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Strategic entry in a triopoly market of firms with asymmetric cost structures

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Abstract

This paper examines the strategic investment timing decision in a triopoly market comprising firms with asymmetric cost structures. We present three novel results. First, in the case where there are relatively small cost asymmetries between firms and a relatively small first-mover advantage, the firm with the lowest cost structure is not always the first investor. In other cases, the firm with the lowest cost structure is the first investor. Second, an increase in volatility increases the possibility that a firm without the lowest cost structure is the first investor. Finally, even in the three-asymmetric-firm model, we show that the first investor threshold is larger in a triopoly than in a duopoly, although it is smaller in a duopoly than in a monopoly.

Keywords: Investment analysis; Real options; Competition; Uncertainty.

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1 Introduction

"Investment under uncertainty" considers the problem of the investment timing decision in a *monopoly* market using a contingent claim approach following the seminal work of McDonald and Siegel (1986). Dixit and Pindyck (1994) provide a thorough review.

"Strategic investment under uncertainty" examines the problem of strategic (preemptive) investment timing decisions in a *duopoly* market using a contingent claim approach. Grenadier (1996), Weeds (2002), Huisman and Kort (2004), and Nishihara and Shibata (2010) examine the strategic investment timing problem in a duopoly market of firms with *symmetric (homogeneous)* cost structures. See Chevalier-Roignant et al. (2011) and Azevedo and Paxson (2014) for a thorough review of "strategic investment under uncertainty."

More recently, there have been various extensions to the literature on "strategic investment under uncertainty." One important extension is to incorporate *cost asymmetry (heterogeneity)* between firms. Pawlina and Kort (2006), Kong and Kwok (2007), Nishihara and Fukushima (2008), and Shibata and Yamazaki (2010) investigate the problem of strategic investment timing decisions in a duopoly market of firms with asymmetric cost structures. We call this model the "two-asymmetric-firm" model. Importantly, all of these studies examine the strategic investment timing decision in a *duopoly* market.

Another important extension of the literature on "strategic investment under uncertainty" is to examine the problem of strategic investment timing decisions in a *triopoly* market. Bouis et al. (2009) consider the strategic investment timing decision of firms with *symmetric* cost structures.[1] We call this model the "three-symmetric-firm" model. Alternatively, Ko and Shibata (2012) considers the strategic investment timing decision of firms with *asymmetric* cost structures. The main result is that the firm without the lowest cost has the possibility to be the first firm (i.e., the first investor) to enter the market. We call this model the "three-asymmetric-firm" model. However, we can extend Ko and Shibata (2012), which we undertake in this analysis.

Our extension is to derive a region where the firm without the lowest cost is the first investor in a triopoly market. Ko and Shibata (2012) find that, under only one specific parameter, the firm without the lowest cost is the first investor. However, we do not know whether such a result is robust to other parameters. In this three-asymmetric-firm model, there are three important parameters. One is a "cost asymmetry" parameter between firms. The second is a "first-mover advantage" parameter where it measures the gain to

[1] In addition, Bouis et al. (2009) provide numerical examples of an n-firm symmetric cost structure model.

the first-moving significant occupant of a market. The third is a "volatility" parameter of cash inflows. Thus, for any combination of cost asymmetry, first-mover advantage, and volatility parameters, we derive a region where the firm without the lowest cost is the first investor. Our contribution is to describe the market environment of a firm without the lowest cost that is a first investor in a triopoly market.

Our model contains elements of the two-asymmetric-firm model in Pawlina and Kort (2006) and the three-symmetric-firm model in Bouis et al. (2009). In the extreme case where the profit for a third investor is zero, our model is equivalent to the two-asymmetric-firm model. The main result of this extreme case is that the low-cost firm is always the first investor at the equilibrium. In the extreme case where the degree of cost asymmetry is zero, our model is equivalent to the three-symmetric-firm model. The important result of this extreme case is that the preemptive (strategic) investment threshold of being the first investor is larger in the three-symmetric-firm model than in the two-symmetric-firm model. In other words, the first investor's investment threshold in a triopoly market lies between those in the monopoly and duopoly markets, such that in their numerical example, the investment thresholds are 2.2078 for the monopoly, 1.3997 for the duopoly, and 1.5115 for the triopoly.[2]

There are two ways in which the three-asymmetric-firm model is complicated, compared with the two-asymmetric-firm and three-symmetric-firm models. First, the three-asymmetric-firm model is more complex than the two-asymmetric-firm model. In the three-asymmetric-firm model, after one of the three firms invests as the first investor, the other two firms compete to be the second investor. This environment differs from that of the two-asymmetric-firm model.[3] Thus, each firm considers its preemptive investment strategy as the first investor on the condition that the preemptive investment strategies of the second investor are through competition. Second, the three-asymmetric-firm model is more complex than the three-symmetric-firm model. In the three-asymmetric-firm model, each firm's preemptive investment threshold to be the first investor is not necessarily the same. This differs from the three-symmetric-firm model. Suppose there are three Firms, A, B, and C, with asymmetric (heterogeneous) cost structures. Consider, for example, Firms A's (B's) preemptive investment thresholds to be the first investor. Firm A's (B's) preemptive investment threshold of being the first investor is then obtained backwardly through dynamic programming on the condition that Firms B (A) and C are the second

[2]In this literature, once the state variable reaches the investment threshold from a sufficiently low level, the firm undertakes the necessary market-entry investment. Following existing studies, we assume that a smaller (larger) threshold implies earlier (later) investment.

[3]In a duopoly market with two asymmetric firms, after one of the two firms invests as a first investor, the other decides upon its (nonstrategic) investment strategy as the second investor without competition.

and third investors, respectively. Consequently, the strategic interaction between Firms B and C differs from that between Firms A and C. This leads to a difference between Firms A's and B's preemptive investment thresholds of being the first investor. Thus, it is difficult to conjecture intuitively the results of the three-asymmetric-firm model by combining the two-asymmetric-firm model with the three-symmetric-firm model. These complexities produce interesting results.

We provide three new insights into the three-asymmetric-firm model. First, for the case of a relatively small first-mover advantage and a relatively small cost asymmetry, a firm without the lowest cost structure is the first investor. For all other cases, the firm with the lowest cost structure is the first investor. These results are new because the solutions are derived for only one parameter in Ko and Shibata (2012). The result that the firm with the lowest cost structure does not always enter the market as the first investor is obtained by increasing the number of asymmetric firms from two to three. This is because the low-cost firm is always the first investor in the two-asymmetric-firm model developed by Pawlina and Kort (2006). Second, we show that an increase in volatility enlarges the region where the firm without the lowest cost structure is the first investor. This result implies that the firm without the lowest cost structure has the possibility to be the first investor by increasing the volatility. Finally, we show that the first investor's preemptive investment threshold is larger in a triopoly than in a duopoly, although it is smaller in a duopoly than in a monopoly. Thus, the "nonmonotonic investment threshold with respect to the number of firms" property is obtained, even in the three-asymmetric-firm model. Moreover, our theoretical results are consistent with the empirical results.

The remainder of the paper is organized as follows. Section 2 describes the setup of the model and derives the value functions given the investment strategies. As benchmarks, we provide the strategic investment decisions in the monopoly and duopoly markets. Section 3 considers the strategic investment decisions in the triopoly market. Section 4 discusses the implications of the model. Section 5 concludes.

2 Model

In this section, we begin with a description of the model. We then provide the value functions for the triopoly market. Finally, as benchmarks, we provide the optimal investment strategies in the monopoly and duopoly markets.

2.1 Setup

Consider three firms, Firm A, Firm B and Firm C. These firms have an investment opportunity. The firms are risk neutral and compete with each other to maximize profit. The risk-free interest rate $r > 0$ is a positive constant.

The cash flow from exercising the investment opportunity depends on the number of firms operating in the market (i.e., monopoly, duopoly, or triopoly). The investment yields a cash flow $D_n Y(t)$, where $D_n > 0$ represents the competition parameter and the subscript "n" represents the number of operating firms ($n \in \{1, 2, 3\}$). If the number of operating firms increases, more firms share the market. Thus, we assume that

$$D_1 > D_2 > D_3 > 0. \tag{1}$$

These conditions imply that the profit rate of each firm is lower when more firms invest (enter the market). Let $Y(t)$ follow a geometric Brownian motion:

$$\mathrm{d}Y(t) = \mu Y(t)\mathrm{d}t + \sigma Y(t)\mathrm{d}W(t), \quad Y(0) = y > 0, \tag{2}$$

where $\mu > 0$, $\sigma > 0$, and $W(t)$ denotes a standard Brownian motion. We denote the risk-free interest rate as $r > 0$. In order to obtain a finite valuation, we assume that $r > \mu$.[4] Throughout our analysis, we assume that the current demand level $Y(0) = y$ is sufficiently low such that all three firms do not enter the market immediately.[5]

When the investment option is exercised, each firm pays a fixed cost I_i ($i \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$). We assume that the cost structures of the three firms are asymmetric (heterogeneous), which leads to the three-asymmetric-firm model. The assumptions of asymmetric cost structures are given by $I_{\mathrm{A}} < I_{\mathrm{B}} < I_{\mathrm{C}}$. Thus, Firm A is the lowest-cost firm, Firm B is the second lowest-cost firm, and Firm C is the highest-cost firm.

2.2 Value functions for the triopoly market

In this subsection, we consider the value functions for the first, second, and third investors. Because the three firms have asymmetric (heterogeneous) cost structures, we can ignore cases of simultaneous entry by two or more firms into the market.

Suppose that Firms i, j, and k enter the market as the first, second, and third investors, respectively ($i, j, k \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}; i \neq j \neq k$). Let $\tau_{ijk}^{(1)}$, $\tau_{jk}^{(2)}$, and $\tau_{k}^{(3)}$ denote the adoption (stopping) times for entering the market for the firms as the first, second, and third

[4] This assumption is the same as in Kort et al. (2010), Shibata and Nishihara (2011), and Shibata and Nishihara (2015).

[5] This assumption is the same as in Kong and Kwok (2007) and is justified because we focus on the preemptive investment, not the joint investment. See Thijssen et al. (2012) for the joint investment.

investors, respectively. Here, the superscript "(n)" indicates that the firm is the nth to enter ($n \in \{1,2,3\}$), and the subscript "ijk" indicates that Firms i, j, and k enter the market as the first, second, and third investors, respectively. Let $y^{(1)}_{ijk}$, $y^{(2)}_{jk}$, and $y^{(3)}_{k}$ denote the associated investment thresholds by $0 < y^{(1)}_{ijk} \leq y^{(2)}_{jk} \leq y^{(3)}_{k}$. Mathematically, we define the stopping times as $\tau^{(1)}_{ijk} := \inf\{t \geq 0 | Y(t) \geq y^{(1)}_{ijk}\}$, $\tau^{(2)}_{jk} := \inf\{t \geq 0 | Y(t) \geq y^{(2)}_{jk}\}$, and $\tau^{(3)}_{k} := \inf\{t \geq 0 | Y(t) \geq y^{(3)}_{k}\}$, respectively. We let $C^{(1)}_{ijk}$, $C^{(2)}_{jk}$, and $C^{(3)}_{k}$ denote the option values of Firms i, j, and k of entering the market as the first, second and third investors, respectively. Let $F^{(1)}_{ijk}$, $F^{(2)}_{jk}$, and $F^{(3)}_{k}$ denote the adoption values of Firms i, j, and k of entering the market as the first, second and third investors, respectively. The function C represents the option value before entering the market, while the function F represents the adoption value after entering the market. Thus, $C^{(1)}_{ijk}$, $C^{(2)}_{jk}$, and $C^{(3)}_{k}$ are the option values of $F^{(1)}_{ijk}$, $F^{(2)}_{jk}$, and $F^{(3)}_{k}$, respectively.

We derive the value functions backwardly through dynamic programming. We begin by deriving the values of the firm as the third investor, $C^{(3)}_{k}$ and $F^{(3)}_{k}$. We then provide the values of the firm as the second investor, $C^{(2)}_{jk}$ and $F^{(2)}_{jk}$. Finally, we obtain the values of the firm as the first investor, $C^{(1)}_{ijk}$ and $F^{(1)}_{ijk}$.

2.2.1 Value function for the third investor

First, we derive the value function for the firm as the third investor. We recall the assumption that Firms i, j, and k enter the market as the first, second, and third investors, respectively ($i, j, k \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}; i \neq j \neq k$). Suppose any time $t \in (\tau^{(2)}_{jk}, \tau^{(3)}_{k}]$, i.e., Firms i and j have already entered the market. Then, Firm k's option value of being the third (last) investor, $C^{(3)}_{k}(Y(t))$, is given by

$$C^{(3)}_{k}(Y(t)) := \mathbb{E}^{Y(t)}\Big[\int_{\tau^{(3)}_{k}}^{+\infty} \mathrm{e}^{-r(u-t)} D_3 Y(u)\mathrm{d}u - \mathrm{e}^{-r(\tau^{(3)}_{k}-t)} I_k\Big], \tag{3}$$

where $\mathbb{E}^{Y(t)}[\cdot]$ denotes the expectation operator given $Y(t)$. As in McDonald and Siegel (1986) and Dixit and Pindyck (1994), $C^{(3)}_{k}(Y(t))$ is rewritten as

$$C^{(3)}_{k}(Y(t)) = \Big(\frac{Y(t)}{y^{(3)}_{k}}\Big)^{\beta} F^{(3)}_{k}(y^{(3)}_{k}), \tag{4}$$

where $\beta := 1/2 - \mu/\sigma^2 + \sqrt{(\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2} > 1$ and

$$F^{(3)}_{k}(y^{(3)}_{k}) = \frac{D_3}{r-\mu} y^{(3)}_{k} - I_k. \tag{5}$$

2.2.2 Value function for the second investor

Second, we now provide the value function for the firm as the second investor. Suppose any time $t \in (\tau^{(1)}_{ijk}, \tau^{(2)}_{jk}]$, i.e., Firm i has already entered the market ($i, j, k \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}; i \neq$

$j \neq k$). Then, Firm j's option value of being the second investor, $C_{jk}^{(2)}(Y(t))$, is given by

$$C_{jk}^{(2)}(Y(t)) := \mathbb{E}^{Y(t)}\Big[\int_{\tau_{jk}^{(2)}}^{\tau_k^{(3)}} \mathrm{e}^{-r(u-t)} D_2 Y(u)\mathrm{d}u - \mathrm{e}^{-r(\tau_{jk}^{(2)}-t)} I_j + \int_{\tau_k^{(3)}}^{+\infty} \mathrm{e}^{-r(u-t)} D_3 Y(u)\mathrm{d}u\Big]. \quad (6)$$

As in Pawlina and Kort (2006) and Shibata and Yamazaki (2010), $C_{jk}^{(2)}(Y(t))$ is rewritten as

$$C_{jk}^{(2)}(Y(t)) = \Big(\frac{Y(t)}{y_{jk}^{(2)}}\Big)^{\beta} F_{jk}^{(2)}(y_{jk}^{(2)}), \quad (7)$$

where

$$F_{jk}^{(2)}(y_{jk}^{(2)}) := \frac{D_2}{r-\mu} y_{jk}^{(2)} - I_j + \Big(\frac{y_{jk}^{(2)}}{y_k^{(3)}}\Big)^{\beta} \frac{D_3 - D_2}{r-\mu} y_k^{(3)}. \quad (8)$$

Note that $C_{jk}^{(2)}$ is the option value of $F_{jk}^{(2)}$. Here, (8) consists of three terms. The first term represents the present value of Firm j's profit before Firm k enters the market. The second term captures the cost of entering the market. The third term measures the option value lost by making Firm k enter the market.

2.2.3 Value function for the first investor

Finally, we provide the value function for the firm as the first investor. Suppose any time $t \in [0, \tau_{ijk}^{(1)}]$, i.e., Firms i, j, and k have not yet entered the market ($i, j, k \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}; i \neq j \neq k$). Then, Firm i's option value of being the first investor, $C_{ijk}^{(1)}(Y(t))$, is defined by

$$\begin{aligned} C_{ijk}^{(1)}(Y(t)) \quad := \quad & \mathbb{E}^{Y(t)}\Big[\int_{\tau_{ijk}^{(1)}}^{\tau_{jk}^{(2)}} \mathrm{e}^{-r(u-t)} D_1 Y(u)\mathrm{d}u - \mathrm{e}^{-r(\tau_{ijk}^{(1)}-t)} I_i \\ & + \int_{\tau_{jk}^{(2)}}^{\tau_k^{(3)}} \mathrm{e}^{-r(u-t)} D_2 Y(u)\mathrm{d}u + \int_{\tau_k^{(3)}}^{+\infty} \mathrm{e}^{-r(u-t)} D_3 Y(u)\mathrm{d}u\Big]. \end{aligned} \quad (9)$$

Similar to Bouis et al. (2009), $C_{ijk}^{(1)}(Y(t))$ is rewritten as

$$C_{ijk}^{(1)}(Y(t)) = \Big(\frac{Y(t)}{y_{ijk}^{(1)}}\Big)^{\beta} F_{ijk}^{(1)}(y_{ijk}^{(1)}), \quad (10)$$

where

$$F_{ijk}^{(1)}(y_{ijk}^{(1)}) := \frac{D_1}{r-\mu} y_{ijk}^{(1)} - I_i + \Big(\frac{y_{ijk}^{(1)}}{y_{jk}^{(2)}}\Big)^{\beta} \frac{D_2 - D_1}{r-\mu} y_{jk}^{(2)} + \Big(\frac{y_{ijk}^{(1)}}{y_k^{(3)}}\Big)^{\beta} \frac{D_3 - D_2}{r-\mu} y_k^{(3)}. \quad (11)$$

Note that $C_{ijk}^{(1)}$ is the option value of $F_{ijk}^{(1)}$, which consists of four components. The first component represents the present value of the profit before Firms j and k enter the market. The second term captures the cost. The third and fourth components measure the option values lost for the second and third investors by making Firms j and k enter the market, respectively.

Note that the optimal investment thresholds, $y_{ijk}^{(1)}$, $y_{jk}^{(2)}$, and $y_k^{(3)}$, have not yet been decided. Before analyzing the optimal investment thresholds, we first review two benchmarks briefly.

2.3 Benchmarks

In this subsection, we briefly review the optimal investment thresholds in the monopoly and duopoly markets, in turn. Note that we have the value functions for the monopoly and duopoly markets from the derivation of those for the triopoly market. See Appendix A for the value functions for the monopoly and duopoly markets.

2.3.1 Monopoly market

We begin by considering the optimal investment strategy in the monopoly market. Here, we derive the optimal investment threshold $y_i^{(1)}$ ($i \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$). This is the standard nonstrategic investment decision problem developed by McDonald and Siegel (1986).

We assume that, in the monopoly market, there exists only Firm i, which has an opportunity to enter ($i \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$). Then, the optimal investment threshold $y_1^{(1)}$ is obtained by

$$y_i^{(1)*} = y_i^{1\mathrm{n}} := \phi \frac{I_i}{D_1}, \tag{12}$$

where $\phi := \beta(r-\mu)/(\beta-1) > 0$.[6] The superscript "1n" of $y_i^{1\mathrm{n}}$ denotes the nonstrategic (nonpreemptive) investment threshold in the monopoly market. The subscript "$*$" of $y_1^{(1)*}$ indicates the equilibrium. We use this threshold as a benchmark.

2.3.2 Duopoly market

We then consider investment strategies in the duopoly market. This problem is the same as in Pawlina and Kort (2006), Kong and Kwok (2007), and Shibata and Yamazaki (2010).

Suppose that in the duopoly market, there exist two firms, i and j, which have an opportunity to enter ($i, j \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}, i \neq j$). Here, we derive the optimal investment thresholds $y_{ij}^{(1)}$ and $y_j^{(2)}$. We solve the optimal investment problem by working backward using dynamic programming. First, we derive the nonstrategic investment threshold for Firm j after Firm i enters the market ($i, j \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}, i \neq j$).[7] Now suppose that Firm

[6] Differentiating $C_i^{(1)}$ given in (A.1) by $y_i^{(1)}$ and arranging gives the investment threshold in (12).

[7] Note that the investment decision of the last investor is nonstrategic.

i has already entered the market as a first investor. Then, Firm j's investment threshold is nonstrategic and is obtained by

$$y_j^{(2)} = y_j^{2\mathrm{n}} := \phi \frac{I_j}{D_2}, \tag{13}$$

where the superscript "2n" represents the nonstrategic investment threshold of the second (final) investor in the duopoly market.

Next, we derive the strategic investment threshold of the first investor. Let y_{ij}^{p} denote the preemptive (strategic) investment threshold of the first investor in the duopoly market, where the superscript "p" denotes the preemptive investment and the subscript "ij" indicates that Firms i and j enter the market as the first and second investors, respectively. Then, Firm i's preemptive investment threshold as the first investor is defined by

$$y_{ij}^{\mathrm{p}} := \inf\{y \in [0, y_j^{(2)}] | F_{ij}^{(1)}(y) \geq C_i^{(2)}(y)\}, \tag{14}$$

where $F_{ij}^{(1)}(y)$ is Firm i's adoption value of entering the market as a first investor given in (A.4), and $C_i^{(2)}(y)$ is Firm i's option value of entering the market as a second investor given in (A.5).[8]

As y_{ij}^{p} does not always exist[9], we define z_{ij} as

$$z_{ij} := \begin{cases} y_{ij}^{\mathrm{p}}, & \text{if } y_{ij}^{\mathrm{p}} \text{ exists,} \\ y_i^{1\mathrm{n}}, & \text{otherwise.} \end{cases} \tag{15}$$

We define the set of preemptive thresholds as Ω_{20} where the subscript "20" indicates that the number of firms currently operating in the duopoly market is zero, where an operating firm signifies that the firm has already entered the market. Suppose that, for example, there are Firms A and B in the duopoly market. Then, we have

$$\Omega_{20} = \{z_{\mathrm{AB}}, z_{\mathrm{BA}}\}. \tag{16}$$

Furthermore, we define

$$\Omega_{20}^{(1)} := \min \Omega_{20}, \quad \Omega_{20}^{(2)} := \max \Omega_{20}. \tag{17}$$

Firm i, being the first investor, has $\Omega_{20}^{(1)}$.

Finally, we consider the threshold at which Firm i invests. By using preemptive threshold (14), we examine the intervals over which Firm i provides a preemptive incentive for Firm j ($i \neq j$). Firm i has an incentive to invest as the first investor in the interval

[8]See Pawlina and Kort (2006) and Shibata and Yamazaki (2010) for the preemptive investment threshold in the duopoly market in detail.

[9]See Appendix B for a numerical example concerning the nonexistence of preemptive thresholds.

$y \in [y_{ij}^{\mathrm{p}}, y_i^{1\mathrm{n}}]$, and invests as closely as possible to $y_i^{1\mathrm{n}}$. Firm j has an incentive to invest as the first investor in the interval $y \in [y_{ji}^{\mathrm{p}}, y_j^{1\mathrm{n}}]$, and invests as closely as possible to $y_j^{1\mathrm{n}}$. Thus, Firm i invests as the first investor at $\min\{y_{ji}^{\mathrm{p}}, y_i^{1\mathrm{n}}\}$. Thus, we have the following result.

Lemma 1 *Suppose the duopoly market consists of a low-cost firm and a high-cost firm. Then, Firm i as the first investor has $\Omega_{20}^{(1)}$. Thus, Firms i and j enter the market as the first and second investors at*

$$y_{ij}^{(1)*} = \min\{\Omega_{20}^{(2)}, y_i^{1\mathrm{n}}\}, \quad y_2^{(2)*} = y_j^{2\mathrm{n}}, \tag{18}$$

respectively.

Note that the profile of strategies $(y_{ij}^{(1)*}, y_j^{(2)*})$ is a Nash equilibrium. The reason is that each firm's choice of strategy is a best response to the strategy played by its opponent. That is, each firm does not have an incentive to deviate from $(y_{ij}^{(1)*}, y_j^{(2)*})$. Moreover, we have the following result.

Observation 1 *In a duopoly market comprising a low-cost firm and a high-cost firm, the low-cost firm enters the market as the first investor.*

This simple and more intuitive identification of the equilibrium strategies with the low-cost firm investing first is also present in Pawlina and Kort (2006), which states that it holds, even for the slightest degree of cost asymmetry. That is, it is always the low-cost firm that takes the role of leader (i.e., we have $y_{ij}^{\mathrm{p}} < y_{ji}^{\mathrm{p}}$ if $I_i < I_j$). To the extent that we have solved (14) numerically for various parameters, we could not find any counterexample for the result in Observation 1. See Appendix C for numerical examples. We conclude that the result in Observation 1 is obtained in all cases. Consider, for example, the duopoly market with Firms A (the low-cost firm) and B (the high-cost firm). Then, Firms A and B are the first and second investors, respectively. From Lemma 1, Firm A enters the market at $y_{\mathrm{AB}}^{(1)*} = \min\{\Omega_{20}^{(2)}, y_{\mathrm{A}}^{1\mathrm{n}}\}$, and Firm B enters at $y_{\mathrm{B}}^{(2)*} = y_{\mathrm{B}}^{2\mathrm{n}}$. We also consider the cases of Firms A and C and Firms B and C. Note that we use these results as a benchmark.

3 Model solution

In this section, we derive the optimal investment strategies in the triopoly market.

We solve the optimal investment problem using backward dynamic programming. The following five steps determine the optimal investment strategies.

(i) On the condition that Firms i and j have already entered the market, we derive Firm k's nonstrategic investment threshold as a third investor $y_k^{3\mathrm{n}}$ $(i, j, k \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}, i \neq j \neq k)$.

(ii) On the condition that Firm i has already entered the market, we derive Firm j's preemptive investment threshold as a second investor y_{jk}^{p} given the strategy of step (i) where Firm k enters the market as the third investor.[10] Firm j is defined as a second investor that has a lower cost structure than Firm k.

(iii) Under the condition that all firms have not yet entered the market, we derive Firm i's preemptive investment threshold as the first investor y_{ijk}^{p} given the strategies of step (ii) where Firms j and k enter the market as the second and third investors, respectively. Here, we assume the preemptive strategies of interactions between Firms j and k for a given Firm i strategy.

(iv) Given the strategies of step (iii), the firm with the smallest preemptive investment threshold is the first investor. We then determine at what threshold level $y_{ijk}^{(1)*}$ the first investor enters the market.

(v) Given the strategies of step (iv), we obtain the thresholds for the second and third investors, $y_{jk}^{(2)*}$ and $y_k^{(3)*}$, respectively.

Suppose that Firm i is the first firm to enter the market. Then, given that Firm i has already entered the market, we consider the strategic interaction between Firms j and k as the second and third firms. As in Lemma 1, the optimal investment threshold of Firm k is nonstrategic, i.e.,

$$y_k^{(3)} = y_k^{3\mathrm{n}} := \phi \frac{I_k}{D_3}, \tag{19}$$

where the superscript "3n" represents the nonstrategic investment threshold of the third investor in the triopoly market. Next, we define the preemptive threshold of being the second investor as

$$z_{jk} := \begin{cases} y_{jk}^{\mathrm{p}}, & \text{if } y_{jk}^{\mathrm{p}} \text{ exists,} \\ y_j^{2\mathrm{n}}, & \text{otherwise,} \end{cases} \tag{20}$$

where

$$y_{jk}^{\mathrm{p}} := \inf\{y \in [0, y_k^{(3)}] | F_{jk}^{(2)}(y) \geq C_j^{(3)}(y)\}, \tag{21}$$

[10]This problem requires us to consider the strategic interaction between Firms j and k and is similar to that in the duopoly market in the previous subsection.

for $i \neq j \neq k$ $(i,j,k \in \{\mathrm{A},\mathrm{B},\mathrm{C}\})$. We define the set of preemptive thresholds of the second investor as Ω_{31} where the subscript "31" indicates that the number of firms operating in the triopoly market is one. For example, suppose that Firms B and C have not entered the market while Firm A has already done. Then we have

$$\Omega_{31} = \{z_{\mathrm{BC}}, z_{\mathrm{CB}}\}. \tag{22}$$

We also define the smaller and larger preemptive thresholds of Ω_{31} as

$$\Omega_{31}^{(1)} := \min \Omega_{31}, \quad \Omega_{31}^{(2)} := \max \Omega_{31}. \tag{23}$$

As in Lemma 1, Firm j with $\Omega_{31}^{(1)}$ is the second investor and enters the market at

$$y_{jk}^{(2)} = \min\{\Omega_{31}^{(2)}, y_j^{2\mathrm{n}}\}. \tag{24}$$

Note that we have $y_{jk}^{(2)} < y_k^{(3)}$ because of the definition of y_{jk}^{P}.

Given that Firms j and k are the second and third firms to enter the market at $y_{jk}^{(2)}$ and $y_k^{(3)}$, we define Firm i's preemptive investment threshold as the first investor as y_{ijk}^{P}. Recall that there are Firms A, B, and C with $I_{\mathrm{A}} < I_{\mathrm{B}} < I_{\mathrm{C}}$. As in Lemma 1, because Firm j is the second investor, it has a lower cost structure than Firm k as the third investor. Based on this result, Firm A's preemptive investment threshold as the first investor in the triopoly market is obtained by $y_{\mathrm{ABC}}^{\mathrm{P}}$, not $y_{\mathrm{ACB}}^{\mathrm{P}}$. Similarly, Firm B's threshold is $y_{\mathrm{BAC}}^{\mathrm{P}}$, and Firm C's threshold is $y_{\mathrm{CAB}}^{\mathrm{P}}$. Firm A's, B's, and C's preemptive investment thresholds are defined by $y_{\mathrm{ABC}}^{\mathrm{P}}$, $y_{\mathrm{BAC}}^{\mathrm{P}}$, and $y_{\mathrm{CAB}}^{\mathrm{P}}$, respectively.

Firm i's preemptive investment threshold is defined by the two value functions for when Firm i invests as the first investor and for when Firm i does not invest as the first investor, respectively. If a firm does not invest as the first investor, the firm is either the second investor or the third investor. As in Lemma 1, the second investor has a lower cost structure than the third investor. Thus, y_{ijk}^{P} is defined depending on the magnitude of I_i and I_k. The preemptive investment threshold for being the first investor, y_{ijk}^{P}, is defined by

$$y_{ijk}^{\mathrm{P}} := \begin{cases} \inf\{y \in [0, y_{jk}^{(2)}] | F_{ijk}^{(1)}(y) \geq C_{ik}^{(2)}(y)\}, & \text{if } I_i \leq I_k, \\ \inf\{y \in [0, y_{jk}^{(2)}] | F_{ijk}^{(1)}(y) \geq C_i^{(3)}(y)\}, & \text{if } I_i > I_k, \end{cases} \tag{25}$$

where $F_{ijk}^{(1)}(y)$ is Firm i's adoption value of entering the market as the first investor given in (11), $C_{ik}^{(2)}(y)$ is Firm i's option value of entering the market as the second investor given in (7), and $C_i^{(3)}(y)$ is Firm i's option value of entering the market as the third investor given in (4). Note that the infimum preemptive incentive threshold (25) is defined by the combination of $F_{ijk}^{(1)}(y)$ and $C_{ij}^{(2)}(y)$ or $F_{ijk}^{(1)}(y)$ and $C_i^{(3)}(y)$.

First, we derive Firm A's preemptive investment threshold of being the first investor $y^{\mathrm{p}}_{\mathrm{ABC}}$. In order to do so, we derive two values. One is Firm A's adoption value where Firm A enters the market as the first investor. The other is Firm A's option value where Firm A does not enter the market as the first investor. On the one hand, suppose that Firm A enters the market as the first investor. Then, Firms B and C compete with each other in order to be the second investor. Firm B is the second investor because $I_{\mathrm{B}} < I_{\mathrm{C}}$. Thus, Firm A's adoption value of being the first investor is $F^{(1)}_{\mathrm{ABC}}(y)$. On the other hand, suppose that Firm A does not enter the market as the first investor. Then there are two cases. One is that Firm B is the first investor. The other is that Firm C is the first investor. Here, we assume that Firm B is the first investor if Firm A is not the first investor. This assumption is reasonable. The reason is as follows. Once Firm A is not the first investor, Firms B and C compete with each other in order to be the first investor. Then Firm B enters the market earlier than Firm C because $I_{\mathrm{B}} < I_{\mathrm{C}}$. In addition, unless Firm B is the first investor once Firm A is not the first investor, we neglect the strategic interaction between Firms B and C for a given strategy of Firm A. From these two reasons, we assume that Firm B is the first investor if Firm A is not the first investor. Thus, if Firm A is not the first investor, Firm A's option value is $C^{(2)}_{\mathrm{AC}}(y)$, where we have used the fact that Firm A is the second investor because $I_{\mathrm{A}} < I_{\mathrm{C}}$. Consequently, $y^{\mathrm{p}}_{\mathrm{ABC}}$ is derived by $F^{(1)}_{\mathrm{ABC}}(y)$ and $C^{(2)}_{\mathrm{AC}}(y)$ in the upper equation of (25). See Appendix D for a detailed discussion.

Second, we derive Firm B's preemptive investment threshold of being the first investor $y^{\mathrm{p}}_{\mathrm{BAC}}$. We derive two values. One is Firm B's adoption value where Firm B enters the market as the first investor. The other is Firm B's option value where Firm B does not enter the market as the first investor. On the one hand, suppose that Firm B enters the market as the first investor. Then, Firms A and C compete with each other in order to be the second investor. Because of $I_{\mathrm{A}} < I_{\mathrm{C}}$, Firm A is the second investor. Thus, Firm B's adoption value of being the first investor is $F^{(1)}_{\mathrm{BAC}}(y)$. On the other hand, suppose that Firm B does not enter the market as the first investor. Then there are two cases. One is that Firm A is the first investor. The other is that Firm C is the first investor. Here, we assume that Firm A is the first investor. The reason is the same as for Firm A's preemptive investment threshold. Thus, if Firm B is not the first investor, Firm B's option value is $C^{(2)}_{\mathrm{BC}}(y)$, where we have used the fact that Firm B is the second investor because $I_{\mathrm{B}} < I_{\mathrm{C}}$. As a result, $y^{\mathrm{p}}_{\mathrm{BAC}}$ is derived by $F^{(1)}_{\mathrm{BAC}}(y)$ and $C^{(2)}_{\mathrm{BC}}(y)$ in the upper equation of (25).

Finally, we derive Firm C's preemptive investment threshold of being the first investor $y^{\mathrm{p}}_{\mathrm{CAB}}$. We derive two values. One is Firm C's adoption value where Firm C enters the market as the first investor. The other is Firm C's option value where Firm C does not enter the market as the first investor. On the one hand, suppose that Firm C enters the market as the first investor. Then, Firms A and B compete with each other in order to be the second investor. Firm A is the second investor because of $I_{\mathrm{A}} < I_{\mathrm{B}}$. Thus, Firm C's adoption value of being the first investor is $F^{(1)}_{\mathrm{CAB}}(y)$. On the other hand, suppose that Firm C does not enter the market as the first investor. Then, Firm A is the first investor because of $I_{\mathrm{A}} < I_{\mathrm{B}}$. In addition, Firms B and C compete with each other in order to be the second investor. Then, because of $I_{\mathrm{B}} < I_{\mathrm{C}}$, Firm B enters the market as the second investor and Firm C is the third investor. Thus, Firm C's option value is $C^{(3)}_{\mathrm{C}}(y)$. As a result, $y^{\mathrm{p}}_{\mathrm{CAB}}$ is derived by $F^{(1)}_{\mathrm{CAB}}(y)$ and $C^{(3)}_{\mathrm{C}}(y)$ in the lower equation of (25).

Then, we have the following result.

Lemma 2 *Consider a triopoly market for firms with asymmetric (heterogeneous) cost structures. Then, the preemptive investment thresholds of being the first investor for Firms A, B, and C are obtained by $y^{\mathrm{p}}_{\mathrm{ABC}}$, $y^{\mathrm{p}}_{\mathrm{BAC}}$, and $y^{\mathrm{p}}_{\mathrm{CAB}}$, respectively.*

Note that these preemptive thresholds for the first investor, y^{p}_{ijk}, do not always exist. Thus, we define z_{ijk} as

$$z_{ijk} := \begin{cases} y^{\mathrm{p}}_{ijk}, & \text{if } y^{\mathrm{p}}_{ijk} \text{ exists,} \\ y^{\mathrm{1n}}_{i}, & \text{otherwise.} \end{cases} \tag{26}$$

From Lemma 2, the set of preemptive investment thresholds as the first investor is defined by

$$\Omega_{30} := \{z_{\mathrm{ABC}}, z_{\mathrm{BAC}}, z_{\mathrm{CAB}}\}, \tag{27}$$

where the subscript "30" of Ω_{30} indicates that the number of firms operating in the triopoly is zero.

Finally, we consider which of the three firms invests as the first investor at the equilibrium. From the definition of y^{p}_{ijk}, Firm i has an incentive as the first investor for the intervals of $y \in [y^{\mathrm{p}}_{ijk}, y^{\mathrm{1n}}_{i}]$ and invests as closely as possible to y^{1n}_{i}. We define the smallest and second-smallest preemptive investment thresholds of Ω_{30} as

$$\Omega^{(1)}_{30} := \min \Omega_{30}, \qquad \Omega^{(2)}_{30} := \min \Omega_{30} \backslash \Omega^{(1)}_{30}, \tag{28}$$

respectively. Then, once the state variable reaches $\Omega^{(1)}_{30}$ starting from a sufficiently small level, the firm with $\Omega^{(1)}_{30}$ has an incentive to enter the market. This implies that the firm with $\Omega^{(1)}_{30}$ is the first investor. As Firm i with $\Omega^{(1)}_{30}$ enters the market as closely as possible to y^{1n}_{i}, Firm i does so as the first investor at

$$y^{(1)}_{ijk} = \min\{\Omega^{(2)}_{30}, y^{\mathrm{1n}}_{i}\}. \tag{29}$$

Note that Firm i with $\Omega_{30}^{(1)}$ invests at $\min\{\Omega_{30}^{(2)}, y_i^{1\mathrm{n}}\}$, not at $\{\Omega_{30}^{(1)}, y_i^{1\mathrm{n}}\}$. After Firm i has invested, the optimal strategies between the two other firms are obtained by Lemma 1. Thus, we have the following result.

Proposition 1 *Consider the triopoly of firms with asymmetric (heterogeneous) cost structures. Then, Firm i as the first investor has $\Omega_{30}^{(1)}$ and Firm j as the second investor has $\Omega_{31}^{(1)}$. Firm i, Firm j, and Firm k enter the market at*

$$y_{ijk}^{(1)*} = \min\{\Omega_{30}^{(2)}, y_i^{1n}\}, \quad y_{jk}^{(2)*} = \min\{\Omega_{31}^{(2)}, y_j^{2\mathrm{n}}\}, \quad y_k^{(3)*} = y_k^{3n}, \tag{30}$$

respectively ($i, j, k \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}, i \neq j \neq k$).

Note that the profile of strategies, $(y_{ijk}^{(1)*}, y_{jk}^{(2)*}, y_k^{(3)*})$, is a Nash equilibrium in that each firm's strategy choice is an optimal response to the strategies of the other firms. Because we cannot derive the closed-form solutions and values, we compare these solutions and values by means of numerical methods in the following section.

4 Model implications

This section considers the investment thresholds and provides several new insights into the three-asymmetric-firm model.

In the numerical examples, we assume the basic parameters as

$$D_1 = 5, D_3 = 2, \sigma = 0.15, r = 0.09, \mu = 0.04, y = 0.1.$$

The two other parameters, $D_2 \in (D_3, D_1)$ and $\theta \geq 0$, are important in our model. On the one hand, if we assume $D_2 = 3.1622$, then we have

$$\frac{D_1}{D_2} = \frac{D_2}{D_3} \approx 1.5811. \tag{31}$$

Thus, $D_2 \in (2, 3.1619)$ and $D_2 \in (3.1619, 5)$ imply

$$\frac{D_1}{D_2} > \frac{D_2}{D_3} > 1, \quad \frac{D_2}{D_3} > \frac{D_1}{D_2} > 1. \tag{32}$$

If the first inequality in (32) is satisfied, the firm's profit rate of a first investor relative to a second investor is larger than that of a second investor relative to a third investor. We call this condition a *relatively large first-mover advantage*. If the second inequality in (32) is satisfied, the firm's profit rate of a first investor relative to a second investor is smaller than that of a second investor relative to a third investor. We call this condition a *relatively small first-mover advantage*. In the numerical examples, we assume $D_2 = 3$ and $D_2 = 3.25$ as large and small first-mover advantages, respectively.

On the other hand, we assume that $I_{\mathrm{A}} = I$, $I_{\mathrm{B}} = (1+\theta)I$, and $I_{\mathrm{C}} = (1+2\theta)I$, where $I = 10$ and $\theta \geq 0$. Then, we assume that $\theta = 0.01$ and $\theta = 0.05$. The larger (smaller) is θ, the larger (smaller) is the degree of cost asymmetry. The parameter θ measures the extent of cost asymmetry across the firms. In the numerical examples, we assume $\theta = 0.01$ and $\theta = 0.05$, implying relatively small and large degrees of cost asymmetry, respectively.

4.1 Who is the first investor?

This subsection examines the investment strategies at the equilibrium.

Consider the case of $D_2 = 3$ and $\theta = 0.01$ (see the first row in Table 1). This case corresponds to a relatively large first-mover advantage (because of $D_1/D_2 \approx 1.6667 > 1.5 = D_2/D_3$) and a relatively small degree of cost asymmetry. First, by following the three steps (i) to (iii), the set of Firm A's, B's, and C's investment thresholds as the first investor Ω_{30} is obtained, i.e.,

$$\Omega_{30} := \{y^{\mathrm{p}}_{\mathrm{ABC}}, y^{\mathrm{p}}_{\mathrm{BAC}}, y^{\mathrm{p}}_{\mathrm{CAB}}\} = \{0.1822, 0.1833, 0.1906\}. \tag{33}$$

Clearly, we have $\Omega^{(1)}_{30} = y^{\mathrm{p}}_{\mathrm{ABC}}$; that is, Firm A has the smallest preemptive investment threshold. Using step (iv), Firm A enters the market as the first investor at $y^{(1)*}_{\mathrm{ABC}} = 0.1833 = \min\{\Omega^{(2)}_{30}, y^{1\mathrm{n}}_{\mathrm{A}}\} = \min\{0.1833, 0.2211\}$. Second, using step (v), we have

$$\Omega_{31} := \{y^{\mathrm{p}}_{\mathrm{BC}}, y^{\mathrm{p}}_{\mathrm{CB}}\} = \{0.2748, 0.2805\}. \tag{34}$$

Because we have $\Omega^{(1)}_{31} = y^{\mathrm{p}}_{\mathrm{BC}}$, Firm B enters the market as the second investor at $y^{(2)*}_{\mathrm{BC}} = 0.2805 = \min\{\Omega^{(2)}_{31}, y^{2\mathrm{n}}_{\mathrm{B}}\} = \min\{0.2805, 0.3722\}$. Finally, Firm C enters the market as the third (last) investor at $y^{(3)*}_{\mathrm{C}} = y^{3\mathrm{n}}_{\mathrm{C}} = 0.5638$. As a result, at the equilibrium, once the state variable starting at $y = 0.1$ reaches $y^{(1)*}_{\mathrm{ABC}} = 0.1833$, $y^{(2)*}_{\mathrm{BC}} = 0.2805$, and $y^{(3)*}_{\mathrm{C}} = 0.5638$, Firms A, B, and C enter the market, respectively.

The upper-left panel of Fig. 1 demonstrates Ω_{30} with respect to θ for fixed $D_2 = 3$. Here, the degree of cost asymmetry θ changes from 0 to 0.05. For $\theta = 0$, as the three firms have the same cost structure, the three thresholds are identical (i.e., $y^{\mathrm{p}}_{\mathrm{ABC}} = y^{\mathrm{p}}_{\mathrm{BAC}} = y^{\mathrm{p}}_{\mathrm{CAB}} = 0.1848$). For $\theta = 0.05$, we have $\Omega_{30} = \{y^{\mathrm{p}}_{\mathrm{ABC}}, y^{\mathrm{p}}_{\mathrm{BAC}}, y^{\mathrm{p}}_{\mathrm{CAB}}\} = \{0.1720, 0.1789, 0.2152\}$ as in the second row in Table 1. Investment thresholds $y^{\mathrm{p}}_{\mathrm{ABC}}$ and $y^{\mathrm{p}}_{\mathrm{BAC}}$ are decreasing with θ while $y^{\mathrm{p}}_{\mathrm{CAB}}$ is increasing with θ. For the regions of $\theta \in (0, 0.05]$, we have $y^{\mathrm{p}}_{\mathrm{ABC}} < y^{\mathrm{p}}_{\mathrm{BAC}} < y^{\mathrm{p}}_{\mathrm{CAB}}$. That is, Firm A has $\Omega^{(1)}_{30}$ and Firm B has $\Omega^{(1)}_{31}$. Thus, Firms A, B, and C are the first, second, and last investors, respectively.

Consider the case of $D_2 = 3.25$ and $\theta = 0.01$ (see the third row in Table 1). The case of $D_2 = 3.25$ implies a relatively small first-mover advantage (i.e., $D_2/D_3 = 1.625 >$

Table 1 Investment thresholds in the triopoly market

		First investor				Second investor				Third investor	
D_2	θ	$y^{\mathrm{p}}_{\mathrm{ABC}}$	$y^{\mathrm{p}}_{\mathrm{BAC}}$	z_{CAB}	Firm	$y^{\mathrm{p}}_{\mathrm{AC}}$	$y^{\mathrm{p}}_{\mathrm{BC}}$	$y^{\mathrm{p}}_{\mathrm{CA}}/y^{\mathrm{p}}_{\mathrm{CB}}$	Firm	$y^{3\mathrm{n}}_{\mathrm{C}}$	Firm
3	0.01	0.1822	0.1833	0.1906	A	–	0.2748	0.2805	B	0.5638	C
3	0.05	0.1720	0.1789	0.2152	A	–	0.2806	0.3092	B	0.6080	C
3.25	0.01	0.2008	0.1998	0.2152	B	0.2402	–	0.2497	A	0.5638	C
3.25	0.05	0.1882	0.1927	0.2432	A	–	0.2492	0.2732	B	0.6080	C

The parameters are $D_1 = 5$, $D_3 = 2$, $\sigma = 0.15$, $r = 0.09$, $\mu = 0.04$, and $y = 0.1$. Under these parameters, $y^{\mathrm{p}}_{\mathrm{CAB}}$ does not always exist while $y^{\mathrm{p}}_{\mathrm{ABC}}$ and $y^{\mathrm{p}}_{\mathrm{BAC}}$ exist. Thus, Firm C's preemptive threshold of being the first investor is defined by z_{CAB}, not $y^{\mathrm{p}}_{\mathrm{CAB}}$.

$1.5385 \approx D_1/D_2$). Then, we have the preemptive investment thresholds of being the first investor as

$$\Omega_{30} := \{y^{\mathrm{p}}_{\mathrm{ABC}}, y^{\mathrm{p}}_{\mathrm{BAC}}, y^{\mathrm{p}}_{\mathrm{CAB}}\} = \{0.2008, 0.1998, 0.2152\}. \tag{35}$$

First, surprisingly, we have $\Omega^{(1)}_{30} = y^{\mathrm{p}}_{\mathrm{BAC}}$, implying that Firm B is the first investor at $y^{(1)*}_{\mathrm{BAC}} = 0.2008 = \min\{\Omega^{(2)}_{30}, y^{1\mathrm{n}}_{\mathrm{B}}\} = \min\{0.2008, 0.2233\}$. Second, we have

$$\Omega_{31} := \{y^{\mathrm{p}}_{\mathrm{AC}}, y^{\mathrm{p}}_{\mathrm{CA}}\} = \{0.2402, 0.2497\}, \tag{36}$$

implying $\Omega^{(1)}_{31} = y^{\mathrm{p}}_{\mathrm{AC}}$. Thus, Firm A is the second investor at $y^{(2)*}_{\mathrm{AC}} = 0.2497 = \min\{\Omega^{(2)}_{31}, y^{2\mathrm{n}}_{\mathrm{A}}\} = \min\{0.2497, 0.3401\}$. Finally, Firm C is the third (last) investor at $y^{(3)*}_{\mathrm{C}} = y^{3\mathrm{n}}_{\mathrm{C}} = 0.5638$.

The upper-right panel of Fig. 1 depicts Ω_{30} with respect to θ for fixed $D_2 = 3.25$. For the same reason as in the upper-left panel, for $\theta = 0$, we have $y^{\mathrm{p}}_{\mathrm{ABC}} = y^{\mathrm{p}}_{\mathrm{BAC}} = y^{\mathrm{p}}_{\mathrm{CAB}} = 0.2030$. For $\theta = 0.05$, we have $\Omega_{30} = \{y^{\mathrm{p}}_{\mathrm{ABC}}, y^{\mathrm{p}}_{\mathrm{BAC}}, z_{\mathrm{CAB}}\} = \{0.1882, 0.1927, 0.2432\}$ as in the fourth row in Table 1. Note that $y^{\mathrm{p}}_{\mathrm{CAB}}$ does not exist for the regions of $\theta \geq 0.015$. That is, we have

$$z_{\mathrm{CAB}} = \begin{cases} y^{\mathrm{p}}_{\mathrm{CAB}}, & \text{if } \theta \in [0, 0.015), \\ y^{1\mathrm{n}}_{\mathrm{C}}, & \text{otherwise.} \end{cases} \tag{37}$$

Most interestingly, Firm B has $\Omega^{(1)}_{30}$ for the regions of $\theta \in (0, 0.0231)$ although Firm A has $\Omega^{(1)}_{30}$ for $\theta \geq 0.0231$.

The lower-left panel of Fig. 1 shows Ω_{30} with respect to D_2 for $\theta = 0.01$. For the regions of $D_2 \in [3, 3.1838)$, Firm A has $\Omega^{(1)}_{30}$. For the other regions, Firm B has $\Omega^{(1)}_{30}$. The lower-right panel depicts Ω_{30} with respect to D_2 for $\theta = 0.05$. For the regions of $D_2 \in [3, 3.39)$, Firm A has $\Omega^{(1)}_{30}$. For the other regions, Firm B has $\Omega^{(1)}_{30}$. Here, because $y^{\mathrm{p}}_{\mathrm{CAB}}$ does not exist for $D_2 \geq 3.1100$, $z_{\mathrm{CAB}} = y^{1\mathrm{n}}_{\mathrm{C}}$ for $D_2 \geq 3.1100$ is constant.

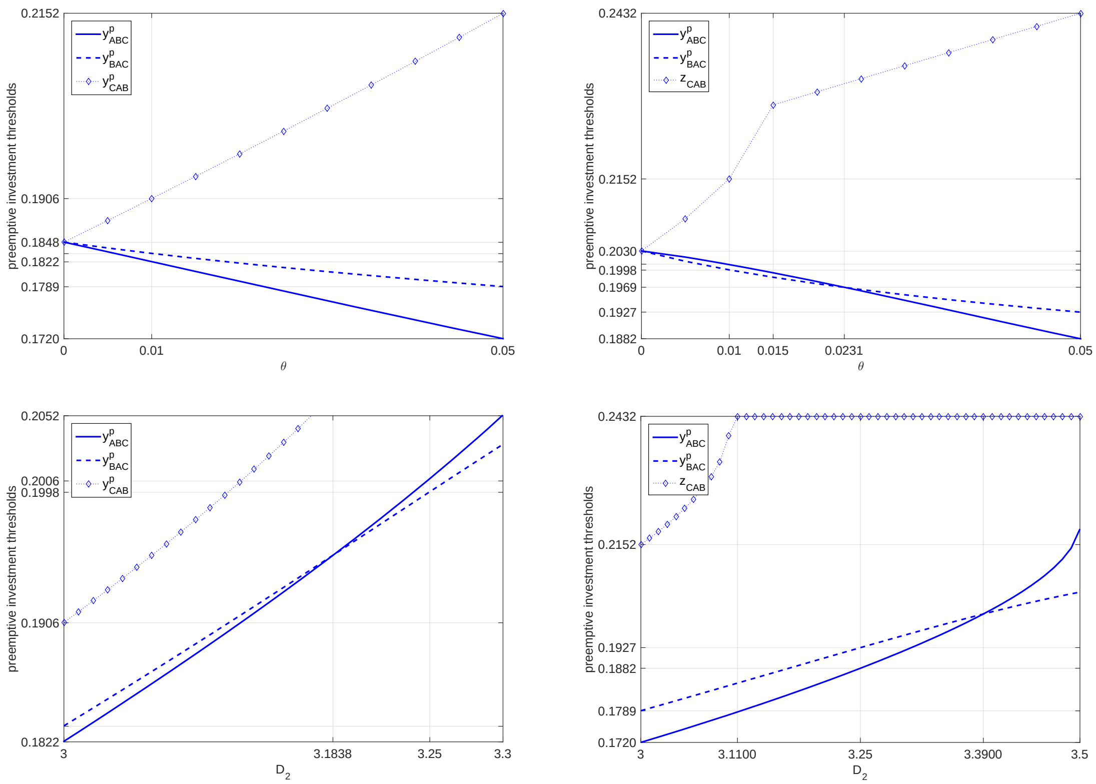

Fig. 1 Effects of cost asymmetry (θ) and first-mover advantage (D_2)

The parameters are $D_1 = 5$, $D_3 = 2$, $\sigma = 0.15$, $r = 0.09$, $\mu = 0.04$, and $y = 0.1$. The upper-left and upper-right panels are assumed to be $D_2 = 3$ and $D_2 = 3.25$, respectively. The lower-left and lower-right panels are assumed to be $\theta = 0.01$ and $\theta = 0.05$, respectively. Note that Firm C's preemptive threshold of being the first investor is defined by z_{CAB}, not $y^{\mathrm{p}}_{\mathrm{CAB}}$ in the upper-right and lower-right panels, because $y^{\mathrm{p}}_{\mathrm{CAB}}$ does not exist for $\theta \geq 0.15$ in the upper-right panel and for $D_2 \geq 3.11$ in the lower-right panel.

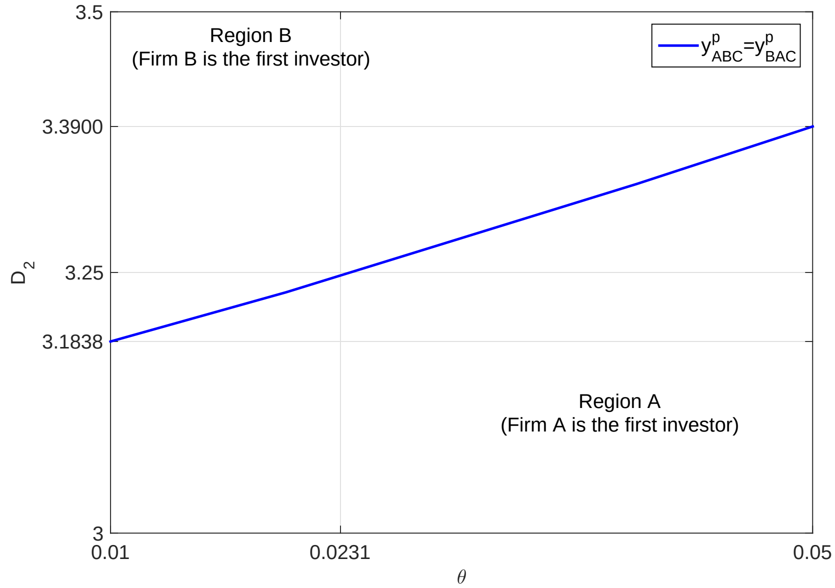

Fig. 2 Region A ($y^{\mathrm{p}}_{\mathrm{ABC}} < y^{\mathrm{p}}_{\mathrm{BAC}}$) and Region B ($y^{\mathrm{p}}_{\mathrm{BAC}} < y^{\mathrm{p}}_{\mathrm{ABC}}$)

The parameters are $D_1 = 5$, $D_3 = 2$, $\sigma = 0.15$, $r = 0.09$, $\mu = 0.04$, and $y = 0.1$. The line represents the boundary of $y^{\mathrm{p}}_{\mathrm{ABC}} = y^{\mathrm{p}}_{\mathrm{BAC}}$. The lower-right and upper-left regions of the boundary are defined by Regions A and B, respectively.

We summarize the results in Fig. 2, which demonstrates the regions where Firms A and B have $\Omega^{(1)}_{30}$ in space (θ, D_2). The line from $(\theta, D_2) = (0.01, 3.1838)$ to $(\theta, D_2) = (0.05, 3.3900)$ indicates the boundary of $y^{\mathrm{p}}_{\mathrm{ABC}} = y^{\mathrm{p}}_{\mathrm{BAC}}$. The lower-right regions of the boundary are defined by Region A, where Firm A has $\Omega^{(1)}_{30}$ (i.e., $y^{\mathrm{p}}_{\mathrm{ABC}} < y^{\mathrm{p}}_{\mathrm{BAC}} < z_{\mathrm{CAB}}$). The upper-left regions of the boundary are defined by Region B, where Firm B has $\Omega^{(1)}_{30}$ (i.e., $y^{\mathrm{p}}_{\mathrm{BAC}} < y^{\mathrm{p}}_{\mathrm{ABC}} < z_{\mathrm{CAB}}$). Note that Fig. 2 corresponds to the four panels of Fig. 1. For example, the line from $(\theta, D_2) = (0.01, 3)$ to $(\theta, D_2) = (0.05, 3)$ corresponds to the upper-left panel, and the line from $(\theta, D_2) = (0.01, 3)$ to $(\theta, D_2) = (0.01, 3.3)$ corresponds to the lower-left panel of Fig. 1, and so on. In addition, see Fig. 3 for Regions A and B in space (θ, D_2) for $\theta \in [0.01, 0.5]$ and $D_2 \in [2, 5]$. Thus, we have the following result.

Observation 2 *Consider the triopoly of firms with asymmetric (heterogeneous) cost structures. The firm with the lowest cost structure (i.e., Firm A) is not always the first firm to enter the market. In particular, when the first-mover advantage and the degree of cost asymmetry are both relatively small, the firm without the lowest cost structure (i.e., Firm B) is likely to be the first investor.*

The result in Observation 2 appears surprising. This result in the three-asymmetric-firm model lies contrary to those in the two-asymmetric-firm model developed by Pawlina and Kort (2006).

The result arises from the strategic interaction between firms. In the two-asymmetric-firm model, Firm i's preemptive (strategic) threshold of being the first investor is obtained as the best response to Firm j's nonpreemptive threshold because the opponent of Firm i is the only other firm. In the three-asymmetric-firm model, by contrast, Firm i's preemptive threshold of being the first investor is decided as the best response to Firm j's preemptive threshold (and Firm k's nonstrategic threshold). That is, the former is derived as the best response to Firm j's nonpreemptive threshold, while the latter is derived as the best response to Firm j's preemptive threshold. As a result, the former is the single preemptive strategy, while the latter is a compound of preemptive strategies. The compound of preemptive strategies provides our interesting result that the firm without the lowest cost structure is possibly the first investor. In other words, the single preemptive strategy does not cause such a result (see Pawlina and Kort (2006)).

4.2 Effects of volatility

This subsection examines the effects of volatility. In the numerical examples, we assume the basic parameters.

Fig. 3 provides the effects of volatility in space (θ, D_2). The line from $(\theta, D_2) = (0.01, 3.1838)$ to $(\theta, D_2) = (0.2340, 5)$ indicates the boundary of $y^{\mathrm{p}}_{\mathrm{ABC}} = y^{\mathrm{p}}_{\mathrm{BAC}}$ for $\sigma = 0.15$. The lower-right regions of the boundary are defined by Region A, where Firm A has $\Omega^{(1)}_{30}$. The upper-left regions of the boundary are defined by Region B, where Firm B has $\Omega^{(1)}_{30}$. The line from $(\theta, D_2) = (0.01, 3.0805)$ to $(\theta, D_2) = (0.2475, 5)$ indicates the boundary of $y^{\mathrm{p}}_{\mathrm{ABC}} = y^{\mathrm{p}}_{\mathrm{BAC}}$ for $\sigma = 0.25$. Thus, we have the following results.

Observation 3 *Consider the triopoly of firms with asymmetric (heterogeneous) cost structures. An increase in volatility enlarges the regions in which the firm without the lowest cost structure is the first investor.*

Consider, e.g., $(\theta, D_2) = (0.1, 3.6)$ corresponding to the point in Fig. 3. Then, Firm A is the first investor for $\sigma = 0.15$, while Firm B is the first investor for $\sigma = 0.25$. Thus, an increase in volatility increases the possibility that the firm without the lowest cost structure is the first investor.

4.3 Practical and economic insights

This subsection provides the practical implication corresponding to our result that the firm without the lowest cost structure enters the market earlier than the firm with the lowest cost structure.

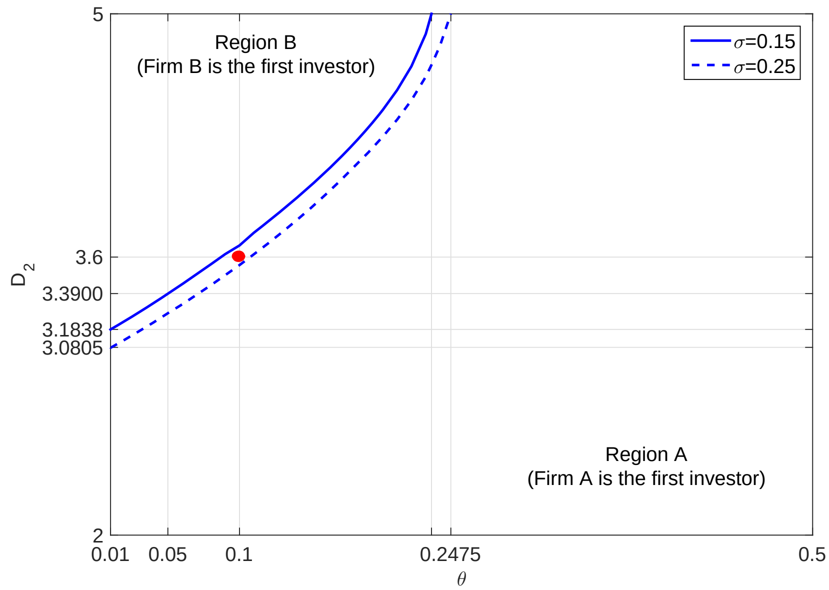

Fig. 3 Effects of volatility

The parameters are $D_1 = 5$, $D_3 = 2$, $\sigma = 0.15$, $r = 0.09$, $\mu = 0.04$, and $y = 0.1$. The two lines indicate the boundary of $y^{\mathrm{p}}_{\mathrm{ABC}} = y^{\mathrm{p}}_{\mathrm{BAC}}$ for a fixed σ ($\sigma = 0.15, 0.25$). We define the lower-right and upper-left regions of the boundary as Regions A and B, respectively.

In practice, our results are consistent with the empirical results. For example, consider the mobile phone market in Japan where the combined market share of three firms (NTT DOCOMO Corporation, KDDI Corporation, and SoftBank Corporation) is more than 95 percent. The incumbent before the liberalization of telecommunications, DOCOMO, possesses a relative cost advantage over any entrants (KDDI and SoftBank). The reason is that the government imposes asymmetric access charges regulation across the three firms. In particular, the access charge for DOCOMO is highest, and the access charge for the remaining two firms is smaller than that for DOCOMO. As technology has spread rapidly in the mobile phone market, the first-mover advantage is *relatively small* although it exists. Thus, whether the cost asymmetry between three firms is relatively small or large decides which of the three firms is the first firm to enter the market.

In such an economic environment, KDDI (the firm with a cost disadvantage) started providing wireless telecommunications services based on "cdmaOne" technology, commonly known as "2.5G" technology, earlier than DOCOMO (the incumbent with a cost advantage) in 1999.[11] This is because there is a relatively small cost asymmetry be-

[11] "2.5G" technology is defined as the state between "2G" (second-generation) and "3G" (third-generation) wireless technology. CDMA stands for Code Division Multiple Access.

tween the three firms. However, DOCOMO was the first firm to start providing those based on "CDMA2000" (and "W-CDMA") technology known as "3G" technology, earlier than the two other companies in 2001. The reason is that there is a relatively large cost asymmetry between the three firms. In addition, DOCOMO plan to start providing wireless telecommunications services based on "LTE-Advanced" ("Long-Term Evolution-Advanced") technology, known as "4G" technology, earlier than any other entrant (KDDI and Softbank) in 2015.[12] This is because there is a relatively large cost asymmetry between the three firms. Thus, our results accord well with the practical findings. To summarize, when there is a relatively small first-mover advantage and relatively small cost asymmetry, KDDI (the firm with a cost disadvantage) was the first firm to start providing the new service. When there is a relatively small first-mover advantage and relatively large cost asymmetry, DOCOMO (the firm with a cost advantage) was the first firm.

In Fig. 3, we have shown that lower volatility reduces the size of the regions where the firm with a cost disadvantage is the first investor. There is a relatively small first mover advantage and a relatively small cost asymmetry for providing "LTE" technology known as "3.9G."[13] The degrees of first-mover advantage and cost asymmetry from "3G" to "3.9G" are similar to those from "2G" to "2.5G." Then, we assumed that KDDI is the first firm to start providing the service based on "3.9G." However, DOCOMO started providing the service based on "3.9G" technology earlier than the other entrants (KDDI and SoftBank) in 2010. The reason is that the volatility is relatively smaller in the shift from "3G" to "3.9G" than in the shift from "2G" to "2.5G." This result corresponds to the point $(\theta, D_2) = (0.1, 3.6)$ in Fig. 3. That is, DOCOMO (the firm with the lowest cost structure) was the first investor under lower volatility, while KDDI (the firm without the lowest cost structure) was the first investor under greater volatility. As a result, the empirical results correspond to those for volatility effects as demonstrated in Fig. 3.

4.4 Nonmonotonic investment threshold

This subsection compares our model (i.e., the three-asymmetric-firm model) with the three-symmetric-firm model developed by Bouis et al. (2009). In the numerical examples, we use the basic parameter values and $D_2 = 3.25$.

As a benchmark, we derive the investment thresholds in the monopoly market and the duopoly market. In the monopoly market model, we assume that there exists only one

[12] One of the requirements for "4G" (also called International Mobile Telecommunications-Advanced as defined by the International Telecommunication Union) is peak data rates up to 1 Gb/s.

[13] "3.9G" is defined as the state between "3G" and "4G."

Table 2 Investment thresholds in the duopoly market

θ	First investor $y^{\mathrm{p}}_{\mathrm{AB}}$	$y^{\mathrm{p}}_{\mathrm{BA}}$	Firm	Second investor $y^{\mathrm{2n}}_{\mathrm{B}}$	Firm
0.01	0.1611	0.1644	A	0.3401	B
0.05	0.1582	0.1748	A	0.3571	B

The parameters are $D_1 = 5$, $D_2 = 3.25$, $D_3 = 2$, $\sigma = 0.15$, $r = 0.09$, $\mu = 0.04$, and $y = 0.1$.

firm, Firm A. Under the basic parameters, Firm A enters the market at the nonpreemptive threshold $y^{(1)*}_{\mathrm{A}} = y^{\mathrm{1n}}_{\mathrm{A}} = 0.2211$.

In the duopoly market model, we assume that there are two firms, Firms A and B. Under $\theta = 0.01$ (corresponding to the first row in Table 2), we have Ω_{20} as

$$\Omega_{20} := \{y^{\mathrm{p}}_{\mathrm{AB}}, y^{\mathrm{p}}_{\mathrm{BA}}\} = \{0.1611, 0.1644\}. \tag{38}$$

Then, Firm A has $\Omega^{(1)}_{20}$, implying that Firm A is the first investor at $y^{(1)*}_{\mathrm{AB}} = 0.1644 = \min\{\Omega^{(2)}_{20}, y^{\mathrm{2n}}_{\mathrm{A}}\} = \{0.1644, 0.2211\}$. Firm B is the second (last) investor at $y^{(2)*}_{\mathrm{B}} = y^{\mathrm{2n}}_{\mathrm{B}} = 0.3401$. Similarly, under $\theta = 0.05$ (corresponding to the second row in Table 2), at the equilibrium, Firms A and B enter the market at $y^{(1)*}_{\mathrm{AB}} = 0.1748$ and $y^{(2)*}_{\mathrm{B}} = 0.3571$.

Suppose that there are three firms, Firms A, B, and C. Under $\theta = 0.01$ (see the third row in Table 1), we have already recognized that, at the equilibrium, Firms B, A, and C enter the market at $y^{(1)*}_{\mathrm{BAC}} = 0.2008$, $y^{(2)*}_{\mathrm{AC}} = 0.2497$, and $y^{(3)*}_{\mathrm{C}} = 0.5638$, respectively. Here, the first investor in the triopoly market is Firm B. Furthermore, under $\theta = 0.05$ (see the fourth row in Table 1), we have already obtained that, at the equilibrium, Firms A, B, and C enter the market at $y^{(1)*}_{\mathrm{ABC}} = 0.1927$, $y^{(2)*}_{\mathrm{BC}} = 0.2732$, and $y^{(3)*}_{\mathrm{C}} = 0.6080$, respectively. Here, the first investor in the triopoly market is Firm A.

We summarize our results in Table 3. Under $\theta = 0.01$, the first investor's investment thresholds are 0.2211 in the monopoly, 0.1644 in the duopoly, and 0.2008 in the triopoly. Under $\theta = 0.05$, the first investor's investment thresholds are 0.2211 in the monopoly, 0.1748 in the duopoly, and 0.1927 in the triopoly. In both cases, $\theta = 0.01$ and $\theta = 0.05$, the first investor's threshold is larger in the triopoly than in the duopoly, although it is smaller in the duopoly than in the monopoly. These results indicate that the preemptive investments need not decrease with an increase in the number of firms from two to three. This property of a "nonmonotonic investment threshold with respect to the number of firms" is the same as in Bouis et al. (2009). Note also that the second investor's threshold is smaller in the triopoly than in the duopoly. We summarize the results as follows.

Table 3 Equilibrium investment thresholds

θ	Market	First investor	Second investor	Third investor
	Monopoly	0.2211	–	–
0.01	Duopoly	0.1644	0.3401	–
	Triopoly	0.2008	0.2497	0.5638
	Monopoly	0.2211	–	–
0.05	Duopoly	0.1748	0.3571	–
	Triopoly	0.1927	0.2732	0.6080

The parameters are $D_1 = 5$, $D_2 = 3.25$, $D_3 = 2$, $\sigma = 0.15$, $r = 0.09$, $\mu = 0.04$, and $y = 0.1$.

Observation 4 *Consider the triopoly of firms with asymmetric (heterogeneous) cost structures. The investment threshold of being the first investor is larger in the triopoly than in the duopoly although it is smaller in the duopoly than in the monopoly.*

Observation 4 supports the presence of a "nonmonotonic investment threshold with respect to the number of firms," even though the model is extended to the asymmetric-three-firm model from the symmetric-three-firm model.

Fig. 4 depicts the investment thresholds of being the first investor for the various values of σ. Here, σ is changed from 0.15 to 0.25 for $\theta \in \{0.01, 0.05\}$. In the left panel ($D_2 = 3.25$, $\theta = 0.01$), $y^{\mathrm{p}}_{\mathrm{ABC}}$ and $y^{\mathrm{p}}_{\mathrm{BAC}}$ exist for all the regions of $\sigma \in [0.15, 0.25]$. In contrast, $y^{\mathrm{p}}_{\mathrm{CAB}}$ exists for $\sigma \in [0.1, 0.1965)$ while $y^{\mathrm{p}}_{\mathrm{CAB}}$ does not exist (i.e., $z_{\mathrm{CAB}} = y^{\mathrm{1n}}_{\mathrm{C}}$) for $\sigma \in [0.1965, 0.25]$. In the right panel ($D_2 = 3.25$, $\theta = 0.05$), $y^{\mathrm{p}}_{\mathrm{ABC}}$ and $y^{\mathrm{p}}_{\mathrm{BAC}}$ exist for all the regions of $\sigma \in [0.15, 0.25]$. We have $z_{\mathrm{CAB}} = y^{\mathrm{1n}}_{\mathrm{C}}$ for all the regions of $\sigma \in [0.15, 0.25]$. Importantly, in both panels, we see that $y^{\mathrm{p}}_{\mathrm{ABC}}$ and $y^{\mathrm{p}}_{\mathrm{BAC}}$ are larger than $y^{\mathrm{p}}_{\mathrm{AB}}$ and $y^{\mathrm{p}}_{\mathrm{BA}}$.

5 Concluding remarks

In this paper, we considered the optimal investment timing strategies for three asymmetric firms. For tractability, Bouis et al. (2009) assume that there are three symmetric firms in the triopoly market. The contribution of our paper is to extend the investment timing decision problem from that of three symmetric firms to three asymmetric firms.

Our paper proves the following novel insight into triopoly markets for firms with asymmetric (heterogeneous) cost structures. First, in the case of a relatively small first-mover advantage and a relatively small cost asymmetry, the firm without the lowest cost structure has the possibility of being the first investor. In other words, the lowest-cost firm does not always enter the market as the first investor at the equilibrium. We obtained this

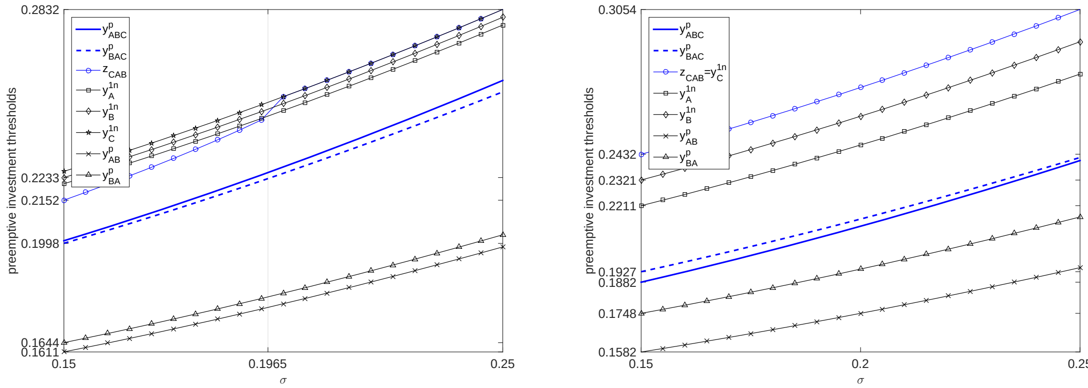

Fig. 4 Nonmonotonic investment thresholds

The parameters are $D_1 = 5$, $D_2 = 3.25$, $D_3 = 2$, $\sigma = 0.15$, $r = 0.09$, $\mu = 0.04$, and $y = 0.1$. The left and right panels are assumed to be $\theta = 0.01$, $\theta = 0.05$, respectively. We see that the preemptive investment thresholds of being the first investor in the triopoly are in between the monopoly and duopoly.

new result by increasing the number of asymmetric firms from two to three. For all other cases, by contrast, the firm with the lowest cost structure is the first investor. This result is the same as in the duopoly market of firms with asymmetric cost structures. Second, we show that an increase in volatility increases the possibility that the firm without the lowest cost structure is the first investor. We also obtained this new result by increasing the number of asymmetric firms from two to three. Finally, we show that the first investor's preemptive investment threshold is larger in the triopoly than in the duopoly. Thus, the three-asymmetric-firm model displays the same "nonmonotonic investment threshold" property as the three-symmetric-firm model developed by Bouis et al. (2009).

For future research, it would be worthwhile extending the investment timing problem from three to four or more asymmetric firms. This analysis provides a suitable foundation for such future research.

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Appendix A. Value functions for the monopoly and duopoly markets

From the derivations of the value functions in the triopoly, it is straightforward to obtain the value functions in the monopoly and duopoly markets.

Value function for the monopoly market

In the monopoly, we assume that there is only a single Firm i ($i \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$). Suppose any time $t \in [0, \tau_i^{(1)}]$, i.e., $Y(t) \leq y_i^{(1)}$. Firm i's option value is given by

$$C_i^{(1)}(Y(t)) = \Big(\frac{Y(t)}{y_i^{(1)}}\Big)^{\beta} F_i^{(1)}(y_i^{(1)}), \tag{A.1}$$

where $F_i^{(1)}(y_i^{(1)})$ is given as

$$F_i^{(1)}(y_i^{(1)}) = \frac{D_1}{r-\mu} y_i^{(1)} - I_i. \tag{A.2}$$

Note that $D_2 = D_3 = 0$ in the monopoly market. In addition, note that there is one component "i" in the subscripts of $C_i^{(1)}$ and $F_i^{(1)}$ in (A.1) and (A.2).

Value function for the duopoly market

In the duopoly, we assume that there are Firms i and j ($i, j \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}; i \neq j$). On the one hand, suppose any time $t \in [0, \tau_{ij}^{(1)}]$, i.e., $Y(t) \leq y_{ij}^{(1)}$. Firm i's option value of being the first investor is

$$C_{ij}^{(1)}(Y(t)) = \Big(\frac{Y(t)}{y_{ij}^{(1)}}\Big)^{\beta} F_{ij}^{(1)}(y_{ij}^{(1)}), \tag{A.3}$$

where $F_{ij}^{(1)}(y_{ij}^{(1)})$ is given as

$$F_{ij}^{(1)}(y_{ij}^{(1)}) = \frac{D_1}{r-\mu} y_{ij}^{(1)} - I_i + \Big(\frac{y_{ij}^{(1)}}{y_j^{(2)}}\Big)^{\beta} \frac{D_2 - D_1}{r-\mu} y_j^{(2)}. \tag{A.4}$$

Note that $D_3 = 0$ in the duopoly market. In addition, note that there are two components "ij" in the subscripts of $C_{ij}^{(1)}$ and $F_{ij}^{(1)}$ in (A.3) and (A.4). On the other hand, suppose any time $t \in [\tau_{ij}^{(1)}, \tau_j^{(2)}]$, i.e., $Y(t) \leq y_j^{(2)}$. Firm j's option value of being the second investor is

$$C_j^{(2)}(Y(t)) = \Big(\frac{Y(t)}{y_j^{(2)}}\Big)^{\beta} F_j^{(2)}(y_j^{(2)}), \tag{A.5}$$

where $F_j^{(2)}(y_j^{(2)})$ is given as

$$F_j^{(2)}(y_j^{(2)}) = \frac{D_2}{r-\mu} y_j^{(2)} - I_j. \tag{A.6}$$

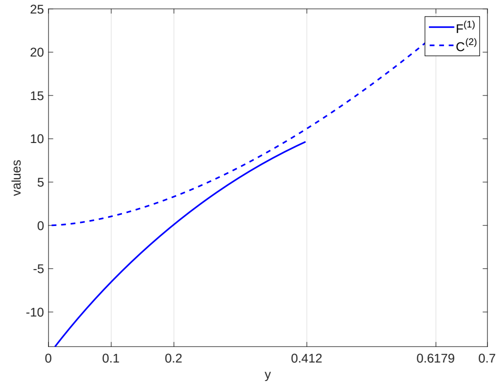

Fig. 5 Nonexistence of preemptive investment thresholds

This figure demonstrates nonexistence of preemptive investment thresholds as the first investor in the duopoly market.

Appendix B. Nonexistence of preemptive thresholds

In this Appendix, we illustrate the nonexistence of preemptive (strategic) investment thresholds.

Suppose Firms A and C are in the duopoly market. Also assume that the degree of cost asymmetry is large; i.e., $\theta = 0.25$ (then we have $I_{\mathrm{A}} = 10$ and $I_{\mathrm{C}} = 15$). The other parameters are $\sigma = 0.2$, $r = 0.09$, $\mu = 0.04$, $D_1 = 5$, $D_2 = 3$, $D_3 = 2$ as the basic parameters. Fig. 5 depicts the value $C_{\mathrm{C}}^{(2)}(y)$ and $F_{\mathrm{CA}}^{(1)}(y)$, where $F_{\mathrm{CA}}^{(1)}(y)$ and $C_{\mathrm{C}}^{(2)}(y)$ are defined by the intervals of $[0, y_{\mathrm{A}}^{2\mathrm{n}}] = [0, 0.4120]$ and $[0, y_{\mathrm{C}}^{2\mathrm{n}}] = [0, 0.6179]$. We see that $F_{\mathrm{CA}}^{(1)}(y)$ is always smaller than $C_{\mathrm{C}}^{(2)}(y)$. Thus, there does not exist the preemptive threshold $y_{\mathrm{CA}}^{\mathrm{p}}$ because Firm C does not have an incentive to preempt. This is likely for the cost-disadvantaged firm when the degree of cost asymmetry is large.

Appendix C. Preemptive thresholds in the two-asymmetric-firm model

In this Appendix, we confirm that $y_{ij}^{\mathrm{p}} < y_{ji}^{\mathrm{p}}$ if $I_i < I_j$ in the two-asymmetric-firm model.

We assume the parameter values as before (i.e., $D_1 = 5$, $D_3 = 2$, $\sigma = 0.15$, $r = 0.09$, $\mu = 0.04$, and $y = 0.1$). The degree of cost asymmetry is small; i.e., $\theta = 0.01$ (then we have $I_{\mathrm{A}} = 10$, $I_{\mathrm{B}} = 10.1$, and $I_{\mathrm{C}} = 10.2$). Fig. 6 depicts Firm i's preemptive investment threshold y_{ij}^{p} when Firms i and j enter the market as the first and second investors, respectively ($i, j \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \}; i \neq j$). The left-panel depicts the preemptive investment threshold in the duopoly market for Firms A and B. We see $y_{\mathrm{AB}}^{\mathrm{p}} < y_{\mathrm{BA}}^{\mathrm{p}}$ with θ. The

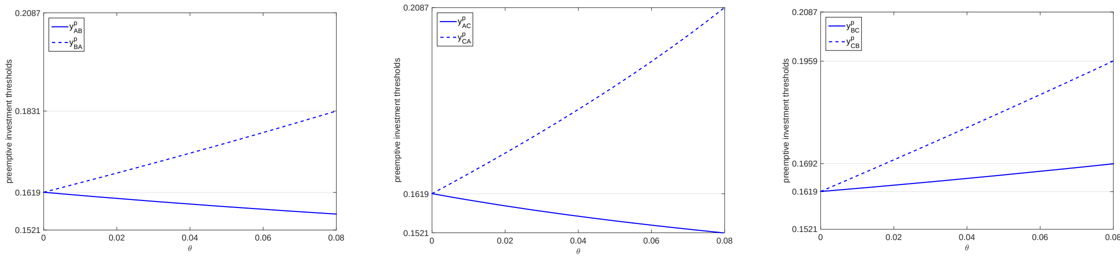

Fig. 6 Preemptive thresholds in the two-asymmetric-firm model

In the duopoly market of firms with asymmetric cost structure, we have $y^{\mathrm{p}}_{ij} < y^{\mathrm{p}}_{ji}$ if $I_i < I_j$ ($i \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}, i \neq j \neq k$).

middle-panel demonstrates the preemptive investment threshold for Firms A and C. We have $y^{\mathrm{p}}_{\mathrm{AC}} < y^{\mathrm{p}}_{\mathrm{CA}}$. The right-panel illustrates the preemptive threshold for Firms B and C. We see $y^{\mathrm{p}}_{\mathrm{BC}} < y^{\mathrm{p}}_{\mathrm{CB}}$.

Appendix D. Preemptive thresholds in the three-asymmetric-firm model

In this Appendix, we consider the preemptive thresholds of being the first investor in the three-asymmetric-firm model. Recall that $y^{\mathrm{p}}_{\mathrm{ABC}}$ and $y^{\mathrm{p}}_{\mathrm{BAC}}$ are derived as

$$y^{\mathrm{p}}_{\mathrm{ABC}} := \inf\{y \in [0, y^{(2)}_{\mathrm{BC}}] | F^{(1)}_{\mathrm{ABC}}(y) \geq C^{(2)}_{\mathrm{AC}}(y)\}, \tag{A.7}$$
$$y^{\mathrm{p}}_{\mathrm{BAC}} := \inf\{y \in [0, y^{(2)}_{\mathrm{AC}}] | F^{(1)}_{\mathrm{BAC}}(y) \geq C^{(2)}_{\mathrm{BC}}(y)\}, \tag{A.8}$$

respectively. Alternatively, $y^{\mathrm{p}}_{\mathrm{ABC}}$ and $y^{\mathrm{p}}_{\mathrm{BAC}}$ may be defined as

$$y^{\mathrm{p2}}_{\mathrm{ABC}} := \inf\{y \in [0, y^{(2)}_{\mathrm{BC}}] | F^{(1)}_{\mathrm{ABC}}(y) \geq C^{(2)}_{\mathrm{AB}}(y)\}, \tag{A.9}$$
$$y^{\mathrm{p2}}_{\mathrm{BAC}} := \inf\{y \in [0, y^{(2)}_{\mathrm{AC}}] | F^{(1)}_{\mathrm{BAC}}(y) \geq C^{(3)}_{\mathrm{B}}(y)\}, \tag{A.10}$$

respectively.

First, we consider Firm A's preemptive investment threshold of being the first investor defined by (A.9). Then, Firm A's option value is defined by $C^{(2)}_{\mathrm{AB}}(y)$, which is different from $C^{(2)}_{\mathrm{AC}}(y)$ in (A.7). This implies that the first investor is assumed to be Firm C if Firm A is not the first investor. This assumption does not make sense. The reason is as follows. If Firm A is not the first investor, Firms B and C compete with each other to be the first investor. Then, because $I_{\mathrm{B}} < I_{\mathrm{C}}$, Firm B is the first investor. Thus, we neglect the strategic interaction between Firms B and C for a given strategy of Firm A under the preemptive threshold defined by (A.9).

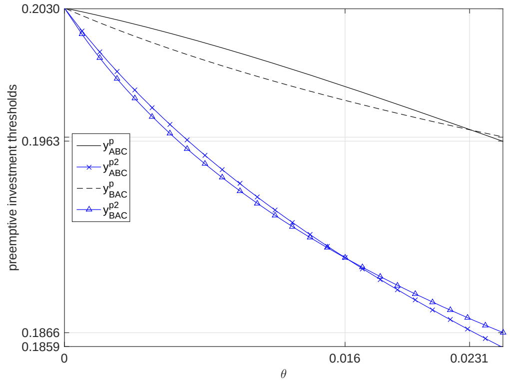

Fig. 7 Preemptive investment thresholds in the three-asymmetric-firm model

The parameters are $D_1 = 5$, $D_2 = 3.25$, $D_3 = 2$, $\sigma = 0.15$, $r = 0.09$, $\mu = 0.04$, and $y = 0.1$. We see that $y^{\mathrm{p2}}_{\mathrm{BAC}} \leq y^{\mathrm{p2}}_{\mathrm{ABC}}$ for $\theta \in [0, 0.016)$.

Next, we consider Firm B's preemptive investment threshold of being the first investor defined by (A.10). Then, Firm B's option value is defined by $C^{(3)}_{\mathrm{B}}(y)$, which is different from $C^{(2)}_{\mathrm{BC}}(y)$ in (A.8). This means that the first investor is assumed to be Firm C if Firm B is not the first investor. This assumption does not make sense for exactly the same reason as for Firm A's preemptive threshold of being the first investor. Thus, we neglect the strategic interaction between Firms A and C for a given strategy of Firm B under the preemptive threshold defined by (A.10).

To summarize, in $y^{\mathrm{p}}_{\mathrm{ABC}}$ in (A.7) and $y^{\mathrm{p}}_{\mathrm{BAC}}$ in (A.8), we consider the strategic interaction between the other two firms for a given firm. By contrast, In $y^{\mathrm{p2}}_{\mathrm{ABC}}$ in (A.9) and $y^{\mathrm{p2}}_{\mathrm{BAC}}$ in (A.10), we neglect the strategic interaction between the other two firms for a given firm. As a result, it is rational that Firms A's and B's preemptive investment thresholds are defined by $y^{\mathrm{p}}_{\mathrm{ABC}}$ and $y^{\mathrm{p}}_{\mathrm{BAC}}$, respectively.

Finally, in order to check that our conclusion drawn in this contribution even applies in a hypothetic case when Firm A's and B's preemptive thresholds were defined by $y^{\mathrm{p2}}_{\mathrm{ABC}}$ and $y^{\mathrm{p2}}_{\mathrm{BAC}}$, we calculate the values of $y^{\mathrm{p2}}_{\mathrm{ABC}}$ and $y^{\mathrm{p2}}_{\mathrm{BAC}}$. Fig. 7 depicts $y^{\mathrm{p2}}_{\mathrm{ABC}}$ and $y^{\mathrm{p2}}_{\mathrm{BAC}}$ with respect to θ for a fixed $D_2 = 3.25$. The other parameters are the same as in the basic parameters defined in Section 4. We see $y^{\mathrm{p2}}_{\mathrm{BAC}} < y^{\mathrm{p2}}_{\mathrm{ABC}}$ for $D_2 \in (0, 0.016)$ although $y^{\mathrm{p2}}_{\mathrm{BAC}} \geq y^{\mathrm{p2}}_{\mathrm{ABC}}$ for $D_2 \geq 0.016$. We see that, from these numerical examples, there is indeed a possibility that the firm without the lowest cost (i.e., Firm B) is the first investor, even if Firms A's and B's preemptive thresholds were defined by $y^{\mathrm{p2}}_{\mathrm{ABC}}$ and $y^{\mathrm{p2}}_{\mathrm{BAC}}$.

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