Research Paper Series

No. 143

Investment timing and quantity strategies under asymmetric information

Xue Cui[†] and Takashi Shibata[‡]

Jan, 2015

[†]Graduate School of Social Sciences, Tokyo Metropolitan University
[‡]Graduate School of Social Sciences, Tokyo Metropolitan University

Investment timing and quantity strategies under asymmetric information

Xue Cui[a*] and Takashi Shibata[a†]

[a] *Graduate School of Social Sciences, Tokyo Metropolitan University, 1-1 Minami-osawa, Hachioji, Tokyo 192-0397, Japan.*

(First Version: January 13, 2015)

Abstract:
We extend the asymmetric information problem by incorporating not only the investment timing decision but also the investment quantity strategy. We show that investment timing is more delayed under asymmetric information than under full information, implying a decrease in the value of equity. However, in order to minimize this inefficiency, investment quantity is larger under asymmetric information than under full information. Thus, there are trade-offs between the efficiencies of investment timing and investment quantities under asymmetric information.

Keywords:
Investment under uncertainty; real options; informational rent; private information.

*Email: xcuiaa@gmail.com
†Corresponding author. Email: tshibata@tmu.ac.jp. Phone:+81-42-677-2310.

1 Introduction

The real options model has become a standard framework for corporate investment decisions. In the standard real options model, the firm is assumed to be managed by owners.[1] In most modern corporations, however, owners delegate the corporate management to managers, taking advantage of managers' special skills and expertise. In this situation, asymmetric information is likely to exist, because managers have private information. Thus, asymmetric information leads to agency conflicts between owners and managers.

Recently, several studies have considered the asymmetric information between owners and managers and incorporated this into the real options model. Grenadier and Wang (2005) develop the real options model in the presence of asymmetric information between owners and managers.[2] In such a situation, owners must design a contract to provide mechanisms for managers to reveal private information truthfully. The implied investment timing is then delayed, compared with that under full (symmetric) information, which leads to a decrease in the stock price (owners' value). Although these strategies turn out to be suboptimal, they reduce the owners' losses arising from asymmetric information. Without any mechanism that induces managers to reveal private information truthfully, owners suffer further distortions.

To the best of our knowledge, there has been no examination of the investment quantity strategy in a real options model under asymmetric information. Therefore, the following questions are unanswered. How does asymmetric information influence investment quantity? How does asymmetric information affect the interaction between the investment timing and quantity strategies? Thus, in this study, we consider the interactions between the investment timing and quantity strategies. In particular, we extend the Grenadier and Wang (2005) model by incorporating the investment quantity decision.

We show that the investment timing is more delayed under asymmetric information than under full information, implying a decrease in the value of equity. However, in order to minimize this inefficiency, the investment quantity is larger under asymmetric information than under full information. These results imply that the efficiency of the investment timing is lower under asymmetric information than under full information, while the efficiency of investment quantity is higher under asymmetric information than under full information. Thus, there are trade-offs between the efficiencies of investment timing and investment quantities under asymmetric information.

This paper proceeds as follows. Section 2 describes the framework of our model. It is useful to consider the full information problem as a benchmark before analyzing the asymmetric information problem. Section 3 provides the solution to the asymmetric

[1] For the interested reader, Dixit and Pindyck (1994) provide an excellent overview of the standard real options approach.

[2] See Nishihara and Shibata (2008) and Shibata (2009) for the extension of the Grenadier and Wang model.

information problem. We then discuss the properties of the solution by using numerical examples. Section 4 concludes. The appendix contains details of the derivation of the solutions.

2 Model setup

In this section, we begin with a description of the model framework. We then, formulate the asymmetric information problem. Finally, as a benchmark, we present the solution to the full information problem.

2.1 Setup

The owner of a firm has the option to invest in a single project. We assume that the owner (principal) delegates the investment decision to a manager (agent). Throughout our analysis, we assume that the owner and the manager are risk neutral and aim to maximize their expected pay-offs.

If the investment option is exercised at time t, the firm pays the one-time fixed cost to initiate the project I and receives cash flow δX_t after time t. Here, X_t follows a geometric Brownian motion:

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}z_t, \quad X_0 = x, \tag{1}$$

where z_t denotes a standard Brownian motion, and where $\mu > 0$ and $\sigma > 0$ are positive constants. For convergence, we assume that $r > \mu$ where $r > 0$ is a constant interest rate.[3]

Alternatively, $\delta \geq 1$ represents the project quantity and incurs a cost $c(\delta) \geq 0$ with $c'(\delta) > 0$ and $c''(\delta) > 0$ for any δ. These conditions are intuitively reasonable. The first and second conditions mean that $c(\delta)$ is strictly increasing and convex with δ. Note that at the time of investment, δ is *endogenously chosen* to maximize the owners' profits.[4]

We assume that the one-time fixed cost to initiate the project, I, takes one of two possible values: I_1 or I_2 with $I_2 > I_1$, where $I_i > 0$ for all $i \in \{1, 2\}$. We denote $\Delta I := I_2 - I_1 > 0$. We assume that I_1 represents "lower cost" expenditure and I_2 represents "higher cost" expenditure. The probability of drawing I_1 equals $\mathbb{P}(I_1) = q > 0$, an exogenous variable.

We assume that the cash flow, X_t, is observed by both the owner and the manager. However, the one-time cost, I, is observed privately only by the manager.[5] Immediately after making a contract with the owner at time zero, the manager observes whether the

[3]The assumption $r > \mu$ is needed to ensure that the value of the firm is finite.

[4]In the standard model in Dixit and Pindyck (1994), $\delta = 1$ is assumed to be exogenous.

[5]The assumption that a portion of the project value is observed privately only by one person (here, the manager) and not observed by the other (here, the owner) is quite common in the asymmetric information

cost expenditure is of "lower cost" or "higher cost." On the other hand, the owner cannot observe the true value of I. Therefore, the owner must induce the manager to reveal private information truthfully at the time when the manager undertakes the investment. Otherwise, the owner suffers from further losses. Suppose, for example, the manager observes $I = I_1$ as the realized value of I. Then the manager diverts the difference $\Delta I > 0$ to himself/herself by reporting $I = I_2$ to the owner. To prevent the diversion, the owner must encourage the manager to report the true value by providing incentives.

It is useful to provide the present value operator of one dollar received at the first moment that X_t reaches the threshold x_i. Let $x_i = x(I_i)$ and $\delta_i = \delta(I_i)$ denote the investment threshold and quantity for $I = I_i$ ($i \in \{1, 2\}$). Let τ_i denote the stopping time at which the investment is exercised once X_t arrives at the trigger x_i, i.e., $\tau_i := \inf\{t \geq 0 | X_t = x_i\}$. Let $\mathbb{E}^x[\mathrm{e}^{-r\tau_i}]$ represent the discounted present value operator where $\mathbb{E}^x[\cdot]$ denotes the expectation operator given that $X_0 = x$. Using standard arguments, $\mathbb{E}^x[\mathrm{e}^{-r\tau_i}]$ is obtained by

$$\mathbb{E}^x[\mathrm{e}^{-r\tau_i}] = \left(\frac{x}{x_i}\right)^{\beta}, \quad i \in \{1, 2\}, \tag{2}$$

where $x < x_i$ and $\beta = 1/2 - \mu/\sigma^2 + \sqrt{(\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2)} > 1$.

2.2 Asymmetric information model

In this subsection, we formulate the owner's maximization problem under asymmetric information. As explained earlier, under asymmetric information, the owner must induce the manager to reveal the manager's private information truthfully.

In this study, we assume that the owner designs a contract at time zero, which commits the owner to give the incentive to the manager at the time of investment. Renegotiation is not allowed. While commitment may cause *ex post* inefficiency in investment timing, it increases the *ex ante* owner's option value. To motivate the manager to reveal private information, we assume that the owner provides a bonus-incentive w_i to the manager at the time of investment.

Thus, the contract in the asymmetric information problem is modeled as a mechanism:

$$\mathcal{M}^{**} = (x_i, \delta_i, w_i), \quad i \in \{1, 2\}.$$

Let superscript "$**$" refer to the *asymmetric information* problem.[6] Note that the contract $\mathcal{M}^{**}$ is composed of the threshold x_i, quantity δ_i, and bonus w_i for any i.

literature. An excellent overview of the analysis of asymmetric information situations is found in Laffont and Martimort (2002).

[6]Because at the equilibrium, the manager reveals the true I_i as private information, we make no distinction between the reported $\tilde{I}_i$ and the true I_i.

Then, the asymmetric information problem is to maximize the owner's option value through choice of the mechanism $\mathcal{M}^{**}$, i.e.,

$$\max_{x_1,x_2,\delta_1,\delta_2,w_1,w_2} \quad \sum_{i=1}^{2} \mathbb{P}(I_i)\left(\frac{x}{x_i}\right)^{\beta}\{\delta_i x_i - I_i - w_i - c(\delta_i)\}, \tag{3}$$

subject to

$$\left(\frac{x}{x_1}\right)^{\beta} w_1 \geq \left(\frac{x}{x_2}\right)^{\beta}(w_2 + \Delta I), \tag{4}$$

$$\left(\frac{x}{x_2}\right)^{\beta} w_2 \geq \left(\frac{x}{x_1}\right)^{\beta}(w_1 - \Delta I), \tag{5}$$

$$q\left(\frac{x}{x_1}\right)^{\beta} w_1 + (1-q)\left(\frac{x}{x_2}\right)^{\beta} w_2 \geq 0, \tag{6}$$

$$w_i \geq 0, \quad i \in \{1,2\}. \tag{7}$$

Here, the objective function (3) is the *ex ante* owner's option value. Note that the problem for a fixed δ (e.g., $\delta = 1$) is the same as that in Grenadier and Wang (2005).

Constraints (4) and (5) are the *ex post* incentive-compatibility constraints for the manager under states I_1 and I_2, respectively. Consider, for example, constraint (4). The manager's payoff in state I_1 is $(x/x_1)^{\beta} w_1$ if he/she tells the truth, but it is $(x/x_2)^{\beta}(w_2+\Delta I)$ if he/she instead claims that it is state I_2. Thus, he/she tells the truth if (4) is satisfied. Constraint (5) follows similarly.

Constraints (6) and (7) are the *ex ante* participation constraint and the *ex post* limited-liability constraints, respectively. Constraint (6) ensures that the manager makes an agreement about employment.

Before analyzing the asymmetric information problem, we first briefly review the symmetric information problem.

2.3 Full information benchmark

In this subsection, we consider the optimization problem when the owner observes the true value of I. This problem is equivalent to the problem in which there is no delegation of the investment decision because the manager has no informational advantage. Then we have $w_i = 0$ for all i ($i \in \{1,2\}$). Thus, the contract $\mathcal{M}^*$ in the full information problem is modeled as

$$\mathcal{M}^* = (x_i, \delta_i), \quad i \in \{1,2\}.$$

Let superscript "$*$" refer to the *full (symmetric) information* problem. The owner's maximization problem is defined as

$$\max_{x_1,x_2,\delta_1,\delta_2} qV(x, x_1, \delta_1; I_1) + (1-q)V(x, x_2, \delta_2; I_2), \tag{8}$$

where $x < x_i$ for any i ($i \in \{1,2\}$) and

$$V(x, x_i, \delta_i; I_i) := \left(\frac{x}{x_i}\right)^{\beta} \{\delta_i x_i - I_i - c(\delta_i)\}. \tag{9}$$

Then, we have the following result (see the proof in the appendix).

Proposition 1 *Suppose the full information problem. For any i ($i \in \{1,2\}$), δ_i^* is obtained by solving the following equation:*

$$c'(\delta_i^*)\delta^* = \frac{\beta}{\beta - 1}(I_i + c(\delta_i^*)), \tag{10}$$

and x_i^* is given by

$$x_i^* = \frac{\beta}{\beta - 1} \frac{I_i + c(\delta_i^*)}{\delta_i^*}. \tag{11}$$

The owner's optimal value is given by

$$o^*(x) := qV(x, x_1^*, \delta_1^*; I_i) + (1-q)V(x, x_2^*, \delta_2^*; I_2). \tag{12}$$

We use the results in Proposition 1 as a benchmark.

3 Model solution

In this section, we provide the solution to the asymmetric information problem that was described in the previous section. We then discuss some properties of the solution.

Although the optimization problem is subject to five inequality constraints, we can simplify the problem through the following three steps. First, (7) implies (6). Second, unlike a manager in state I_1, a manager in state I_2 does not have the incentive to tell a lie. This is because the manager in state I_2 suffers a loss from such a false announcement. Thus, (5) is satisfied automatically, and $w_2^{**} = 0$ is obtained at the optimum. Finally, suppose that (4) holds as a strict inequality. Then, by decreasing w_1, the owner's value is increased. Thus, (4) is binding, implying that we have $w_1^{**} = (x_1/x_2)^{\beta}\Delta I$.

As a result, the simplified optimization problem is as follows:

$$\max_{x_1, x_2, \delta_1, \delta_2} qV(x, x_1, \delta_1; I_1) + (1-q)V(x, x_2, \delta_2; I_2 + \phi\Delta I), \tag{13}$$

where $\phi := q/(1-q) > 0$ and $x < x_i$ for all i ($i \in \{1,2\}$). We obtain the following results (see the proof in the appendix).

Proposition 2 *Suppose the asymmetric information problem. The solutions for $I = I_1$ are $x_1^{**} = x_1^*$, $\delta_1^{**} = \delta_1^*$, and $w_1^{**} = (x_1^*/x_2^{**})^{\beta}\Delta I$. For $I = I_2$, δ_2^{**} is obtained by solving the following equation:*

$$c'(\delta_2^{**})\delta_2^{**} = \frac{\beta}{\beta - 1}\Big(I_2 + \phi\Delta I + c(\delta_2^{**})\Big), \tag{14}$$

x_2^{**} is given by

$$x_2^{**} = \frac{\beta}{\beta - 1} \frac{1}{\delta_2^{**}} \Big(I_2 + \phi \Delta I + c(\delta_2^{**}) \Big), \tag{15}$$

and $w_2^{**} = 0$. The optimal owner's value is obtained by

$$o^{**}(x) = qV(x, x_1^*, \delta_1^*; I_1) + (1 - q)V(x, x_2^{**}, \delta_2^{**}; I_2 + \phi \Delta I), \tag{16}$$

and the optimal manager's value is obtained by

$$m^{**}(x) = q \Big(\frac{x}{x_2^{**}} \Big)^{\beta} \Delta I. \tag{17}$$

In Proposition 2, there are three important remarks. First, we have $x_1^{**} = x_1^*$, $\delta_1^{**} = \delta_1^*$, $x_2^{**} \neq x_2^*$, and $\delta_2^{**} \neq \delta_2^*$. It is less costly for the owner to distort (x_2^{**}, δ_2^{**}) away from (x_2^*, δ_2^*) than to distort (x_1^{**}, δ_1^{**}) away from (x_1^*, δ_1^*). Second, we have $w_1^{**} \in (0, \Delta I)$ and $w_2^{**} = 0$. Because $\Delta I > 0$ can be regarded as the *informational rent* for the manager in I_1, the owner gives the manager in I_1 a portion of the informational rent to reveal the private information. Finally, we have $o^{**}(x) < o^*(x)$ which is caused by $V(x, x_2^{**}, \delta_2^{**}; I_2 + \phi \Delta I) < V(x, x_2^*, \delta_2^*; I_2)$. Thus, inefficiency is caused by the informational rent of $\phi \Delta I > 0$.

We discuss the properties of the solution to the asymmetric information problem using endogenous investment quantity. We obtain the following results (see the proof in the appendix).

Proposition 3 *We obtain $\delta_2^{**} > \delta_2^*$. For the optimal investment quantity δ_2^{**}, we have $x_2^{**} > x_2^*$ and $V(x, x_2^{**}, \delta_2^{**}; I_2 + \phi \Delta I) < V(x, x_2^*, \delta_2^*; I_2)$. In addition, an increase in ΔI increases x_2^{**} and δ_2^{**} and decreases $V(x, x_2^{**}, \delta_2^{**}; I_2 + \phi \Delta I)$.*

Proposition 3 implies that there are trade-offs between efficiencies in the investment timing and quantity strategies. The ordering of $x_2^{**} > x_2^*$ means that the firm will exercise the investment *later under asymmetric information* than under full information. The ordering of $\delta_2^{**} > \delta_2^*$ implies that the firm will undertake a *larger investment quantity under asymmetric information* than under full information. Thus, under asymmetric information, because the firm suffers from losses due to delayed investment, the firm makes a larger investment quantity to compensate for the losses. That is, there are trade-offs between efficiencies in the investment timing and quantity strategies. These properties are similar to those in Shibata and Nishihara (2011) where there are trade-offs between efficiencies in the investment timing and management effort.

To confirm the properties of the solutions, we consider numerical examples. In order to do so, the cost of investment quantity is assumed to be

$$c(\delta_i) = \delta_i^2, \quad i \in \{1, 2\}. \tag{18}$$

Suppose that the basic parameters are $r = 0.09$, $\mu = 0.02$, $I_1 = 5$, $I_2 = 10$, $\sigma = 0.15$, and $x = 5$.

The top-left panel of Figure 1 shows the equity (owner's) value with respect to the state variable. We denote the full information value as $V_i^* = V(x, x_i^*, \delta_i^*; I_i)$ where $x < x_i^*$ for any i ($i \in \{1, 2\}$) and the asymmetric information value as $V_2^{**} = V(x, x_2^{**}, \delta_2^{**}; I_2 + \phi\Delta I)$ where $x < x_2^{**}$. We have optimal investment thresholds as $x_1^* = 10.2864$, $x_2^* = 14.5472$, and $x_2^{**} = 17.8165$. In addition, we have

$$V_2^{**} < V_2^* < V_1^*.$$

Thus, asymmetric information leads to a decrease in the equity value because of an increased investment threshold.

[Insert Figure 1 about here]

The top-right panel illustrates the owner's value (stock price) with I_2 (i.e., ΔI). For a fixed $I_1 = 5$, the parameter of I_2 is changed from 5 to 10, which correspond to $\Delta I = 0$ and $\Delta I = 5$, respectively. Recall that $\Delta I > 0$ is defined by informational rent for the manager in I_1. Thus, the larger the I_2 (i.e., ΔI) is, the larger is the magnitude of private information. We confirm that an increase in I_2 (i.e., ΔI) decreases the owner's value.

The middle-left panel shows the effects of I_2 on investment thresholds. We see $x_2^{**} > x_2^*$, implying that even for an endogenous investment quantity, we have the same result of $x_2^{**} > x_2^*$ as in Grenadier and Wang (2005). In addition, an increase in I_2 increases x_2^* and x_2^{**}. The middle-right panel depicts the investment quantity with I_2. The most important result is $\delta_2^{**} > \delta_2^*$. This means that the investment quantities under asymmetric information are larger than those under full information. In addition, an increase in I_2 increases δ_2^* and δ_2^{**}.

The bottom-left panel depicts the ratios x_2^{**}/x_2^*, δ_2^{**}/δ_2^*, and V_2^{**}/V_2^* with ΔI. An increase in ΔI increases $x_2^{**}/x_2^* > 1$ and $\delta_2^{**}/\delta_2^* > 1$. Most interestingly, $x_2^{**}/x_2^* > 1$ is exactly the same as $\delta_2^{**}/\delta_2^* > 1$. An increase in ΔI decreases $V_2^{**}/V_2^* < 1$. Thus, a larger magnitude of private information leads to more delayed investment timing, larger investment quantity, and lower stock price.

To measure the inefficiency arising from asymmetric information, we define the loss L as

$$\begin{aligned} L &:= o^*(x) - o^{**}(x) - m^{**}(x) \\ &= (1-q)\Big(V(x, x_2^*, \delta_2^*; I_2) - V(x, x_2^{**}, \delta_2^{**}; I_2)\Big) \geq 0. \end{aligned} \tag{19}$$

The bottom-right panel demonstrates the loss L with ΔI. We see that L is increasing with ΔI, i.e., the larger the magnitude of private information, the larger the loss. This property is the same as in Shibata and Tian (2010) and Shibata and Tian (2012).

Recall that our model becomes the model of Grenadier and Wang (2005) when investment quantity δ is exogenously given. The three panels of Figure 2 demonstrate the effects of the endogenous investment quantity δ. The upper-left panel shows investment thresholds. For the asymmetric information threshold x_2^{**}, we take $\delta_2 = 5.1432$ and $\delta_2 = 8.9072$ as a benchmark, which correspond to the optimal quantities δ_2^{**} for $\Delta I = 0$ and $\Delta I = 5$, respectively. We have $x_2^{**}(\delta_2 = 5.1432) \leq x_2^{**}(\delta_2^{**}) \leq x_2^{**}(\delta_2 = 8.9072)$,[7] where $x_2^{**}(\delta_2)$ is the investment threshold for a fixed δ_2. For any δ_2 ($\delta_2 \in \{5.1432, \delta_2^{**}, 8.9072\}$), $x_2^{**}(\delta_2)$ is increasing with ΔI for any δ_2.

[Insert Figure 2 about here]

In the upper-right panel, we depict $V_2^*(\delta_2)$ and $V_2^{**}(\delta_2)$, where $V_2^*(y) := (x, x_2^*, y; I_2)$ and $V_2^{**}(y) := (x, x_2^{**}, y; I_2 + \phi\Delta I)$. It is straightforward to have $V_2^{**}(\delta_2^{**}) \geq V_2^{**}(\delta_2)$ for any δ_2 (e.g., $\delta_2 \in \{5.1432, 8.9072\}$). $V_2^{**}(\delta_2)$ is decreasing with ΔI for any δ_2.

In the lower panel, we depict $L(\delta_2)$, where $L(y) := (1-q)(V(x, x_2^*, \delta_2^*; I_2) - V(x, x_2^{**}, y; I_2))$. It is clear to have $L(\delta_2^{**}) \leq L(\delta_2)$ for any δ_2 (e.g., $\delta_2 \in \{5.1432, 8.9072\}$). Interestingly, $L(\delta_2)$ is decreasing with ΔI for $\delta_2 = 8.9072$, while $L(\delta_2)$ is increasing with ΔI for $\delta_2 = 5.1432$. For the optimal δ_2^{**}, $L(\delta_2)$ is increasing with ΔI.

4 Concluding remarks

Our paper has examined the investment timing and quantity strategies under asymmetric information between the owner and the manager where the manager has an informational advantage. We find that investment timing is later under asymmetric information than under full information, while investment quantity is greater under asymmetric information than under full information. We conclude that under asymmetric information, efficiency in investment quantity is higher although efficiency in investment timing is lower, when compared with that under full information.

Acknowledgments

This research was supported by the Asian Human Resources Fund of the Tokyo Metropolitan Government, JSPS KAKENHI (Grant nos. 26242028 and 26285071), and the Telecommunications Advancement Foundation.

[7]Similarly, for the full information threshold x_2^*, we take $\delta_2 = 5.1432$ and $\delta_2 = 7.2736$ for $\Delta I = 0$ and $\Delta I = 5$, respectively. We have $x_2^*(\delta_2 = 5.1432) \leq x_2^*(\delta_2^*) \leq x_2^*(\delta_2 = 7.2736)$.

Appendix

Proof of Propositions 1 and 2

Because the proofs of Propositions 1 and 2 are similar, we present the proof of Proposition 2.

Here, we derive the solution for I_2. Differentiating $V(x, x_2, \delta_2; I_2 + \phi\Delta I)$ with x_2 and δ_2 gives

$$\frac{\partial V}{\partial x_2^{**}} = \left(\frac{x}{x_2^{**}}\right)^{\beta}\left(\frac{-\beta}{x_2^{**}}\Big\{\delta_2^{**}x_2^{**} - I_2 - \phi\Delta I - c(\delta_2^{**})\Big\} + \delta_2^{**}\right) = 0, \tag{A.1}$$

$$\frac{\partial V}{\partial \delta_2^{**}} = \left(\frac{x}{x_2^{**}}\right)^{\beta}\{x_2^{**} - c'(\delta_2^{**})\}, \tag{A.2}$$

respectively. The second order condition is satisfied as

$$M_{22} < 0 \text{ and } |M| > 0, \tag{A.3}$$

where the matrix M is defined by

$$M = \begin{pmatrix} \dfrac{\partial^2 V}{\partial x_2^{**2}} & \dfrac{\partial^2 V}{\partial \delta_2^{**}\partial x_2^{**}} \\ \dfrac{\partial^2 V}{\partial x_2^{**}\delta_2^{**}} & \dfrac{\partial^2 V}{\partial \delta_2^{**2}} \end{pmatrix} = \theta \begin{pmatrix} \dfrac{\delta_2^{**}}{x_2^{**}}(1-\beta) & 1 \\ 1 & -c''(\delta_2^{**}) \end{pmatrix} \tag{A.4}$$

where $\theta := (x/x_2^{**})^{\beta} \in (0, 1)$. We have used (A.1) and (A.2) to derive (A.4). Rearranging (A.2) and (A.3) gives δ_2^{**} and x_2^{**} as (14) and (15), respectively. Similarly, we derive δ_1^{*} and x_1^{*}.

Proof of Proposition 3

We begin by giving the proofs of $\mathrm{d}x_2^{**}/\mathrm{d}\Delta I > 0$ and $\mathrm{d}\delta_2^{**}/\mathrm{d}\Delta I > 0$. By totally differentiating (A.1) and (A.2), we have $My = p$, where M is defined in (A.4) and y and p are defined by

$$y = \begin{pmatrix} \mathrm{d}x_2^{**} \\ \mathrm{d}\delta_2^{**} \end{pmatrix}, \quad p = \theta \begin{pmatrix} \dfrac{-\beta}{x_2^{**}}\phi \mathrm{d}\Delta I \\ 0 \end{pmatrix}. \tag{A.5}$$

By solving with y, we obtain

$$\frac{\mathrm{d}x_2^{**}}{\mathrm{d}\Delta I} = \frac{\theta}{|M|}\frac{\beta}{x_2^{**}}\phi c''(\delta_2^{**}) > 0, \quad \frac{\mathrm{d}\delta_2^{**}}{\mathrm{d}\Delta I} = \frac{\theta}{|M|}\frac{\beta}{x_2^{**}}\phi > 0, \tag{A.6}$$

where we have used the fact of $|M| > 0$ in (A.3).

Next, we give the proof of $\mathrm{d}V/\mathrm{d}\Delta I < 0$. By differentiating $V(x, x_2^{**}, \delta_2^{**}; I_2 + \phi\Delta I)$ with ΔI, we have

$$\frac{\mathrm{d}V(x, x_2^{**}, \delta_2^{**}; I_2 + \phi\Delta I)}{\mathrm{d}\Delta I} = \frac{\partial V}{\partial x_2^{**}}\frac{\mathrm{d}x_2^{**}}{\mathrm{d}\Delta I} + \frac{\partial V}{\partial \delta_2^{**}}\frac{\mathrm{d}\delta_2^{**}}{\mathrm{d}\Delta I} + \frac{\partial V}{\partial \Delta I} \tag{A.7}$$

$$= \frac{\partial V}{\partial \Delta I} \tag{A.8}$$

$$= -\left(\frac{x}{x_2^{**}}\right)^{\beta}\phi < 0, \tag{A.9}$$

where we have used the envelope theorem from (A.7) to (A.8).

References

Dixit, A., Pindyck, R. S., 1994. Investment under uncertainty: Priceton University Press, Princeton, NJ.

Grenadier, S. R., Wang, N., 2005. Investment timing, agency, and information. Journal of Financial Economics, 75(3), 493–533.

Laffont, J. J., Martimort, D., 2002. The theory of incentives: the principal-agent model: Prenceton University Press, Princeton, NJ.

Nishihara, M., Shibata, T., 2008. The agency problem between the owner and the manager in real investment: The bonus-audit relationship. Operations Research Lettters, 36(3), 291–296.

Shibata, T., 2009. Investment timing, asymmetric information, audit structure: A real options framework. Journal of Economic Dynamics and Control, 33(4), 903–921.

Shibata, T., Nishihara, M., 2011. Interactions between investment timing and management effort under asymmetric information: Costs and benefits of privatized firm. European Journal of Operational Research, 215(3), 688–696.

Shibata, T., Tian, Y., 2010. Reorganization strategies and securities valuation under asymmetric information. International Review of Economics and Finance, 19(3), 412–426.

Shibata, T., Tian, Y., 2012. Debt reorganization strategies with complete verification under information asymmetry. International Review of Economics and Finance, 22(1), 141–160.

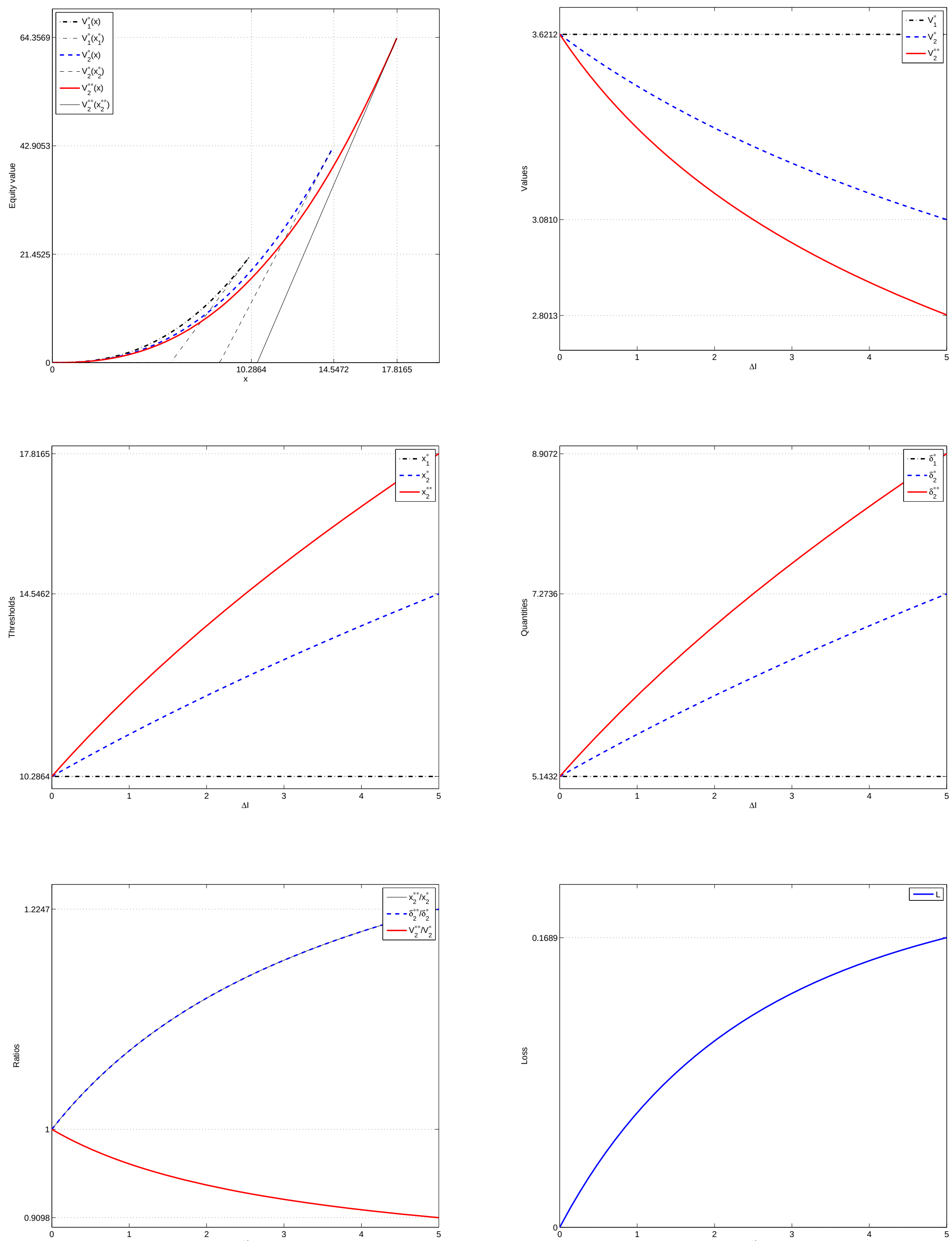

Figure 1: Numerical examples

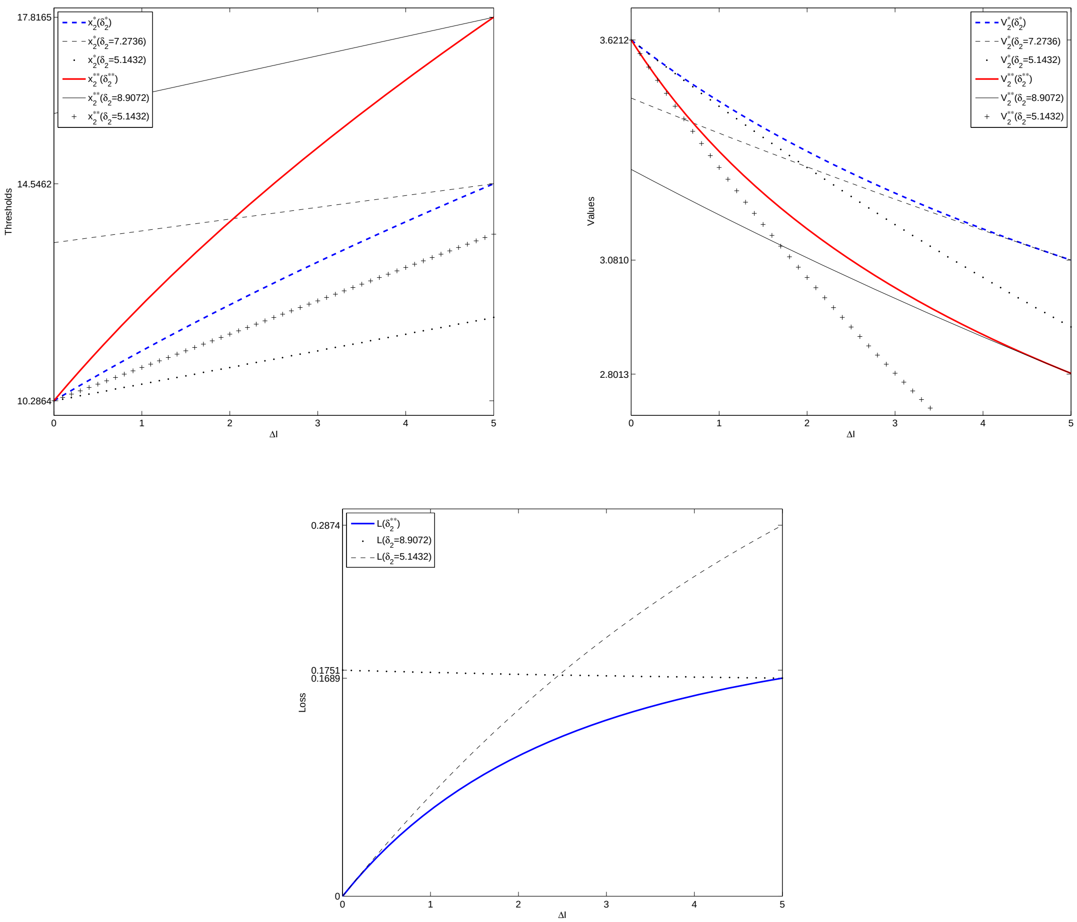

Figure 2: Comparisons with the Grenadier and Wang (2005) model