independent of θ. In addition, $x_1^{\mathrm{d}}(c)$ is a linear function of $c > 0$. This result is originally obtained by Black and Cox (1976). We denote by $T_1^{\mathrm{d}}(c) := \inf\{t \geq T_1^{\mathrm{i}} | X(t) \leq x_1^{\mathrm{d}}(c)\}$ the optimal stopping time for default for a fixed $c > 0$. In our model, we do not consider debt renegotiation during financial distress.[6]

We derive the value of debt backward. Let $E^{\mathrm{b}}(X(t))$ denote the value of equity after default where the superscript "b" represents the value function *after default*. For a fixed $c > 0$, the equity value after default, $E^{\mathrm{b}}(X(T_1^{\mathrm{d}}(c)))$, is defined as

$$\begin{aligned} &E^{\mathrm{b}}(X(T_1^{\mathrm{d}}(c))) \\ &:= \sup_{T_1^{\mathrm{s}}(\geq T_1^{\mathrm{d}}(c))} \mathbb{E}^{X(T_1^{\mathrm{d}}(c))} \Big[\int_{T_1^{\mathrm{d}}(c)}^{T_1^{\mathrm{s}}} \mathrm{e}^{-r(u - T_1^{\mathrm{d}}(c))} (1-\tau) X(u) \mathrm{d}u + \mathrm{e}^{-r(T_1^{\mathrm{s}} - T_1^{\mathrm{d}}(c))} \theta \Big]. \end{aligned} \tag{9}$$

Using standard arguments, we have

$$E^{\mathrm{b}}(x_1^{\mathrm{d}}(c)) = \begin{cases} v x_1^{\mathrm{d}}(c) + \Big(\theta - v x_1^{\mathrm{s}*}(\theta) \Big) \Big(\frac{x_1^{\mathrm{d}}(c)}{x_1^{\mathrm{s}}} \Big)^{\gamma}, & \text{if } \theta \in [0, \theta_1(c)), \\ \theta, & \text{if } \theta \in [\theta_1(c), \theta_2(c)), \end{cases} \tag{10}$$

where $x_1^{\mathrm{s}} := \mathrm{argmax}_{x_1^{\mathrm{s}}} \{ v x_1^{\mathrm{d}}(c) + (\theta - v x_1^{\mathrm{s}})(x_1^{\mathrm{d}}(c)/x_1^{\mathrm{s}})^{\gamma} \}$, i.e.,

$$x_1^{\mathrm{s}} = \lambda \theta \geq 0, \tag{11}$$

[6] See Mella-Barral and Perraudin (1997), Fan and Sundaresan (2000), Broadie et al. (2007), and Sundaresan and Wang (2007) regarding coupon reduction as debt renegotiation during financial distress. See Nishihara and Shibata (2016a) regarding asset sale as debt renegotiation during financial distress.

and $\theta_1(c) \geq 0$ is defined by

$$\theta_1(c) := \inf\left\{\theta \geq 0 \mid \theta \geq \frac{1-\tau}{r}c\right\} < \theta_2(c), \tag{12}$$

for a fixed $c > 0$. The inequality of $\theta_1(c) < \theta_2(c)$ for a fixed $c > 0$ is obtained by the inequalities of $1 - \tau < 1 < 1/(1-\alpha)$ because $\tau > 0$ and $\alpha \in (0, 1)$. Note that

$$\begin{aligned} x_1^{\mathrm{d}}(c) &> x_1^{\mathrm{s}}, && \text{if } \theta \in [0, \theta_1(c)), \\ x_1^{\mathrm{d}}(c) &= x_1^{\mathrm{s}}, && \text{if } \theta = \theta_1(c), \\ x_1^{\mathrm{d}}(c) &< x_1^{\mathrm{s}}, && \text{if } \theta \in (\theta_1(c), \theta_2(c)), \end{aligned}$$

for a fixed $c > 0$.

The upper equation of (10) corresponds to the value at the time of default ("operating concern bankruptcy") where debt holders will operate the firm after changing its ownership. Because $\gamma < 0$, we have $\lim_{\theta \downarrow 0} E^{\mathrm{b}}(x_1^{\mathrm{d}}(c)) = vx_1^{\mathrm{d}}(c) > 0$, which is consistent with Shibata and Nishihara (2012). Note that $(\theta - vx_1^{\mathrm{s}}) = \theta/(1-\gamma) \geq 0$. The lower equation of (10) is the value when shutdown ("liquidation bankruptcy") is exercised at the time of default. We summarize the above results as follows.

Lemma 1 *Suppose that debt is risky, i.e., $\theta \in [0, \theta_2(c))$ for a fixed coupon payment $c > 0$. Then, default ("operating concern bankruptcy") and shutdown ("liquidation bankruptcy") are exercised sequentially for $\theta \in [0, \theta_1(c))$, while default and shutdown are exercised simultaneously for $\theta \in [\theta_1(c), \theta_2(c))$.*

From Lemma 1, we understand that bankruptcy strategies (i.e., $E^{\mathrm{b}}(x_1^{\mathrm{d}}(c))$) are defined according to the magnitude of collateral θ ($\theta \in [0, \theta_2(c))$ for a fixed $c > 0$). For $\theta \in [0, \theta_1(c))$, sequential strategies imply that default and shutdown are exercised one by one, on the condition that both are exercised. To be more precise, for $\theta \in [0, \theta_1(c))$, suppose that $X(t)$ arrives at $x_1^{\mathrm{d}}(c)$. Then, debt holders with ownership transferred from equity holders continue to operate the management after default. If $X(t)$ is further decreased to a level x_1^{s} lower than $x_1^{\mathrm{d}}(c)$, shutdown is exercised. If $X(t)$ does not arrive at x_1^{s} after default, shutdown is not exercised. For $\theta \in [\theta_1(c), \theta_2(c))$, once $X(t)$ arrives at $x_1^{\mathrm{d}}(c) \leq x_1^{\mathrm{s}}$, the equity holders declare the default, and at the same time, debt holders declare the shutdown (liquidation). Thus, default and shutdown are exercised simultaneously.

We define the risky debt value $D_1^{\rm a}(X(t),c)$ as a function of $E^{\rm b}(x_1^{\rm d}(c))$. For a fixed $c>0$, the risky debt value $D_1^{\rm a}(X(t),c)$ is given by

$$\begin{aligned} D_1^{\rm a}(X(t),c) &:= \mathbb{E}^{X(t)}\Big[\int_t^{T_1^{\rm d}(c)} {\rm e}^{-r(u-t)} c{\rm d}u + {\rm e}^{-r(T_1^{\rm d}(c)-t)}(1-\alpha)E^{\rm b}(x_1^{\rm d}(c))\Big] \\ &= \frac{c}{r} - \Big(\frac{c}{r} - (1-\alpha)E^{\rm b}(x_1^{\rm d}(c))\Big)\Big(\frac{X(t)}{x_1^{\rm d}(c)}\Big)^{\gamma}. \end{aligned} \tag{13}$$

Note that we have $F^{\rm a}(c) \geq D_1^{\rm a}(X(t),c)$ for a fixed $c>0$, implying that the value of debt issuance is discounted below face value if debt is risky.

2.3.2 Value functions when debt is riskless

Suppose that debt is riskless, i.e., $\theta \in [\theta_2(c),+\infty)$ for a fixed coupon payment $c>0$. Consider any time $t \geq T^{\rm i}$ after investment. The value of riskless debt, $D_2^{\rm a}(X(t),c)$, is equal to face value of debt, i.e.,

$$D_2^{\rm a}(X(t),c) = \frac{c}{r}, \tag{14}$$

for a fixed $c>0$. Note that $D_2^{\rm a}(X(t),c)$ is independent of $\theta \geq 0$. As shown in the Appendix, we have the following result.

Lemma 2 *Suppose that debt is riskless, i.e., $\theta \in [\theta_2(c),+\infty)$ for a fixed coupon payment $c>0$. Then, default ("operating concern bankruptcy") and shutdown ("liquidation bankruptcy") are exercised simultaneously.*

Lemma 2 implies $T_2^{\rm d} = T_2^{\rm s}$ a.s. (i.e., $x_2^{\rm d} = x_2^{\rm s}$) and $E^{\rm b}(x_2^{\rm s}) = \theta$ for $\theta \in [\theta_2(c),+\infty)$ for a fixed $c>0$. Thus, the equity value after investment, $E_2^{\rm a}(X(t),c)$, is obtained by

$$\begin{aligned} E_2^{\rm a}(X(t),c) &= \sup_{T_2^{\rm s}(>t)} \mathbb{E}^{X(t)}\Big[\int_t^{T_2^{\rm s}} {\rm e}^{-r(u-t)}((1-\tau)X(u)+\tau c) + {\rm e}^{-r(T_2^{\rm s}-t)}(1-\alpha)\theta\Big] - \frac{c}{r} \\ &= vX(t) + \frac{\tau}{r}c + \Big(-vx_2^{\rm s}(c) - \frac{\tau}{r}c + (1-\alpha)\theta\Big)\Big(\frac{X(t)}{x_2^{\rm s}(c)}\Big)^{\gamma} - \frac{c}{r}, \end{aligned} \tag{15}$$

where the optimal default ("liquidation bankruptcy") threshold is obtained by

$$x_2^{\rm s}(c) = \lambda\Big((1-\alpha)\theta - \frac{\tau}{r}c\Big) \geq 0, \tag{16}$$

for a fixed $c>0$. Note that $E_2^{\rm a}(x_2^{\rm s}(c),c) = (1-\alpha)\theta - c/r \geq 0$ for a fixed $c>0$. In addition, $x_2^{\rm s}(c)$ is decreasing with $c>0$ (although $x_1^{\rm d}(c)$ is increasing with $c>0$).

2.3.3 Two value functions after investment for a levered firm

In the two previous sections, we have derived two value functions $g_1(X(t), c)$ and $g_2(X(t), c)$ for a fixed $c > 0$ ($g_k \in \{D_k^{\mathrm{a}}, E_k^{\mathrm{a}}, V_k^{\mathrm{a}}\}$ for $k \in \{1, 2\}$). This subsection shows that $g_1(X(t), c) = g_2(X(t), c)$ for $\theta = \theta_2(c)$ for a fixed $c > 0$, implying that all the value functions after investment are continuous with θ for a fixed $c > 0$.

The properties of two value functions are summarized in Table 1. Recall that for a risky debt, i.e., $\theta \in [\theta_2(c), +\infty)$, $E_1^{\mathrm{a}}(X(t), c)$ is independent of $\theta \geq 0$, while $D_1^{\mathrm{a}}(X(t), c)$ is dependent on $\theta \geq 0$. In contrast, for a riskless debt, i.e., $\theta \in [\theta_2(c), +\infty)$, $E_1^{\mathrm{a}}(X(t), c)$ is dependent on $\theta \geq 0$, while $D_1^{\mathrm{a}}(X(t), c)$ is independent of $\theta \geq 0$.

[Insert Table 1 about here]

In addition, as in Table 1, the properties of bankruptcy strategies are summarized according to $\theta \geq 0$. When $\theta \in [0, \theta_1(c))$, we have $x_1^{\mathrm{d}}(c) > x_1^{\mathrm{s}}$ for a fixed $c > 0$. This implies that default and shutdown are exercised *sequentially*. When $\theta \in [\theta_1(c), \theta_2(c))$, default and shutdown are excised *simultaneously* at $x_1^{\mathrm{d}}(c)(\leq x_1^{\mathrm{s}})$, for a fixed $c > 0$. When $\theta \in [\theta_2(c), +\infty)$, only shutdown is exercised at $x_2^{\mathrm{s}}(c)$ because there is no default for a riskless debt.

As shown in the Appendix, we have the following results.

Lemma 3 *For a fixed $c > 0$, we have*

$$x_1^{\mathrm{d}}(c) = \kappa^{-1} c = \lim_{\theta \downarrow \theta_2(c)} x_2^{\mathrm{s}}(c). \tag{17}$$

In addition, we obtain

$$E_1^{\mathrm{a}}(X(t), c) = \lim_{\theta \downarrow \theta_2(c)} E_2^{\mathrm{a}}(X(t), c), \tag{18}$$

$$\lim_{\theta \uparrow \theta_2(c)} D_1^{\mathrm{a}}(X(t), c) = D_2^{\mathrm{a}}(X(t), c), \tag{19}$$

and

$$\lim_{\theta \uparrow \theta_2(c)} V_1^{\mathrm{a}}(X(t), c) = \lim_{\theta \downarrow \theta_2(c)} V_2^{\mathrm{a}}(X(t), c), \tag{20}$$

for a fixed $c > 0$.

Lemma 3 implies that all values and thresholds are continuous with θ for a fixed $c \geq 0$. These results are similar to those in Mella-Barral and Perraudin (1997).[7]

[7] In Mella-Barral and Perraudin (1997), $\tau = 0$ and $\alpha = 0$ are assumed.

2.4 Problem formulation

In this subsection, we formulate the financing and investment decisions problem under a debt issuance constraint.

The firm faces a credit friction that limits the maximum amount of debt issuance as

$$D^{\mathrm{a}}(X(t), c) \le \min\{q\theta, I\}, \tag{21}$$

where $q \ge 0$. Here, if $q\theta = \min\{q\theta, I\}$ is satisfied, inequality (21) constitutes *collateral constraints* because the maximum amount of a firm's debt issuance is given by the value of its collateral θ. Our collateral constraints are similar to the ones in Kiyotaki and Moore (1997).[8] In addition, our collateral constraints are supported by empirical studies such as Whited (1992) and Gan (2007).[9] The parameter $q \ge 0$ represents the ceiling of debt issuance in collateral constraints. If we assume $q \ge 1$, the firm can issue debt greater than the amount of collateral $\theta \ge 0$. The condition $q < 1$ implies that the firm can issue debt less than the amount of collateral $\theta \ge 0$.[10]

In addition, note that we assume $D^{\mathrm{a}}(X(t), c) \le I$ in (21). Otherwise (i.e., $D^{\mathrm{a}}(X(t,), c) > I$), the firm issues an amount of debt larger than the amount of investment cost at the time of investment. Then, in this model, the excess $D^{\mathrm{a}}(X(t), c) - I > 0$ is distributed to equity holders. At the time of financing, the distribution of the excess financed amount is illegal in practice. Thus, we need at least the mathematical condition $D^{\mathrm{a}}(X(t), c) \le I$. See Shibata and Nishihara (2015a) for details.

Let us denote by $E_{\mathrm{C}}(x)$ the equity option value before investment when the firm has an upper limit of debt issuance under debt-equity financing, where the subscript "C" denotes the "financially constrained" firm. We formulate the financing and investment decision problem for a financially constrained firm. The equity option value $E_{\mathrm{C}}(x)$ is obtained by

$$E_{\mathrm{C}}(x) = \max\{E^{**}(x), E_0^{*}(x)\}, \tag{22}$$

[8]An incomplete list of models incorporating collateral constraints is Bianchi (2010), Jeanne and Korinek (2010), Rampini and Viswanathan (2010), and Gottardi and Kubler (2015).

[9]Whited (1992) shows that the introduction of a debt constraint greatly improves the performance of the Euler equation in optimizing a model of investment, compared to the standard specification.

[10]The condition of $q \in [0, 1]$ corresponds to the condition in the repo market where the amount of a loan is less than the market value of the collateral. This difference $1 - q$ is referred to as the "margin" or "haircut."

where

$$E^{**}(x) = \max_{x^{\mathrm{i}}(>0), c(>0)} \left(\frac{x}{x^{\mathrm{i}}}\right)^{\beta} \Big\{E^{\mathrm{a}}(x^{\mathrm{i}}, c) - (I - D^{\mathrm{a}}(x^{\mathrm{i}}, c))\Big\}, \tag{23}$$

$$\text{subject to } \; D^{\mathrm{a}}(x^{\mathrm{i}}, c) \leq \min\{q\theta, I\}, \tag{24}$$

and

$$E_0^*(x) := \max_{x_0^{\mathrm{i}}(>0)} \left(\frac{x}{x_0^{\mathrm{i}}}\right)^{\beta} \Big\{E_0^{\mathrm{a}}(x_0^{\mathrm{i}}) - I\Big\}, \tag{25}$$

for $x < \min\{x^{\mathrm{i}}, x_0^{\mathrm{i}}\}$ and $\beta := 1/2 - \mu/\sigma^2 - ((\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2)^{1/2} > 1$. Here, $E^{**}(x)$ denotes the equity option value when the firm issues debt under the upper limit of debt issuance (i.e., *constrained levered firm*), where the superscript "$**$" represents the optimum for the constrained problem, and $E_0^*(x)$ denotes the equity option value when the firm does not issue any debt (i.e., *an unlevered firm*), where the superscript "$*$" represents the optimum for the *unconstrained* problem. See the Appendix for the derivations of (23) and (25).

3 Model solution

This section provides the solution to our model. First, before solving our (constrained) problem, we briefly review the unconstrained problem where firm does not have an upper limit of debt issuance under debt-equity financing. We then provide the solution to our problem.

3.1 Unconstrained problem as a benchmark

This subsection provides the solutions to the unconstrained problem as a benchmark.

Let us denote by $E_{\mathrm{U}}(x)$ the equity option value before investment when the firm does not have an upper limit of debt issuance under debt-equity financing, where the subscript "U" denotes the "financially unconstrained" firm. We formulate the financing and investment decision problem for an unconstrained firm. The equity option value $E_{\mathrm{U}}(x)$ is obtained by

$$E_{\mathrm{U}}(x) = \max\{E^*(x), E_0^*(x)\}, \tag{26}$$

where

$$E^*(x) \quad = \quad \max_{x^{\mathrm{i}}(>0),c(>0)} \left(\frac{x}{x^{\mathrm{i}}}\right)^{\beta} \Big\{E^{\mathrm{a}}(x^{\mathrm{i}}, c) - (I - D^{\mathrm{a}}(x^{\mathrm{i}}, c))\Big\}, \tag{27}$$

and $E_0^*(x)$ is defined by (25) where $x < \min\{x_0^{\mathrm{i}}, x^{\mathrm{i}}\}$. Here, $E^*(x)$ denotes the equity option value when the firm does not issue debt under the upper limit of debt issuance (i.e., *unconstrained levered firm*). Note that the superscript "$*$" in $E^*(x)$ in (27) represents the optimum for the *unconstrained* problem.

We derive the solution in two steps. We first derive $E^*(x)$ and $E_0^*(x)$, respectively. By comparing the magnitudes of $E^*(x)$ and $E_0^*(x)$, we then have $E_{\mathrm{U}}(x) = \max\{E^*(x), E_0^*(x)\}$.

We then have the following results (these two proofs are provided in the Appendix).

Proposition 1 *Consider the investment decision problem for an unlevered firm. For any θ, the optimal investment threshold $x_0^{\mathrm{i}*}$ is obtained by solving the following equation:*

$$(\beta - 1)vx_0^{\mathrm{i}*} + \frac{\beta - \gamma}{1 - \gamma}((1 - \alpha)\theta)^{1-\gamma}\lambda^{-\gamma}{x_0^{\mathrm{i}*}}^{\gamma} - \beta I = 0. \tag{28}$$

For any θ, the equity option value before investment is given by

$$E_0^*(x) = \left(\frac{x}{x_0^{\mathrm{i}*}}\right)^{\beta} \Big\{E_0^{\mathrm{a}}(x_0^{\mathrm{i}*}) - I\Big\}. \tag{29}$$

Proposition 2 *Consider the financing and investment decision problem for an unconstrained levered firm. For $\theta \in [0, \theta_1^*)$ where there exists a unique θ_1^* that satisfies $\theta = (1 - \tau)c^*/r$, the optimal investment threshold $x^{\mathrm{i}*}$ is given by solving the following equation:*

$$(\beta - 1)v\psi^{-1}x^{\mathrm{i}*} + \frac{\beta - \gamma}{1 - \gamma}(1 - \alpha)\theta^{1-\gamma}\lambda^{-\gamma}{x^{\mathrm{i}*}}^{\gamma} - \beta I = 0, \tag{30}$$

where $\psi := (1 + \tau(h(1 - \tau))^{-1})^{-1} \leq 1$ and $h := (1 - \gamma(1 + \alpha(1 - \tau)/\tau))^{-1/\gamma} \geq 1$. The optimal coupon payment is given by $c^* = c(x^{\mathrm{i}*}) := (\kappa/h)x^{\mathrm{i}*}$. For $\theta \in [\theta_1^*, \theta_2^*)$ where there exists a unique θ_2^* that satisfies $\theta = c^*/(r(1 - \alpha))$, the solution $(x^{\mathrm{i}*}, c^*)$ is given by solving the following simultaneous equations:

$$(\beta - 1)vx^{\mathrm{i}*} + \beta\frac{\tau}{r}c^* + (\beta - \gamma)\Big\{-\frac{1}{1 - \gamma}\frac{\tau - \gamma}{r}c^* + (1 - \alpha)\theta\Big\}\kappa^{\gamma}{c^*}^{-\gamma}{x^{\mathrm{i}*}}^{\gamma} - \beta I = 0, \tag{31}$$

$$\frac{\tau}{r}c^* - \Big\{\frac{\tau - \gamma}{r}c^* + \gamma(1 - \alpha)\theta\Big\}\kappa^{\gamma}{c^*}^{-\gamma}{x^{\mathrm{i}*}}^{\gamma} = 0. \tag{32}$$

For $\theta \in [\theta_2^*, +\infty)$, we do not obtain x^* and c^*. The equity option value before investment is

$$E^*(x) = \left(\frac{x}{x^{\mathrm{i}*}}\right)^{\beta}\left\{V^{\mathrm{a}}(x^{\mathrm{i}*}, c^*) - I\right\}, \tag{33}$$

where $x \leq x^*$ for $\theta \in [0, \theta_2^*)$.

From Propositions 1 and 2, we have the following (the proof is provided in the Appendix).

Corollary 1 *Consider the investment decision problem without any financing constraints. Then, for $\theta \in [0, \theta_2^*)$, we obtain*

$$x^{\mathrm{i}*} \leq x_0^{\mathrm{i}*}, \quad E^*(x) \geq E_0^*(x), \tag{34}$$

where $x \leq x^{\mathrm{i}*}$. That is, we have $E_{\mathrm{U}}(x) = E^*(x)$.

Corollary 1 means that, if there are no financing constraints, debt financing decreases the investment threshold (accelerates the investment) and increases the equity option value before investment. We call this property the *symmetric relationship* between investment thresholds and equity option values.

In addition, we consider the extreme case of $\theta = 0$. Substituting $\theta = 0$ into (28) and (30) gives

$$x^{\mathrm{i}*} = \psi x_0^{\mathrm{i}*}, \tag{35}$$

where $x_0^{\mathrm{i}*} := \beta/((\beta-1)v)I > 0$ for $\theta = 0$. In addition, as shown in the Appendix, we have

$$E^*(x) = \psi^{-\beta} E_0^*(x), \tag{36}$$

where $E_0^*(x) := (x/x_0^{\mathrm{i}*})^{\beta}(\beta-1)^{-1}I > 0$ for $\theta = 0$. Because $\psi \leq 1$ and $\beta > 1$, these two equations (35) and (36) lead to

$$x^{\mathrm{i}*} \leq x_0^{\mathrm{i}*}, \quad E^*(x) \geq E_0^*(x), \tag{37}$$

for $\theta = 0$. Thus, in the extreme case of $\theta = 0$ in (34), we have the *symmetric relationship* between investment thresholds and equity option values. See Shibata and Nishihara (2012), Shibata and Nishihara (2015a), and Shibata and Nishihara (2015b) for discussion of such a symmetric relationship.

3.2 Constrained problem

This subsection provides the solution to the problem for a *constrained levered (debt financed)* firm. As shown in the Appendix, we obtain the following proposition.

Proposition 3 *Consider the financing and investment decision problem for the constrained levered firm. Suppose that the firm is financially constrained (i.e., the constraint is binding). For the extreme case of* $\theta = 0$*, we have the explicit solution* $(x^{\mathrm{i}**}, c^{**}) = (x_0^{\mathrm{i}*}, 0)$ *and the equity option value* $E^{**}(x) = E_0^*(x)$*. For* $\theta \in (0, \theta_2^{**})$ *where* θ_2^{**} *is given by satisfying* $\theta = c^{**}/(r(1-\tau))$*, the solution* $(x^{\mathrm{i}**}, c^{**})$ *is obtained by solving the two simultaneous equations:*

$$\frac{f_{j1}(x^{\mathrm{i}**}, c^{**})}{f_{j2}(x^{\mathrm{i}**}, c^{**})} = \frac{f_{j3}(x^{\mathrm{i}**}, c^{**})}{f_{j4}(x^{\mathrm{i}**}, c^{**})}, \qquad \min\{q\theta, I\} = D_j^{\mathrm{a}}(x^{\mathrm{i}**}, c^{**}), \tag{38}$$

where $f_{jk}(x^{\mathrm{i}}, c)$ *for* $j \in \{1, 2\}$ *and* $k \in \{1, 2, 3, 4\}$ *is given by*

$$f_{11}(x^{\mathrm{i}}, c) := (\beta - 1)vx^{\mathrm{i}} + \beta\frac{\tau}{r}c + (\beta - \gamma)\Big\{ - \Big(\frac{\tau}{r} + \alpha\frac{v}{\kappa}\Big)\kappa^{\gamma}c^{1-\gamma} + \frac{1-\alpha}{1-\gamma}\theta^{1-\gamma}\lambda^{-\gamma}\Big\}x^{\mathrm{i}\gamma} - \beta I, \tag{39}$$

$$f_{12}(x^{\mathrm{i}}, c) := \gamma\Big\{\Big(- \frac{1}{r} + (1-\alpha)\frac{v}{\kappa}\Big)\kappa^{\gamma}c^{1-\gamma} + \frac{1-\alpha}{1-\gamma}\theta^{1-\gamma}\lambda^{-\gamma}\Big\}x^{\mathrm{i}\gamma}, \tag{40}$$

$$f_{13}(x^{\mathrm{i}}, c) := -\frac{\tau}{r} + (1-\gamma)\Big(\frac{\tau}{r} + \alpha\frac{v}{\kappa}\Big)\kappa^{\gamma}c^{-\gamma}x^{\mathrm{i}\gamma}, \tag{41}$$

$$f_{14}(x^{\mathrm{i}}, c) := \frac{1}{r} + (1-\gamma)\Big(- \frac{1}{r} + (1-\alpha)\frac{v}{\kappa}\Big)\kappa^{\gamma}c^{-\gamma}x^{\mathrm{i}\gamma}, \tag{42}$$

for $\theta \in (0, \theta_1^{**})$ *where* θ_1^{**} *is given by satisfying* $\theta = (1-\tau)c^{**}/r$ *and*

$$f_{21}(x^{\mathrm{i}}, c) := (\beta - 1)vx^{\mathrm{i}} + \beta\frac{\tau}{r}c + (\beta - \gamma)\Big\{ - \frac{1}{1-\gamma}\frac{\tau - \gamma}{r}c + (1-\alpha)\theta\Big\}\kappa^{\gamma}c^{-\gamma}x^{\mathrm{i}\gamma} - \beta I, \tag{43}$$

$$f_{22}(x^{\mathrm{i}}, c) := \gamma\Big(- \frac{c}{r} + (1-\alpha)\theta\Big)\kappa^{\gamma}c^{-\gamma}x^{\mathrm{i}\gamma}, \tag{44}$$

$$f_{23}(x^{\mathrm{i}}, c) := -\frac{\tau}{r} + \Big(\frac{\tau - \gamma}{r} + \gamma(1-\alpha)\theta c^{-1}\Big)\kappa^{\gamma}c^{-\gamma}x^{\mathrm{i}\gamma}, \tag{45}$$

$$f_{24}(x^{\mathrm{i}}, c) := \frac{1}{r} + \Big(- (1-\gamma)\frac{1}{r} + \gamma(1-\alpha)\theta c^{-1}\Big)\kappa^{\gamma}c^{-\gamma}x^{\mathrm{i}\gamma}, \tag{46}$$

for $\theta \in [\theta_1^{**}, \theta_2^{**})$*. The equity option value before investment is given by*

$$E^{**}(x) = \Big(\frac{x}{x^{\mathrm{i}**}}\Big)^{\beta}\Big\{V^{\mathrm{a}}(x^{\mathrm{i}**}, c^{**}) - I\Big\}, \tag{47}$$

where $x < x^{\mathrm{i}**}$ *for* $\theta \in (0, \theta_2^{**})$*. On the other hand, suppose that the firm is not financially constrained (the constraint is not binding). The solution* (x^{**}, c^{**}) *is obtained by* (x^*, c^*) *in Proposition 2.*

4 Model implications

This section examines more important implications of our model. The two key parameters are collateral θ and financing friction q.

Section 4.1 investigates the effects of θ and q on financially constrained regions. From Sections 4.2 and 4.3, we consider the effects of θ for a fixed $q = 1$. Section 4.2 discusses the effects of θ on entry and exit strategies when a firm is financially constrained. Section 4.3 examines the effects of θ on values, leverages, credit spreads, and default probabilities. Section 4.4 considers the effects of q for a fixed $\theta > 0$. Section 4.5 shows comparative statics that account for the volatility ($\sigma > 0$), risk-free interest rate ($r > 0$) corporate tax ($\tau > 0$), and bankruptcy cost parameter ($\alpha > 0$).

To consider the properties of the solution, we consider some numerical examples. To do so, the basic parameters are assumed to be

$$r = 9\%,\ \sigma = 20\%,\ \mu = 1\%,\ I = 5,\ \tau = 15\%,\ \alpha = 40\%,\ \text{and } x = 0.2.$$

Recall that in our model the firm makes the investment once $X(t)$, starting at the sufficiently low level of $X(0) = x$, arrives at the investment threshold x^{i} from below. Following existing studies, we define the notion of delayed (accelerated) investment as follows: a larger (smaller) investment threshold implies delayed (accelerated) investment.

4.1 Financially constrained regions

In this subsection, we consider the effects of θ and q on financially constrained regions.

Figure 1 demonstrates regions where a firm is financially constrained by the collateral-based upper limit of debt financing in the space (q, θ). Here, we consider the space (q, θ) for $q \in [0.75, 1.25]$ and $\theta \in [3.5, 6.6666]$. The dotted line from $(q, \theta) = (0.75, 6.66)$ to $(q, \theta) = (1.25, 4)$ indicates the boundary of satisfying $q\theta = I$. The lower-left region to the dotted line represents the region of $q\theta \leq I$. That is, the region of $q\theta \leq I$ is defined for $\theta \in [0, 6.6666]$ when $q = 0.75$, while it is defined for $[0, 4]$ when $q = 1.25$. The upper-right region to the dotted line of $q\theta = I$ is denoted by Region C where we have $q\theta > I$. Note that we do not consider Region C.

[Insert Figure 1 about here]

The solid line from $(q, \theta) = (0.89, 5.59)$ to $(q, \theta) = (1.25, 3.68)$ indicates the boundary of satisfying $D^{\mathrm{a}}(x^{\mathrm{i}*}, c^*) = q\theta$. Note that $D^{\mathrm{a}}(x^{\mathrm{i}*}, c^*)$ is monotonically increasing with θ. Thus, the lower-left region of the lines from $(q, \theta) = (0.75, 6.66)$ to $(q, \theta) = (1.25, 3.68)$ via $(q, \theta) = (0.89, 5.59)$ is denoted by Region A, which corresponds to the constrained region where $D^{\mathrm{a}}(x^{\mathrm{i}*}, c^*) > q\theta$. The region between the two lines is denoted by Region B, which corresponds to the unconstrained region where $D^{\mathrm{a}}(x^{\mathrm{i}*}, c^*) < q\theta$. Thus, we have the following result.

Observation 1 *Whether the collateral-based upper limit of debt financing is binding depends on two key parameters: q (financing friction) and θ (collateral). For a larger q and a larger θ, the firm is not likely to be financially constrained.*

Observation 1 implies that the firm is likely to be financially constrained for a smaller q or a smaller θ. This result is supported by empirical studies such as Almeida and Campello (2007) and Gan (2007).[11]

4.2 Effect of collateral on entry and exit strategies

In this subsection, we consider the effects of θ. Here, we assume $q = 1$ and $\theta \in [0, 5]$, implying that $q\theta \leq I = 5$. Then, we consider two extreme cases: $\theta = 0$ and $\theta \geq 4.75$. For $\theta = 0$, the firm cannot issue debt, implying that $x^{\mathrm{i}**} = x_0^{\mathrm{i}*}$, $c^{**} = 0$, and $E^{**}(x) = E_0^*(x)$ where $x < x^{\mathrm{i}**}$. For $\theta \geq 4.75$, the firm is not financially constrained, implying that $x^{\mathrm{i}**} = x^{\mathrm{i}*}$, $c^{**} = c^*$, and $E^{**}(x) = E^*(x)$ where $x < x^{\mathrm{i}*}$.

Additionally, there exists $\theta_1^* := \theta_1(c^*) = 4.1822$ because $((1-\tau)/r)c^* = \theta$ for $\theta = 4.1822$. By contrast, there does not exist $\theta_1^{**} := \theta_1(c^{**})$ because $((1-\tau)/r)c^{**} < \theta$ for any $\theta \in [0, 5]$. Moreover, there does not exist $\theta_2^j := \theta_2(c^j)$ because $c^j/(r(1-\alpha)) > \theta$ for any $\theta \in [0, 5]$ $(j \in \{*, **\})$.

Recall that $x^{\mathrm{i}*} \leq x_0^{\mathrm{i}*}$, $c^* \geq 0$, and $E^*(x) \geq E_0^*(x)$ for any θ and $x < x^{\mathrm{i}*}$. Thus, we intuitively conjecture the following inequalities:

$$x^{\mathrm{i}**} \in [x^{\mathrm{i}*}, x_0^{\mathrm{i}*}], \quad c^{**} \in [0, c^*], \quad E^{**}(x) \in [E_0^*(x), E^*(x)], \tag{48}$$

[11] Gan (2007) shows that firms with greater collateral losses are likely to obtain a smaller amount of bank credit.

for any θ and $x < \min\{x^{i*}, x^{i**}\}$. These three inequalities in (49) mean that financing constraints delay the investment, reduce the coupon payment, and decrease the equity option value before investment. However, contrary to our intuition, we show that the first inequality is not always correct, although the second and third inequalities are always correct.

[Insert Figure 2 about here]

The top-left panel of Figure 2 depicts x^{ij} (investment thresholds) with θ ($j \in \{*, **\}$). The investment thresholds x^{i**} (x^{i*} and x_0^{i*}) are decreasing with θ. As already described, we have $x^{i**} = x_0^{i*}$ for $\theta = 0$ and $x^{i**} = x^{i*}$ for $\theta \geq 4.75$. For $\theta \in (0, 4.75)$, our intuitive conjecture is $x^{i**} \in [x^{i*}, x_0^{i*}]$. Interestingly, our conjecture is not always correct. To be more precise, for $\theta \in (2.1881, 4.75)$, we have $x^{i**} \notin [x^{i*}, x_0^{i*}]$. Note, however, that we always obtain $x^{i**} \leq x_0^{i*}$.

The top-right panel demonstrates $((1-\tau)/r)c^j$ with θ where $((1-\tau)/r) > 0$ ($j \in \{*, **\}$). Recall that $\theta_1(c^*) = 4.1822$ exists. The unconstrained coupon payment c^* is decreasing with θ for $\theta \in [0, 4.1822)$, while c^* is increasing with θ for $\theta \in [4.1822, 5]$. By contrast, because $\theta_1(c^{**})$ does not exist, the constrained coupon payment c^{**} is increasing with θ. In addition, we have $c^{**} \in [0, c^*]$ for any θ, which confirms our intuition.

The middle-left panel shows $E^j(x)$ (equity option values before investment) with θ ($j \in \{*, **\}$). $E^{**}(x)$ is monotonically increasing with θ and that $E^{**}(x) \in [E_0^*(x), E^*(x)]$ with $\lim_{\theta \downarrow 0} E^{**}(x) = E_0^*(x)$ and $\lim_{\theta \uparrow \hat{\theta}} E^{**}(x) = E^*(x)$.

These three panels suggest the following results.

Observation 2 *Suppose that a firm is financially constrained by the collateral-based upper limit of debt financing. In such a case, financing constraints do not always delay investment, although financing constraints decrease the coupon payments and equity option values before investment.*

Observation 2 implies that

$$x^{i**} \notin [x^{i*}, x_0^{i*}], \quad c^{**} \in [0, c^*], \quad E^{**}(x) \in [E_0^*(x), E^*(x)],$$

Recall that we have two control variables at the time of investment: x^i and c. As a result, it is less costly to distinguish x^{i**} from x^{i*} than to distinguish c^{**} from c^*. These mechanism are similar to those in previous papers such as Shibata and Nishihara (2012), Shibata and Nishihara (2015a), and Shibata and Nishihara (2015b).

The middle-right panel demonstrates $x^{\mathrm{d}j}$ and $x^{\mathrm{s}j}$ (default and shutdown thresholds) with θ ($j \in \{*, **\}$). We confirm that the unconstrained firm exercises default and shutdown sequentially if $\theta \in [0, 4.1822)$, while it exercises them simultaneously otherwise. To be more precise, if $\theta \in [0, 4.1822)$, the unconstrained firm defaults once $X(t)$ arrives from the above at $x^{\mathrm{d}*}$. Then, corporate ownership is transferred from equity holders to debt holders. After $X(t)$ is decreased further and reaches $x^{\mathrm{s}*} (< x^{\mathrm{d}*})$, the firm is liquidated. Otherwise (if $\theta \geq 4.1822$), the unconstrained firm is liquidated once $X(t)$ declines to $x^{\mathrm{d}*}$, not $x^{\mathrm{s}*}$. By contrast, the constrained firm exercises default and shutdown simultaneously when it declines to $x^{\mathrm{d}**}$ (not $x^{\mathrm{s}**}$) for any $\theta \in [0, 5]$. Thus, we have the following result.

Observation 3 *Suppose that a firm is financially constrained by the collateral-based upper limit of debt financing. Consequently, financing constraints may change default and shutdown strategies from sequential to simultaneous.*

Observation 3 means that financing constraints affect corporate exit strategies via the change of capital structure. This result is similar to that of Nishihara and Shibata (2016b), where asymmetric information changes corporate exit strategies.

The bottom panel depicts the agency cost $ac(x)$ with θ. The agency cost $ac(x)$ is defined by

$$ac(x) := \frac{E^*(x) - E^{**}(x)}{E^*(x)} \geq 0. \tag{49}$$

The agency cost $ac(x)$ is the loss of equity option value caused by debt financing constraints. We see that $ac(x)$ is decreasing with θ. That is, the larger the collateral, the less the financing constraint.

Observation 4 *Suppose that a firm is financially constrained by the collateral-based upper limit of debt financing. In such a case, an increase in collateral decreases agency cost.*

Observation 4 means that firms with less collateral are likely to have larger agency costs than firms with more collateral. This result corresponds to empirical studies such as Almeida and Campello (2007) and Gan (2007).

4.3 Effects of collateral on values, leverage, credit spread, and default probability

This subsection examines the effect of θ on debt, equity, total firm values, leverage, credit spreads, and default probabilities.

The top-left panel depicts $D^{\mathrm{a}j} := D^{\mathrm{a}}(x^{\mathrm{i}j}, c^j)$ (debt values) and $F^{\mathrm{a}j} := F^{\mathrm{a}}(c^j)$ (face values of debt) with θ ($j \in \{*, **\}$). We identify the following four properties. First, we have $D^{\mathrm{a}**} \leq D^{\mathrm{a}*}$ and $F^{\mathrm{a}**} \leq F^{\mathrm{a}*}$ for any $\theta \in [0, 5]$. These inequalities imply that financing constraints decrease debt values and face values. Second, we have $D^{\mathrm{a}j} \leq F^{\mathrm{a}j}$ for any j ($j \in \{*, **\}$) because debt is risky for any $\theta \in [0, 5]$ (i.e., see Equation (13)). Third, debt and face values of the unconstrained levered firm, $D^{\mathrm{a}*}$ and $F^{\mathrm{a}*}$, are decreasing with $\theta \in [0, 4.1822)$, while they are increasing with $\theta \in [4.1822, 5]$. By contrast, debt and face values of the constrained levered firm, $D^{\mathrm{a}**}$ and $F^{\mathrm{a}**}$, are increasing with $\theta \in [0, 5]$. Finally, debt discounts for a constrained levered firm, $F^{\mathrm{a}**} - D^{\mathrm{a}**} > 0$, are smaller than those for an unconstrained levered firm, $F^{\mathrm{a}*} - D^{\mathrm{a}*} > 0$. This implies that financing constraints decrease the discount value of debt.

[Insert Figure 3 about here]

The top-right panel demonstrates $E^{\mathrm{a}j} := E^{\mathrm{a}}(x^{\mathrm{i}j}, c^j)$ (equity values) with θ ($j \in \{*, **\}$). Importantly, we have $E^{\mathrm{a}**} \geq E^{\mathrm{a}*}$ for any $\theta \in [0, 5]$. The inequality of $E^{\mathrm{a}**} \geq E^{\mathrm{a}*}$ is contrary to that of $D^{\mathrm{a}**} \leq D^{\mathrm{a}*}$. Equity values for a constrained levered firm are larger than those for an unconstrained levered firm because the debt issuance for a constrained levered firm is limited.

The middle-left panel displays $V^{\mathrm{a}j} := V^{\mathrm{a}}(x^{\mathrm{i}j}, c^j)$ (total firm values) with θ ($j \in \{*, **\}$). We clearly have $V^{\mathrm{a}**} \leq V^{\mathrm{a}*}$ for any $\theta \in [0, 5]$. On the one hand, $V^{\mathrm{a}*}$ is monotonically decreasing with $\theta \in [0, 5]$. On the other hand, $V^{\mathrm{a}**}$ has a U-shaped curve with $\theta \in [0, 4.75)$.

The top-left, top-right, and bottom-left panels give the following results.

Observation 5 *Suppose that a firm is financially constrained by the collateral-based upper limit of debt financing. In such a case, values after investment have the following three results. First, the constrained debt values are monotonic with the collateral, while the unconstrained debt values are non-monotonic with the collateral. Second, the constrained and* *unconstrained equity values are monotonic with the collateral. Finally, the constrained total firm values are not always monotonic with the collateral, while the unconstrained total firm values are monotonic with the collateral.*

The middle-right panel shows leverages with θ, where the leverage is defined by

$$l^j := \frac{D^{\mathrm{a}}(x^{\mathrm{i}j}, c^j)}{V^{\mathrm{a}}(x^{\mathrm{i}j}, c^j)} \in [0, 1], \tag{50}$$

for any j ($j \in \{*, **\}$). We see that l^j is increasing with θ for any j. In addition, we have $l^{**} \leq l^*$ for any $\theta \in [0, 5]$. This implies that financing constraints reduce leverages.

The bottom-left panel depicts credit spreads with θ, where the credit spread is defined by

$$cs^j := \frac{c^j}{D^{\mathrm{a}}(x^{\mathrm{i}j}, c^j)} - r \geq 0, \tag{51}$$

for any j ($j \in \{*, **\}$). We obtain $cs^{**} \leq cs^*$ for any $\theta \in [0, 5]$. This leads to the finding that the upper limit of debt issuance decreases the credit spreads. Interestingly, the unconstrained credit spreads, cs^*, are decreasing with θ, while the constrained credit spreads, cs^{**}, are increasing with θ.

The bottom-right panel shows default probabilities with θ, where the default probability is defined by

$$p^j := \left(\frac{x^{\mathrm{i}j}}{x^{\mathrm{d}j}}\right)^{\gamma} \in [0, 1], \tag{52}$$

for any j ($j \in \{*, **\}$). We obtain $p^{**} \leq p^*$ for any $\theta \in [0, 5]$. This means that the upper limit of debt issuance decreases the default probabilities. On the one hand, the unconstrained default probabilities, p^*, are constant with θ for $\theta \leq \theta_1(c^*) = 4.1822$, although they are increasing with θ for $\theta > \theta_1(c^*) = 4.1822$. On the other hand, the constrained credit spreads, p^{**}, are monotonically increasing with θ ($\theta \in [0, 5]$).

The middle-right, bottom-left, and bottom-right panels give the following results.

Observation 6 *Suppose that a firm is financially constrained by the collateral-based upper limit of debt financing. In such a case, financing constraints decrease leverages, credit spreads, and default probabilities.*

Thus, the more severe the debt issuance bounds, the lower are the leverages, credit spreads, and default probabilities. These relationships are the same as those traditionally suggested in the literature, and they also match the results of Ericsson and Renault (2006) and Gomes and Schmid (2010).

4.4 Effect of financing frictions

In this subsection, we investigate the effect of financial frictions. Although we assume $q = 1$ in the previous subsection, we now assume $q = 0.75$ and $q = 1.25$.

[Insert Figure 4 about here]

In the two upper panels of Figure 4, suppose that $q = 0.75$. This assumption implies that the upper limit of debt issuance is restricted by three-fourths of collateral value. Then, if $\theta \in [0, 6.6666]$ for $I = 5$, we have $q\theta \leq I$. In the two lower panels, suppose that $q = 1.25$, which means that the upper limit of debt issuance is constrained by five-fourths of collateral value. Then, if $\theta \in [0, 4]$ for $I = 5$, we have $q\theta \leq I$. The two left and two right panels depict $x^{\mathrm{i}j}$ and c^j with θ, respectively ($j \in \{*, **\}$). From these four panels, inequalities of $x^{\mathrm{i}**} \in [x^{\mathrm{i}*}, x_0^{\mathrm{i}*}]$ are not always correct, although inequalities of $c^{**} \in [0, c^*]$ are always correct. Interestingly, in the lower-left panel, $x^{\mathrm{i}**}$ has a U-shaped curve with θ. In the upper-right panel, the constrained firm exercises the default and shutdown simultaneously for any θ ($\theta \in [0, 6.6666]$). In the lower-right panel, the constrained firm exercises the default and shutdown sequentially for any θ ($\theta \in [0, 4]$).

[Insert Figure 5 about here]

In the two panels of Figure 5, we assume $\theta = 5$ and $I = 5$. If $q \in [0, 1]$, we have $q\theta \leq I$. Accordingly, the firm is constrained by the upper limit of debt issuance for $q \in [0, 0.9703)$ while it is not so constrained for $q \in [0.9703, 1]$. The left panel displays $x^{\mathrm{i}j}$ (investment thresholds) with $q \in [0, 1]$ where we have $q\theta \leq I$. Interestingly, $x^{\mathrm{i}**}$ has a U-shaped curve with θ. This result is similar to the results of theoretical studies by Boyle and Guthrie (2003), Shibata and Nishihara (2012), Shibata and Nishihara (2015a), and Shibata and Nishihara (2015b), and to an empirical study by Cleary et al. (2007). Thus, the property of $x^{\mathrm{i}**} \in [x^{\mathrm{i}*}, x_0^{\mathrm{i}*}]$ is not always correct. The right panel demonstrates $E^j(x)$

(equity values) with $q \in [0, 1]$ ($j \in \{*, **\}$). Clearly, $E^{**}(x)$ is monotonically increasing with q.

Recall that, if there are no financing constraints, we obtain a symmetric relationship in (34), where debt financing decreases the investment threshold (accelerates the investment) and increases the equity option value before investment. Interestingly, if there are financing constraints, such a symmetric relationship is *not always* obtained. In the two panels of Figure 5, we see

$$x^{\mathrm{i}**} > x_0^{\mathrm{i}*}, \quad E^{**}(x) > E_0^*(x),$$

for $\theta \in (0.0848, 0.1435)$. These two inequalities lead to the following results.

Observation 7 *Suppose that a firm is financially constrained by the collateral-based upper limit of debt financing. In such a case, debt financing does not always accelerate investment, compared with all-equity financing, although the firm prefers debt financing even under the upper limit constraints.*

Observation 3 implies that financial frictions distort the symmetric relationship that is always obtained in a frictionless market. This result is similar to that in the Modigliani and Miller (1958) theorem, where financial frictions distort the independence between investment and capital structure that is obtained in a frictionless market.

4.5 Comparative statics

In this subsection, we consider the effects of σ (cash-flow volatility), r (risk-free rate), τ (corporate tax), and α (bankruptcy cost) on $x^{\mathrm{i}j}$ (investment thresholds), cs^j (credit spreads), and $ac(x)$ (agency costs) where $x \leq x^{\mathrm{i}j}$ for any j ($j \in \{*, **\}$). In this subsection, we assume that $q = 1.25$ and $\theta = 3.725$. The other parameters are the same as in the previous subsection.

[Insert Figure 6 about here]

The three left panels of Figure 6 display the effects of σ (cash-flow volatility) on $x^{\mathrm{i}j}$, cs^j, and $ac(x)$, respectively ($j \in \{*, **\}$). The top-left panel shows $x^{\mathrm{i}j}$ with σ. To show clearly whether the firm is financially constrained, we define x^{p} as x satisfying

$$D^{\mathrm{a}}(x, c(x)) = q\theta, \tag{53}$$

where $c(x) := (\kappa/h)x$. Then, if $x^{i*} > x^{p}$, the firm is financially constrained. Otherwise, it is not. We obtain an interesting result. The firm is financially constrained for $\sigma \in [0.1, 0.118]$ or $\sigma \in [0.247, 0.3]$, while the firm is not constrained for $\sigma \in (0.118, 0.247)$. In addition, x^{ij} is monotonically increasing with σ. This result is the same as in Dixit and Pindyck (1994). The middle-left panel displays cs^{j} with σ. We see that cs^{j} is increasing with σ. That is, an increase in σ increases the risk of bankruptcy, implying that debt holders increase the rate of return of debt. These results are consistent with the empirical findings of Collin-Dufresne et al. (2001) and Ericsson and Renault (2006). The bottom-left panel illustrates $ac(x)$ with σ. We see that $ac(x)$ is decreasing with σ for $\sigma \in [0.1, 0.118]$, while $ac(x)$ is increasing with σ for $\sigma \in [0.247, 0.3]$. The next observation characterizes the properties of volatility on the corporate financial constraints.

Observation 8 *A firm is likely to be financially constrained when the value of volatility is small or large. Agency costs of financing constraints are not monotonic with volatility.*

Observation 8 is the same as the result in Shibata and Nishihara (2012).

The three right panels of Figure 6 illustrate the effects of r (risk-free interest rate). The top-right panel depicts x^{ij}. We have $x^{i*} > x^{p}$ if $r \in [0.07, 0.0827]$, while $x^{i*} \leq x^{p}$ otherwise. Thus, the firm is financially constrained for $r \in [0.07, 0.0827]$ while the firm is not constrained for $r \in (0.0827, 0.09]$. This is because an increase in r decreases debt value. Thus, when r is increasing, the firm is not likely to be financially constrained. The middle-right panel displays cs^{j}. We see that cs^{j} is monotonically increasing with r. The bottom-right panel shows $ac(x)$. We find that $ac(x)$ is decreasing with r for $r \in [0.07, 0.0827]$. The three right panels of Figure 6 gives the following properties of the risk-free interest rate on the corporate financing constraints.

Observation 9 *A firm tends to be financially constrained when the value of risk-free interest rate is small.*

[Insert Figure 7 about here]

The three left panels of Figure 7 show the effects of τ (corporate tax). The top-left panel depicts x^{ij}. For $\tau \in [0.1, 0.155]$, the firm is not financially constrained because of $x^{i*} \leq x^{p}$. For $\tau \in (0.155, 0.2]$, the firm is constrained because $x^{i*} > x^{p}$. This result is intuitive as follows. An increase in τ leads to an increase in the benefit of tax shields. Thus, when τ is large, the firm is likely to be constrained because the firm would prefer to issue a large amount of debt. The middle-left panel demonstrates cs^j. For $\tau \in [0.1, 0.155]$, cs^* and cs^{**} are increasing with τ. By contrast, For $\tau \in (0.155, 0.2]$, cs^* is increasing with τ, while cs^{**} is constant with τ. The bottom-left panel shows $ac(x)$. We see that $ac(x)$ is increasing with τ for $\tau \in (0.155, 0.2]$. The next observation characterizes the effect of the corporate tax on corporate financing constraints.

Observation 10 *A firm is likely to be financially constrained when the value of corporate tax is large.*

The three right panels of Figure 7 depict the effects of α. The top-right panel demonstrates $x^{\mathrm{i}j}$. We have $x^{\mathrm{i}*} > x^{\mathrm{p}}$ for $\tau \in [0.35, 0.3882)$, while $x^{\mathrm{i}*} \leq x^{\mathrm{p}}$ for $\tau \in [0.382, 0.45]$. Thus, the firm is financially constrained for $\alpha \in [0.35, 0.382)$ while the firm is not constrained for $\alpha \in [0.382, 0.45]$. The reason for this is that a smaller α induces a smaller cost of bankruptcy. Thus, when α is small, the firm is likely to be constrained because the firm would like to issue a large amount of debt. The middle-right panel shows cs^j. Interestingly, cs^* is decreasing with α for $\alpha \in [0.35, 0.45]$. On the other hand, cs^{**} is increasing with α for $\alpha \in [0.35, 0.45]$, while cs^{**} is decreasing with α for $\alpha \in [0.382, 0.45]$. The bottom-right panel demonstrates $ac(x)$. We see that ac is decreasing with α for $\alpha \in [0.35, 0.382)$. The three right panels of Figure 7 provide the properties of the bankruptcy cost on the corporate financial constraints.

Observation 11 *A firm tends to be financially constrained when the value of the corporate tax is small.*

5 Concluding remarks

We have investigated the optimal financing and investment timing decisions problem of a firm constrained by an upper limit of debt financing, depending on the collateral. The originality of this paper is its incorporation of collateral-based financing constraints in the investment timing decision problem. We show the effects of collateral-based debt financing constraints on financing and investment decisions.

We provide five important results. First, collateral-based financing constraints do not always delay investment. Second, collateral-based financing constraints may change exit (default and shutdown) strategies from sequential to simultaneous. Third, collateral-based financing constraints decrease leverages, credit spreads, and default probabilities. Fourth, the agency costs due to collateral-based financing constraints are not monotonic with volatility. Fifth, debt financing with upper limit constraints does not always accelerate investment, compared with all-equity financing, even when the firm prefers debt financing with upper limit constraints to all-equity financing.

Appendix

Proof of Lemma 2

To show the proof, we assume that default ("operating concern bankruptcy") and shutdown ("liquidation bankruptcy") will be exercised sequentially, i.e., $T_2^{\mathrm{d}} < T_2^{\mathrm{s}}$ a.s. Then, for a fixed $c \geq 0$, the equity value after investment, $E_2^{\mathrm{a}}(X(t), c)$, is defined by

$$\begin{aligned} E_2^{\mathrm{a}}(X(t), c) &= \sup_{T_2^{\mathrm{d}}(\geq t), T_2^{\mathrm{s}}(\geq T_2^{\mathrm{d}})} \Big\{ \mathbb{E}^{X(t)} \Big[\int_t^{T_2^{\mathrm{d}}} \mathrm{e}^{-r(u-t)}((1-\tau)X(u) + \tau c)\mathrm{d}t \Big] \\ &\quad + \mathbb{E}^{X(t)} \Big[\mathrm{e}^{-r(T_2^{\mathrm{d}}-t)}(1-\alpha)E^{\mathrm{b}}(X(T_2^{\mathrm{d}})) \Big] - \frac{c}{r} \Big\}, \qquad \text{(A.1)} \\ &= \max_{x_2^{\mathrm{d}}(\geq 0), x_2^{\mathrm{s}}(\leq x_2^{\mathrm{d}})} \Big\{ vX(t) + \frac{\tau}{r}c + \Big(-\alpha v x_2^{\mathrm{d}} - \frac{\tau}{r}c \Big) \Big(\frac{X(t)}{x_2^{\mathrm{d}}} \Big)^{\gamma} \\ &\quad + (1-\alpha)(\theta - v x_2^{\mathrm{s}}) \Big(\frac{X(t)}{x_2^{\mathrm{s}}} \Big)^{\gamma} - \frac{c}{r} \Big\}. \qquad \text{(A.2)} \end{aligned}$$

The optimal default threshold is given by

$$x_2^{\mathrm{d}}(c) = \frac{-\gamma}{\gamma - 1} \frac{\tau}{\alpha v r} c \leq 0, \qquad \text{(A.3)}$$

for a fixed $c \geq 0$. Because the realized value of $X(t)$ is always nonnegative, we do not define the default threshold as (A.3), on the condition that default and shutdown are never exercised sequentially. This is a contradiction. As a result, we have $T_2^{\mathrm{d}} = T_2^{\mathrm{s}}$ a.s. (i.e., $x_2^{\mathrm{d}} = x_2^{\mathrm{s}}$), implying that default and shutdown are exercised simultaneously when debt is riskless ($\theta \in [\theta_2(c), +\infty)$ for a fixed $c \geq 0$).

Proof of Lemma 3

First, recall that $x_1^{\mathrm{d}}(c)$ in (8) is constant with $\theta \geq 0$ for $\theta \in [0, \theta_2(c))$, while $x_2^{\mathrm{s}}(c)$ in (16) is increasing with $\theta \geq 0$ for $\theta \in [\theta_2(c), +\infty)$. In the case of $\theta = \theta_2(c)$ for a fixed $c \geq 0$, we obtain (17).

Next, we consider the values after investment. Suppose that debt is risky, i.e., $\theta \in [0, \theta_2(c))$ for a fixed $c \geq 0$. Substituting (8) into (7) and (13) gives

$$E_1^{\mathrm{a}}(X(t), c) = vX(t) - \frac{1-\tau}{r}c + \frac{1}{1-\gamma}\frac{1-\tau}{r}\kappa^{\gamma}c^{1-\gamma}X(t)^{\gamma}, \tag{A.4}$$

$$D_1^{\mathrm{a}}(X(t), c) = \begin{cases} \dfrac{c}{r} + \Big\{\Big(-\dfrac{1}{r} + (1-\alpha)\dfrac{v}{\kappa}\Big)\kappa^{\gamma}c^{1-\gamma} + \dfrac{1-\alpha}{1-\gamma}\lambda^{-\gamma}\theta^{1-\gamma}\Big\}X(t)^{\gamma}, & \theta \in [0, \theta_1(c)), \\ \dfrac{c}{r} + \Big(-\dfrac{c}{r} + (1-\alpha)\theta\Big)\kappa^{\gamma}c^{-\gamma}X(t)^{\gamma}, & \theta \in [\theta_1(c), \theta_2(c)). \end{cases} \tag{A.5}$$

The sum of these two values are

$$V_1^{\mathrm{a}}(X(t), c) = \begin{cases} vX(t) + \dfrac{\tau}{r}c + \Big\{-\Big(\dfrac{\tau}{r} + \alpha\dfrac{v}{\kappa}\Big)\kappa^{\gamma}c^{1-\gamma} + \dfrac{1-\alpha}{1-\gamma}\lambda^{-\gamma}\theta^{1-\gamma}\Big\}X(t)^{\gamma}, & \theta \in [0, \theta_1(c)), \\ vX(t) + \dfrac{\tau}{r}c + \Big(-\dfrac{\tau-\gamma}{1-\gamma}\dfrac{1}{r}c + (1-\alpha)\theta\Big)\kappa^{\gamma}c^{-\gamma}X(t)^{\gamma}, & \theta \in [\theta_1(c), \theta_2(c)). \end{cases} \tag{A.6}$$

In the extreme case, $\lim_{\theta \downarrow 0} D_1^{\mathrm{a}}(X(t), c)$ and $\lim_{\theta \downarrow 0} V_1^{\mathrm{a}}(X(t), c)$ are the same as those in Shibata and Nishihara (2012), Shibata and Nishihara (2015a), and Shibata and Nishihara (2015b).

Suppose that debt is riskless, i.e., $\theta \in [\theta_2(c), +\infty)$ for a fixed $c \geq 0$. Substituting (16) into (15) gives

$$E_2^{\mathrm{a}}(X(t), c) = vX(t) - \frac{1-\tau}{r}c + \frac{1}{1-\gamma}\Big(-\frac{\tau}{r}c + (1-\alpha)\theta\Big)^{1-\gamma}\lambda^{-\gamma}X(t)^{\gamma}. \tag{A.7}$$

The debt and the total firm value are

$$D_2^{\mathrm{a}}(X(t), c) = \frac{c}{r}, \tag{A.8}$$

$$V_2^{\mathrm{a}}(X(t), c) = vX(t) + \frac{\tau}{r}c + \frac{1}{1-\gamma}\Big(-\frac{\tau}{r}c + (1-\alpha)\theta\Big)^{1-\gamma}\lambda^{-\gamma}X(t)^{\gamma}. \tag{A.9}$$

By using six equations, (A.4) to (A.9), in the case of $\theta = \theta_2(c)$ for a fixed $c \geq 0$, we obtain (18), (19), and (20).

Derivation of Equations (23) and (25)

The equity option value for the constrained debt-equity financed firm, $E^{**}(x;\theta)$, is defined by

$$E^{**}(x) := \sup_{T^{\mathrm{i}}(>0),c(>0)} \mathbb{E}^x\left[\mathrm{e}^{-rT^{\mathrm{i}}}\left\{E^{\mathrm{a}}(X(T^{\mathrm{i}}),c) - \Big(I - D^{\mathrm{a}}(X(T^{\mathrm{i}}),c)\Big)\right\}\right]. \tag{A.10}$$

By the strong Markovian property, (A.10) is given by

$$E^{**}(x) := \sup_{T^{\mathrm{i}}(>0),c(>0)} \mathbb{E}^x[\mathrm{e}^{-rT^{\mathrm{i}}}]\left\{E^{\mathrm{a}}(X(T^{\mathrm{i}}),c) - \Big(I - D^{\mathrm{a}}(X(T^{\mathrm{i}}),c)\Big)\right\}. \tag{A.11}$$

Using standard arguments, we have $\mathbb{E}^x[\mathrm{e}^{-rT^{\mathrm{i}}}] = (x/x^{\mathrm{i}})^{\beta}$ where $\beta := 1/2 - \mu/\sigma^2 - ((\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2)^{1/2} > 1$ and $x < x^{\mathrm{i}} = X(T^{\mathrm{i}})$. Thus, we obtain (23).

The equity option value for the all-equity financed firm, $E_0^*(x;\theta)$, is given by

$$E_0^*(x) := \sup_{T_0^{\mathrm{i}}(>0)} \mathbb{E}^x\left[\mathrm{e}^{-rT_0^{\mathrm{i}}}\left\{E_0^{\mathrm{a}}(X(T_0^{\mathrm{i}})) - I\right\}\right]. \tag{A.12}$$

As as in (23), we have (25).

Proof of Proposition 1

Substituting (4) into (3) gives

$$E_0^{\mathrm{a}}(X(t)) = vX(t) + \frac{1}{1-\gamma}(1-\alpha)^{1-\gamma}\lambda^{-\gamma}\theta^{1-\gamma}X(t)^{\gamma}. \tag{A.13}$$

Note that $E_0^{\mathrm{a}}(X(t);\theta)$ in (3) is defined by (A.13). Differentiating (25) with x_0^{i} yields

$$\left(\frac{x}{x_0^{\mathrm{i}}}\right)^{\beta}\left[(-\beta){x_0^{\mathrm{i}}}^{-1}\left\{E_0^{\mathrm{a}}(x_0^{\mathrm{i}}) - I\right\} + v + \gamma\frac{1}{1-\gamma}(1-\alpha)^{1-\gamma}\lambda^{-\gamma}\theta^{1-\gamma}{x_0^{\mathrm{i}}}^{\gamma-1}\right] = 0. \tag{A.14}$$

By arranging (A.14), we obtain (28).

Proof of Proposition 2

First, suppose that $\theta \in [0, \theta_1(c))$ for a fixed $c \geq 0$. Then, $V^{\mathrm{a}}(x^{\mathrm{i}}, c) := E^{\mathrm{a}}(x^{\mathrm{i}}, c) + D^{\mathrm{a}}(x^{\mathrm{i}}, c)$ in (27) is defined by the upper equation of (A.6). Differentiating $V_1^{\mathrm{a}}(x^{\mathrm{i}}, c)$ with c gives

$$\frac{\tau}{r} - \left(\frac{\tau}{r} + \alpha\frac{v}{\kappa}\right)(1-\gamma)\kappa^{\gamma}c^{-\gamma}{x^{\mathrm{i}}}^{\gamma} = 0. \tag{A.15}$$

Rearranging (A.15) gives $c(x^{\mathrm{i}}) = (\kappa/h)x^{\mathrm{i}}$. By substituting $c(x^{\mathrm{i}})$ into $V_1^{\mathrm{a}}(x^{\mathrm{i}}, c)$ and differentiating (27) with x^{i}, we have

$$(\beta - 1)\Big\{v + \Big(\frac{\tau}{r} - \Big(\frac{\tau}{r} + \alpha\frac{v}{\kappa}\Big)h^{\gamma}\Big)\frac{\kappa}{h}\Big\}x^{\mathrm{i}*} + (\beta - \gamma)\frac{1-\alpha}{1-\gamma}\lambda^{-\gamma}\theta^{1-\gamma}x^{\mathrm{i}*\gamma} - \beta I = 0. \quad \text{(A.16)}$$

Note that we have

$$v + \Big(\frac{\tau}{r} - \Big(\frac{\tau}{r} + \alpha\frac{v}{\kappa}\Big)h^{\gamma}\Big)\frac{\kappa}{h} = v\psi^{-1}, \quad \text{(A.17)}$$

where $\psi := (1 + \tau(h(1-\tau))^{-1})^{-1} \leq 1$. Thus, $x^{\mathrm{i}*}$ should be satisfied with (30) and $c^* = c(x^{\mathrm{i}*}) = (\kappa/h)x^{\mathrm{i}*}$ for $\theta \in [0, \theta_1^*)$, where $\theta_1^* = \theta_1(c^*)$, i.e., θ_1^* is given by θ satisfying with $\theta = (1-\tau)c^*/r$.

Next, suppose that $\theta \in [\theta_1(c), \theta_2(c))$ for a fixed $c \geq 0$. Then, $V^{\mathrm{a}}(x^{\mathrm{i}}, c)$ in (27) is defined by the lower equation of (A.6). Recall that we have $E^{\mathrm{b}}(x_1^{\mathrm{d}}(c)) = \theta \geq 0$. By differentiating (27) with x^{i} and c, $x^{\mathrm{i}*} \geq 0$ and $c^* \geq 0$ are obtained by solving simultaneous equations of (31) and (32) for $\theta \in [\theta_1^*, \theta_2^*)$ where $\theta_2^* = \theta_2(c^*)$, i.e., θ_2^* is given by θ satisfying with $\theta = (1/r(1-\alpha))c^*$.

Finally, suppose that $\theta \in [\theta_2(c), +\infty)$ for a fixed $c \geq 0$. Recall that $V^{\mathrm{a}}(X(t), c)$ in (27) is given by (A.9). Differentiating $V_2^{\mathrm{a}}(X(t), c)$ with c yields $c(X(t)) = (r/\tau)[(1-\alpha)\theta - \lambda^{-1}X(t)]$. Then, we have

$$x_2^{\mathrm{s}}(c(X(t))) = X(t),$$

which implies that investment and shutdown are exercised simultaneously. Thus, we do not obtain $x^{\mathrm{i}*}(\theta)$ and $c^*(\theta)$ for $\theta \in [\theta_2^*, +\infty)$.

Proof of Corollary 1

Recall that $V_1^{\mathrm{a}}(X(t), c)$ and $E_0^{\mathrm{a}}(X(t))$ are given in (A.6) and (A.13), respectively. To show the proof, it is enough to confirm that, for any $X(t)$ and $\theta \in [0, \theta_2^*)$, we have

$$V_1^{\mathrm{a}}(X(t), c(X(t)); \theta) \geq E_0^{\mathrm{a}}(X(t)), \quad \text{(A.18)}$$

where $c(X(t)) = \mathrm{argmax}_c V_1^{\mathrm{a}}(X(t), c)$.

Now suppose that $c(X(t)) \geq 0$ and $V_1^{\mathrm{a}}(X(t), c(X(t))) < E_0^{\mathrm{a}}(X(t); \theta)$ for any $X(t)$ and θ. Then, we obtain $c(X(t)) = 0$ and $\lim_{c \to 0} V_1^{\mathrm{a}}(X(t), c(X(t))) = E_0^{\mathrm{a}}(X(t))$. Thus, the firm can increase the value by choosing $c(X(t))=0$, which is a contradiction. As a result, (A.18) implies that we have $x^{\mathrm{i}*} \le x_0^{\mathrm{i}*}$ and $E^*(x) \ge E_0^*(x)$ for any θ. To confirm that these results are correct, see the top-left and middle-right panels of Figure 2 and the left and right panels of Figure 5, respectively.

Derivation of Equation (36)

Substituting $\theta=0$ into (A.13) yields

$$E_0^{\mathrm{a}}(X(t)) = vX(t). \tag{A.19}$$

In addition, substituting $\theta=0$ and $c(X(t))=(\kappa/h)X(t)$ into the upper equation of (A.7) gives

$$V_1^{\mathrm{a}}(X(t), c(X(t))) = \psi v X(t), \tag{A.20}$$

because of (A.17). Since (A.19) and (A.20) imply $V^{\mathrm{a}}(X(t), c(X(t))) = \psi E_0^{\mathrm{a}}(X(t))$, we obtain (36) by using (35).

Proof of Proposition 3

Suppose that $\theta \in [0, \theta_1(c))$ for a fixed $c \ge 0$. The Lagrangian is formulated as

$$\mathcal{L} \;=\; x^{\mathrm{i}^{-\beta}}\Big\{V_1^{\mathrm{a}}(x^{\mathrm{i}}, c) - I\Big\} + \delta\Big\{q\theta - D_1^{\mathrm{a}}(x^{\mathrm{i}}, c)\Big\}, \tag{A.21}$$

where $\delta \ge 0$ denotes the multiplier on the constraint. Recall that $D_1^{\mathrm{a}}(x^{\mathrm{i}}, c)$ and $V_1^{\mathrm{a}}(x^{\mathrm{i}}, c)$ are defined by the upper equations of (A.5) and (A.6), respectively. The Karush-Kuhn-Tucker (KKT) conditions are given by

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x^{\mathrm{i}}} \;=\; & x^{\mathrm{i}^{-\beta}}\Big\{(-\beta)x^{\mathrm{i}^{-1}}\{V_1^{\mathrm{a}}(x^{\mathrm{i}}, c) - I\} + v \\
& + \Big(- \Big(\frac{\tau}{r} + \alpha\frac{v}{\kappa}\Big)\kappa^{\gamma} c^{1-\gamma} + \frac{1-\alpha}{1-\gamma}\theta^{1-\gamma}\lambda^{-\gamma}\Big)\gamma x^{\mathrm{i}^{\gamma-1}}\Big\} \\
& -\delta\Big\{\Big(- \frac{1}{r} + (1-\alpha)\frac{v}{\kappa}\Big)\kappa^{\gamma} c^{1-\gamma} + \frac{1-\alpha}{1-\gamma}\theta^{1-\gamma}\lambda^{-\gamma}\Big\}\gamma x^{\mathrm{i}^{\gamma-1}} = 0, \qquad \text{(A.22)} \\
\frac{\partial \mathcal{L}}{\partial c} \;=\; & x^{\mathrm{i}^{-\beta}}\Big\{\frac{\tau}{r} - (1-\gamma)\Big(\frac{\tau}{r} + \alpha\frac{v}{\kappa}\Big)\kappa^{\gamma} c^{-\gamma} x^{\mathrm{i}^{\gamma}}\Big\} \\
& -\delta\Big\{\frac{1}{r} + (1-\gamma)\Big(- \frac{1}{r} + (1-\alpha)\frac{v}{\kappa}\Big)\kappa^{\gamma} c^{-\gamma} x^{\mathrm{i}^{\gamma}}\Big\} = 0, \qquad \text{(A.23)}
\end{aligned}$$

and

$$\delta\Big\{\min\{q\theta, I\} - D^{\rm a}_1(x^{\rm i}, c)\Big\} = 0. \tag{A.24}$$

Removing $\delta \geq 0$ from (A.22) and (A.23) and rearranging gives the first equation of (38) with f_{11}, f_{12}, f_{13}, and f_{14}.

For $\theta \in [\theta_1(c), \theta_2(c))$ for a fixed $c \geq 0$, recall that $D^{\rm a}(x^{\rm i}, c)$ and $V^{\rm a}(x^{\rm i}, c)$ are given by the lower equations of (A.5) and (A.6), respectively. Similarly, we derive KTT conditions. By arranging the KKT conditions, we obtain the first equation of (38) with f_{21}, f_{22}, f_{23}, and f_{24}.

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	debt-equity financing			all-equity financing
	risky debt		riskless debt	–
	$\theta \in [0, \theta_1(c))$	$\theta \in [\theta_1(c), \theta_2(c))$	$\theta \in [\theta_2(c), +\infty)$	$\theta \in [0, +\infty)$
Debt value	$D_1^{\mathrm{a}}(X(t), c)$		$D_2^{\mathrm{a}}(X(t), c)$	–
Equity value	$E_1^{\mathrm{a}}(X(t), c)$		$E_2^{\mathrm{a}}(X(t), c)$	$E_0^{\mathrm{a}}(X(t))$
Default	equity holders $x_1^{\mathrm{d}}(c)$	equity holders $x_1^{\mathrm{d}}(c)$	– –	– –
Shutdown	debt holders x_1^{s}		equity holders $x_2^{\mathrm{s}}(c)$	equity holders x_0^{s}
Exit strategies	sequential	simultaneous	only shutdown	

Table 1: Values and bankruptcy strategies for a fixed $c \geq 0$

If $\theta \in [0, \theta_1(c))$ for a fixed $c \geq 0$, default and shutdown are exercised sequentially. If $\theta \in [\theta_1(c), \theta_2(c))$ for a fixed $c \geq 0$, they are exercised simultaneously. Otherwise, shutdown only is done because there is no default.

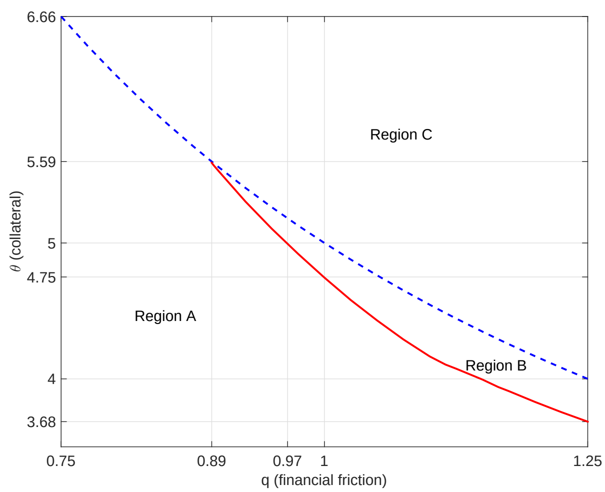

Figure 1: Constrained regions in space (q, θ) for $I = 5$

The other parameters are $r = 9\%$, $\sigma = 20\%$, $\mu = 1\%$, $\tau = 15\%$, $\alpha = 40\%$, $I = 5$, and $x = 0.4$. The dotted line from $(q, \theta) = (0.75, 6.66)$ to $(q, \theta) = (1.25, 4)$ indicates the boundary of satisfying $q\theta = I = 5$. The upper-right region to the dotted line of $q\theta = I$ is denoted by Region C, where $q\theta > I$. The solid line from $(q, \theta) = (0.89, 5.59)$ to $(q, \theta) = (1.25, 3.68)$ indicates the boundary of satisfying $D^{\mathrm{a}}(x^{\mathrm{i}*}, c^*) = q\theta$. Note that $D^{\mathrm{a}}(x^{\mathrm{i}*}, c^*)$ is monotonically increasing with θ. Thus, the lower-left region of the line from $(q, \theta) = (0.75, 6.66)$ to $(q, \theta) = (1.25, 3.68)$ via $(q, \theta) = (0.89, 5.59)$ is denoted by Region A, which corresponds to the constrained region where $D^{\mathrm{a}}(x^{\mathrm{i}*}, c^*) > q\theta = \min\{q\theta, I\}$. The region between the two lines is denoted by Region B, which corresponds to the unconstrained region where $D^{\mathrm{a}}(x^{\mathrm{i}*}, c^*) \leq q\theta = \min\{q\theta, I\}$.

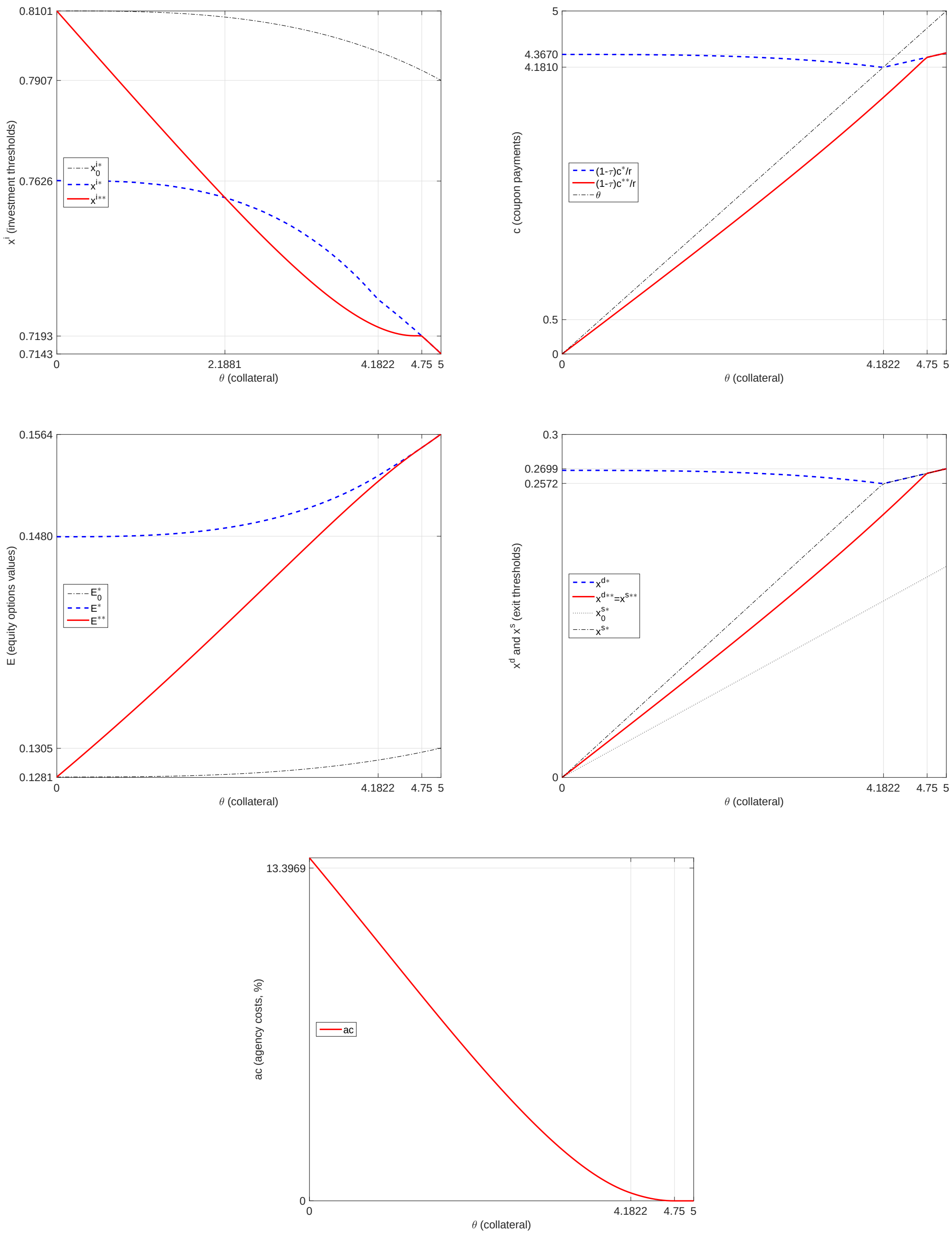

Figure 2: Effects of collateral θ on financing and investment thresholds

Suppose $q = 1$ and $I = 5$. We then have $q\theta \le I$ for any $\theta \in [0, 5]$. The other parameters are $r = 9\%$, $\sigma = 20\%$, $\mu = 1\%$, $\tau = 15\%$, $\alpha = 40\%$, and $x = 0.4$.

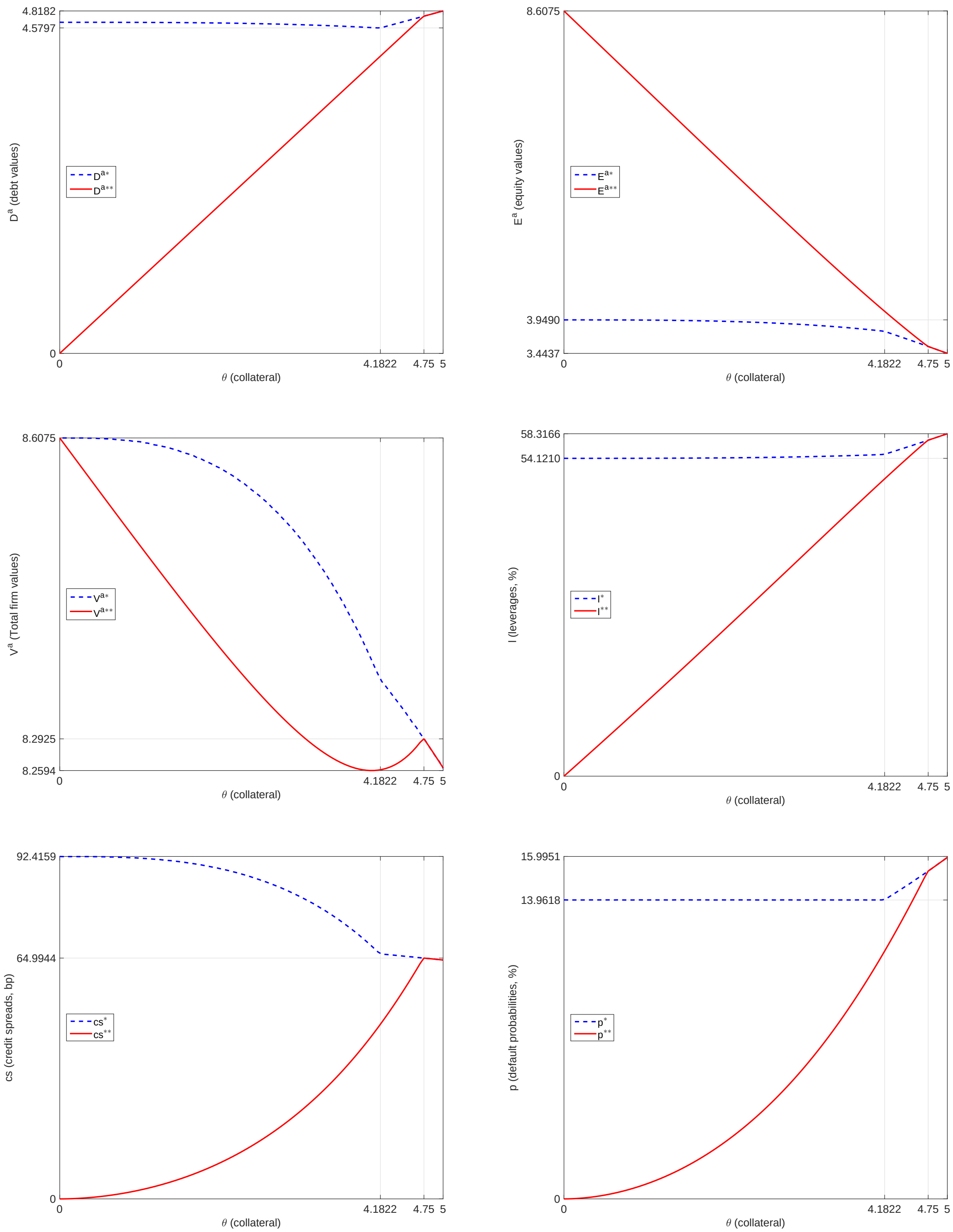

Figure 3: Effects of θ on values, leverage, credit spread, and default probability

Suppose $q = 1$ and $I = 5$. We then have $q\theta \leq I$ for $\theta \in [0, 5]$ The other parameters are $r = 9\%$, $\sigma = 20\%$, $\mu = 1\%$, $\tau = 15\%$, $\alpha = 40\%$, and $x = 0.4$.

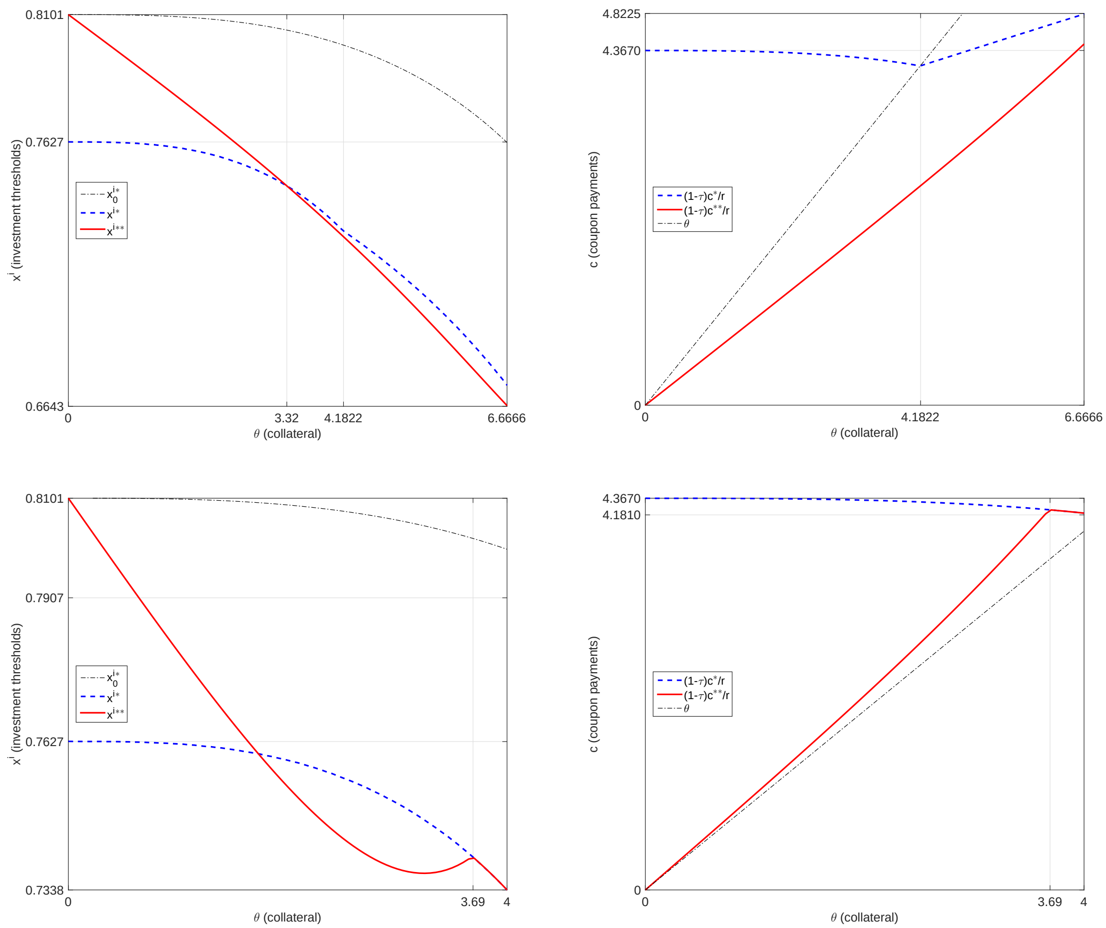

Figure 4: Effects of θ for $q = 0.75$ and $q = 1.25$

Suppose $q \in \{0.75, 1.25\}$ and $I = 5$. For $q = 0.75$ in the two upper panels, we have $q\theta \leq I$ for $\theta \in [0, 6.6666]$. For $q = 1.25$ in the two lower panels, we have $q\theta \leq I$ for $\theta \in [0, 4]$. The other parameters are $r = 9\%$, $\sigma = 20\%$, $\mu = 1\%$, $\tau = 15\%$, $\alpha = 40\%$, and $x = 0.4$.

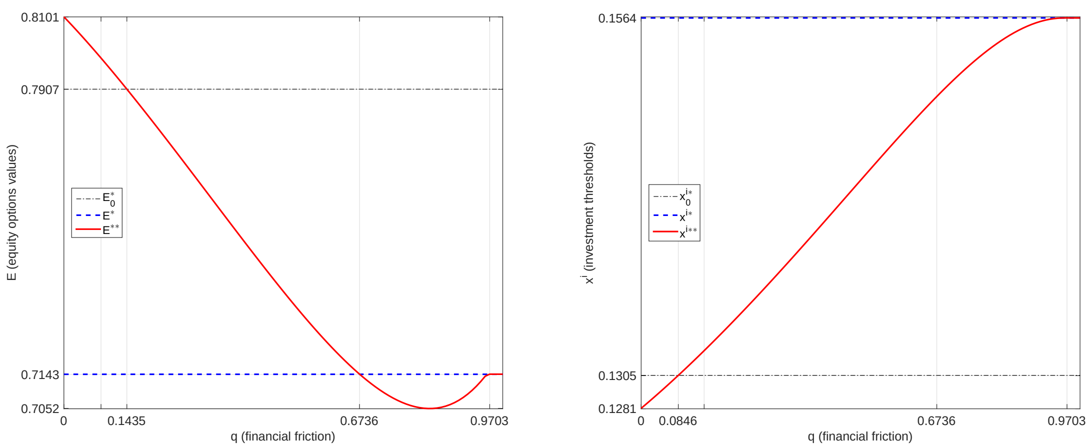

Figure 5: Effects of financing friction q for $\theta = 5$

Suppose $\theta = 5$ and $I = 5$. We then have $q\theta \leq I$ for $q \in [0, 1]$. The other parameters are $r = 9\%$, $\sigma = 20\%$, $\mu = 1\%$, $\tau = 15\%$, $\alpha = 40\%$, and $x = 0.4$.

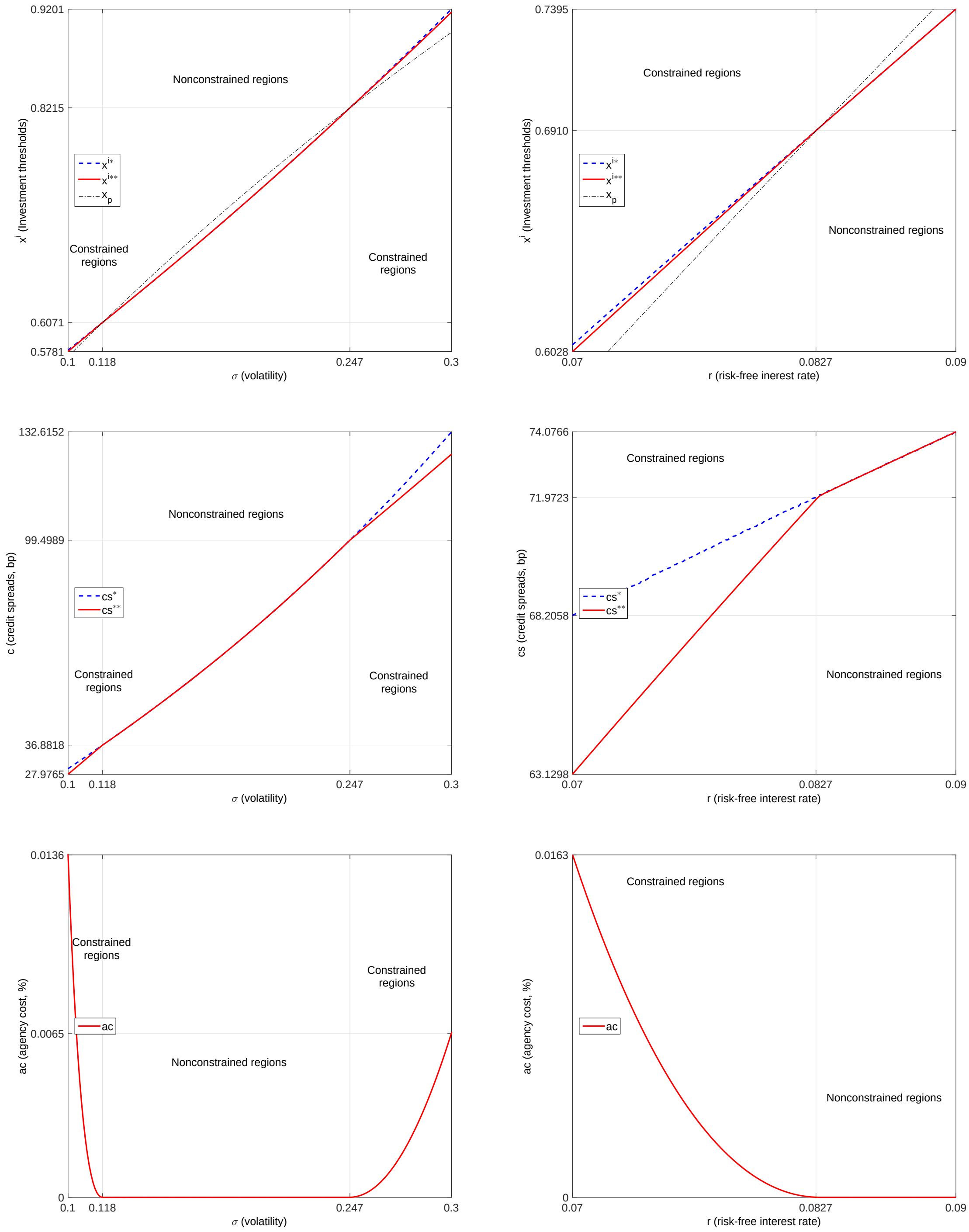

Figure 6: Effects of volatility (σ) and risk-free interest rate (r)

Suppose $q = 1.25$, $\theta = 3.725$, and $I = 5$. The other parameters are $\mu = 1\%$, $\tau = 15\%$, $\alpha = 40\%$, and $x = 0.4$.

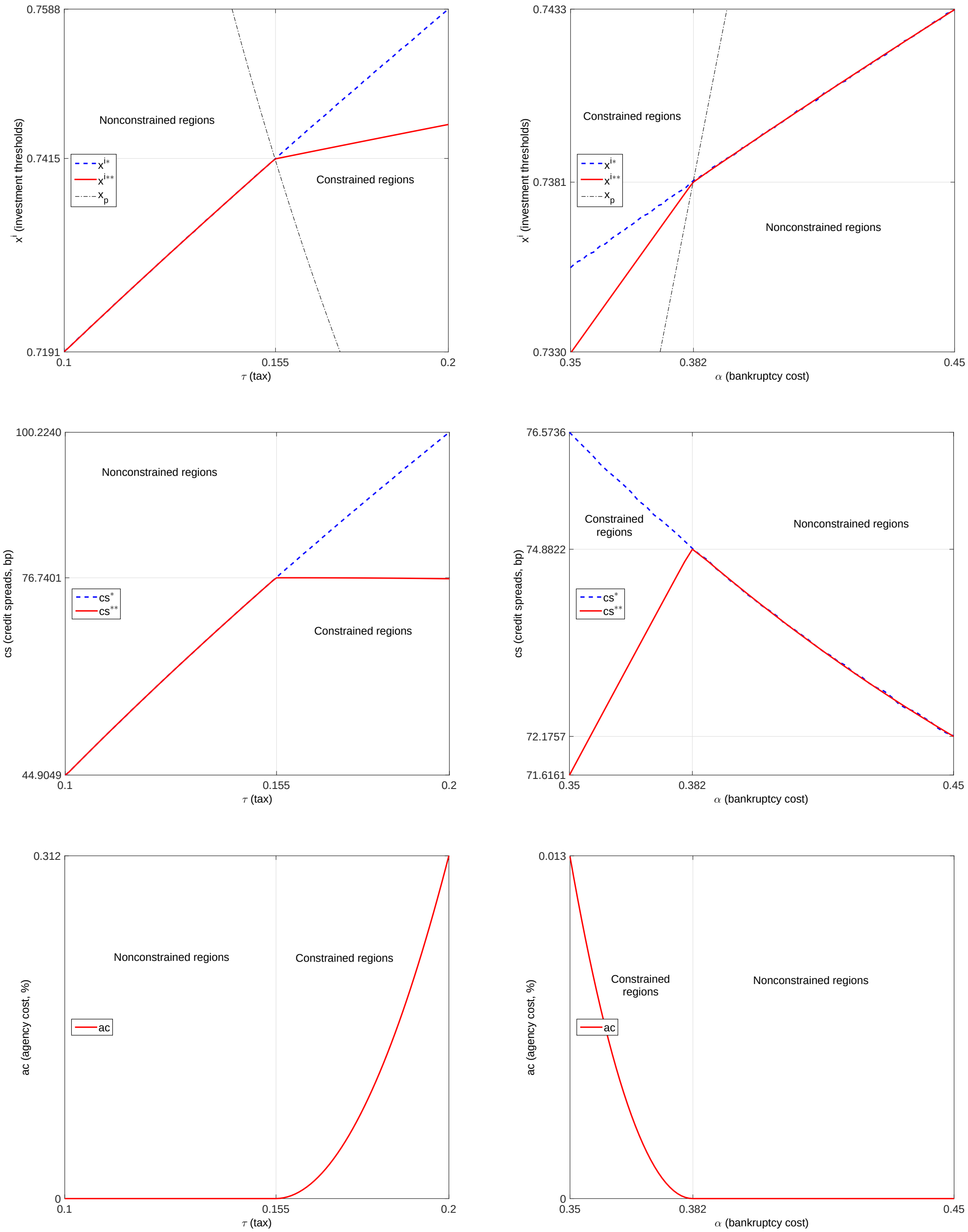

Figure 7: Effects of bankruptcy cost (α) and corporate income tax (τ)

Suppose $q = 1.25$, $\theta = 3.725$, and $I = 5$. The other parameters are $r = 9\%$, $\sigma = 20\%$, $\mu = 1\%$, and $x = 0.4$.