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Investment timing, strategic debt service, and financing constraints

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Investment timing, strategic debt service, and financing constraints*

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Abstract: This paper considers how financial frictions influence investment threshold (timing) and financing decisions by introducing debt renegotiation and endogenous investment quantity. We obtain four novel results. First, even under debt renegotiation, investment thresholds have a U-shaped relationship with financial frictions, while investment quantities do not vary according to financing frictions. Second, when upper limit of debt issuance decreases, firms tend to prefer to issue debt with renegotiation rather than that without renegotiation. Third, financial frictions have the possibility of breaking down the symmetric relationship between investment and financing decisions. Fourth, upper limits of debt issuance decrease credit spreads and default probabilities. Our theoretical results fit well with the stylized facts and empirical results.

Keywords: Real option; U-shaped investment threshold; debt financing constraints; private workout.

JEL classification: G32; G33, G21.

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1 Introduction

Modigliani and Miller (1958) argue that in a frictionless financial market, investment and financing decisions are completely irrelevant. Once financial frictions are introduced, however, investment and financing decisions are not necessarily separate from each other. In contemporary corporate finance, how financial frictions influence the interaction between investment and financing decisions is an important subject.[1]

Brennan and Schwartz (1984), Mauer and Triantis (1994), and Sundaresan and Wang (2007) examine investment and financing decisions in contingent claim models. These models are developed in a frictionless financial market. Boyle and Guthrie (2003) and Nishihara and Shibata (2013) investigate investment and internal financing decisions by incorporating financing constraints. In contrast, Wong (2010) and Shibata and Nishihara (2012) investigate investment and external (debt) financing decisions by introducing financing constraints. An interesting result of these papers is that investment strategies are nonmonotonic with financial frictions.[2]

In this article, we extend Sundaresan and Wang (2007) to incorporate financing friction and endogenous investment quantity. In addition, our model extends that of Wong (2010) by incorporating debt renegotiation. He does not examine debt renegotiation during periods of financial distress, assuming that debt holders are disperse (e.g., public debt). However, if debt holders are not disperse (e.g., private debt), debt holders are willing to reorganize debt during periods of financial distress, with equity holders to avoid costly liquidation. Thus, our contribution is to show the effects of financing friction, endogenous investment quantity, and debt renegotiation on the interaction between investment and financing decisions.

In this paper, we base our analysis on a dynamic real options model in which investment and financing decisions are endogenously determined. We consider the following model. A firm has an investment opportunity. By exercising this opportunity, the firm obtains the stochastic cash in-flows defined by the product of the constant quantity and the price following a geometric Brownian motion. The investment quantity is constant and

[1]An incomplete list includes Gomes (2001), Clementi and Hopenhayn (2006), Hennessy and Whited (2007), and Chava and Roberts (2008).

[2]See Cleary et al. (2007) for an empirical analysis.

is endogenously determined, in that an increase in the quantity increases the cash in-flows as well as the investment cost at the time of investment. To finance the investment cost, the firm issues a mix of equity and debt. As in Sundaresan and Wang (2007), we assume that the debt is reorganized during a period of financial distress through renegotiation between equity and debt holders. These are general descriptions after investment. Then, by drawing up any stochastic scenario, equity holders determine the value-maximizing investment threshold (timing), investment quantity, and amount of issued debt (coupon payment). The most important assumption is that the amount of issued debt is restricted to a certain exogenous upper limit. The relevance of this assumption is based on risk-shifting. Debt holders are reluctant to lend beyond a certain upper limit because issuing debt encourages risk to shift from equity to debt holders (see Jensen and Meckling (1976)). Our model addresses three key questions. First, how does the debt issuance upper limit influence the interdependence between investment and financing decisions? Second, how does debt renegotiation during periods of financial distress affect the relationship between investment and financing decisions? Third, how does the endogenous investment quantity influence the relationship between investment and financing decisions? Consequently, we examine the effects of financial friction, debt renegotiation, and endogenous investment quantity on the interdependence between investment and financing decisions.

Our model provides the following four novel results. First, investment thresholds (the first of the three solutions) have a U-shaped relationship with the upper limit of debt issuance, even when equity holders renegotiate the debt contract with debt holders during financial distress. Although Wong (2010) and Shibata and Nishihara (2012) have already demonstrated the U-shaped relationship, they do not consider debt renegotiation during financial distress. Thus, even under debt renegotiation, the U-shaped relationship between investment thresholds and financial frictions is a new result. The intuition behind this finding is as follows. Suppose that the upper limit of debt issuance is binding. Relaxing the upper limit increases the debt which generates two conflicting effects. The first effect is to increase the threshold by increasing the debt. The reason for this is that the debt increases as a function of the threshold. We call this effect the "financing constraint effect." The second effect is to decrease the threshold by increasing the debt. This is because an increase in the debt enables a constrained firm to raise its leverage, which leads to a decrease in the threshold. We call this effect the "leverage effect." When the limit is sufficiently low, the second decreasing effect dominates the first increasing effect. When the upper limit is close to the bound that is not constrained although it is constrained, the first increasing effect dominates the second decreasing effect. When the limit is sufficiently high, the constraint is immaterial, implying that the constrained threshold is the same as the nonconstrained threshold. Thus, we observe the U-shaped relationship between investment thresholds and financing frictions. The U-shaped curve implies that it is possible for the constrained thresholds to be smaller than the nonconstrained thresholds. We show that, on the other hand, the investment quantity (the second of the three solutions) does not vary according to financing frictions, and that the coupon payment (the final of the three solutions) is monotonically increasing with financing frictions. That is, the investment thresholds are not between those of the two extreme cases (all-equity financing and nonconstrained debt financing), while the investment quantities and coupon payments are between those of the two extreme cases. Consequently, it is less costly to distort the investment thresholds than the investment quantities and coupon payments.

Second, to consider the effects of debt renegotiation, suppose that equity holders have a choice between debt with and without renegotiation. If equity holders issue debt with renegotiation, equity holders reorganize the debt during periods of financial distress by negotiating with debt holders (i.e., the firm continues operating). If equity holders issue debt without renegotiation, on the other hand, since equity holders do not have the option to reorganize the debt, the firm is liquidated during the period of financial distress (i.e., the firm stops operating). Although our intuition is that equity holders always prefer to issue debt with renegotiation, this is not always correct. As shown in Sundaresan and Wang (2007), equity holders prefer to choose debt without renegotiation for cases in which volatility is low and debt holders have low bargaining power (equity holders have high bargaining power) during the period of financial distress. We show that as the upper limit of debt issuance becomes more severe, equity holders become more likely to issue debt with renegotiation. Thus, a decrease in the debt issuance upper limit significantly influences the probability that equity holders will issue debt with renegotiation. In practice, firms with lower (higher) debt issuance upper limits are generally small/young (large/mature) corporations, and debt with (without) renegotiation is regarded as bank (market) debt.

Based on these definitions, our result implies that small/young (large/mature) corporations are more likely to issue bank (market) debt. This segmentation is broadly consistent with the stylized facts and the empirical findings of Blackwell and Kidwell (1988), Denis and Mihov (2003), and Rauh and Sufi (2010).

Third, suppose that equity holders have the option of choosing between two types of debts: debt with or without renegotiation. Financing constraints distort the symmetric relationship between the equity option value and the investment threshold. Under no financing friction, there is a symmetric relationship: if the investment thresholds are smaller under debt with renegotiation than under debt without renegotiation, equity values are larger under debt financing with renegotiation than under that without renegotiation (i.e., equity holders prefer debt with renegotiation to that without renegotiation). We show that under financing friction, however, even though the investment thresholds are smaller under debt with renegotiation than under that without renegotiation, equity values may be larger than debt financing without renegotiation than that with renegotiation (i.e., equity holders may prefer debt without renegotiation). Thus, debt issuance limit constraints have the possibility of breaking down the symmetric relationship between investment and financing decisions.

Finally, we show that severe debt issuance upper limits lead to low risks (low default probabilities) and low returns (low credit spreads) for debt holders. In addition, we show that the credit spreads and default probabilities for debt with renegotiation are higher than those for debt without renegotiation. Notably, even when equity holders prefer to issue debt with renegotiation, the credit spreads and default probabilities for debt with renegotiation are relatively high compared with those for that without renegotiation. These results fit well with the empirical results of Davydenko and Strebulaev (2007).

The remainder of the paper is organized as follows. Section 2 describes the setup of our model. Section 3 provides the solution to our model and examines its properties. Section 4 considers the effects of debt renegotiation. Section 5 concludes. Technical developments are included in two appendices. Appendix A provides proofs of the results in the paper. Appendix B contains the solutions to the optimization problem for a firm that is financed by debt without renegotiation and subject to issuance limits.

2 Model

In this section, we begin by describing the model. We then provide the value functions after investment and formulate our model as the investment problem for a firm financed by debt with issuance limit constraints. As a benchmark, we derive solutions for two extreme cases in our model. One is the problem of an unlevered firm (i.e., a firm that is completely financed by equity). The other is the problem of a nonconstrained levered firm (i.e., a firm that is financed by debt without issuance limit constraints).

2.1 Setup

Equity holders possess the option to invest in a single project at any time. If the investment option is exercised at time t, equity holders receive an instantaneous cash in-flow δX_t after time t. Here, X_t is the stochastic price and is given by the following geometric Brownian motion:

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}z_t^{\mathbb{Q}}, \quad X_0 = x > 0, \tag{1}$$

where $\mu > 0$ and $\sigma > 0$ are positive constants and $z_t^{\mathbb{Q}}$ denotes a standard Brownian motion defined by a risk-neutral probability space $(\Omega, \mathcal{F}, \mathbb{Q})$. For convergence, we assume that $r > \mu$, where $r > 0$ is the risk-free interest rate. We assume that the current price level $X_0 = x$ is sufficiently low such that equity holders do not undertake the investment immediately.

Alternatively, $\delta \geq 1$ represents the project quantity and affects a one-time fixed investment cost denoted by $I(\delta)$. We assume that the investment cost $I(\delta)$ satisfies the conditions $I(1) > 0$, $I'(\delta) > 0$, and $I''(\delta) > 0$ for any δ. These conditions are intuitively reasonable. The first condition states that the investment cost is required to initiate the project. The second and third conditions mean that $I(\delta)$ is strictly increasing and convex in δ. Note that at the time of investment, δ is endogenously chosen to maximize the firm's profits. To ensure a unique optimal solution for δ, we further assume that the elasticity of the investment cost to project quantity, $\delta I'(\delta)/I(\delta)$, is increasing with δ.

2.2 Value functions under all-equity financing after investment

In this subsection, we provide the value function under all-equity financing. Let us denote by $T_0^{\rm i}$ the investment time, where the superscript "i" and the subscript "0" indicate the investment strategy and all-equity financing, respectively. Mathematically, the investment timing is defined as $T_0^{\rm i} = \inf\{s \geq 0, X_s \geq x_0^{\rm i}\}$, where $x_0^{\rm i}$ denotes the associated investment threshold.

We now assume that any time $t > T_0^{\rm i}$, which means that the investment is exercised. The equity value of the all-equity financed firm after investment is defined by

$$\mathbb{E}_t^{\mathbb{Q}}\Big[\int_t^{+\infty} \mathrm{e}^{-r(u-t)}(1-\tau)\delta_0 X_t \mathrm{d}t\Big] = \Pi\delta_0 X_t, \tag{2}$$

where $\mathbb{E}_t^{\mathbb{Q}}$ denotes the expectation operator at time t, $\tau > 0$ represents the corporate tax, and $\Pi := (1-\tau)/(r-\mu) > 0$. Note that δ_0 is the investment quantity under all-equity financing, which is endogenously determined.

2.3 Value functions under debt financing after investment

This subsection provides the value functions after the debt is issued at the time of investment.

We assume that the firm issues risky debt at the time of investment. For analytical convenience, we limit the condition that the risky debt is perpetual. If the firm issues the debt, equity holders obtain $\delta_1 X_t - c_1$, where δ_1 is the investment quantity under debt financing and $c_1 \geq 0$ is the coupon payment. Once X_t becomes sufficiently small after investment, the firm should be liquidated because equity holders cannot pay c_1. Following previous research such as Leland (1994), because there is a liquidation cost defined by $\alpha\Pi\delta_1 X_t$, where $\alpha \in (0,1)$, debt holders collect the residual value $(1-\alpha)\Pi\delta_1 X_t$. Then, since liquidation (formally bankruptcy) is costly, when X_t is small after investment, debt holders will renegotiate the terms of the debt contract with equity holders. As in Sundaresan and Wang (2007), we assume that equity and debt holders negotiate and divide the surplus generated by avoiding liquidation between them. Thus, during periods of financial distress, the coupon payments to debt holders are reduced through a costless private workout.

Let us denote by $T_1^{\rm i}$ and $T_1^{\rm d}$ the investment (indicated by superscript "i") and coupon switching (indicated by superscript "d") timings, respectively. Mathematically, the investment and coupon switching timings are defined as $T_1^{\mathrm{i}} = \inf\{s \geq 0, X_s \geq x_1^{\mathrm{i}}\}$ and $T_1^{\mathrm{d}} = \inf\{s \geq T_1^{\mathrm{i}}; X_s \leq x_1^{\mathrm{d}}\}$, where x_1^{i} and x_1^{d} denote the associated investment and coupon switching thresholds, respectively. Let us denote by $E_1^j(X_t, c_1, \delta_1)$, $D_1^j(X_t, c_1, \delta_1)$, and $V_1^j(X_t, c_1, \delta_1)$ the equity, debt, and total firm values, respectively ($j \in \{\mathrm{a}, \mathrm{b}\}$). Here, the superscripts "a" and "b" indicate the normal and bankruptcy (coupon reduction) regions, respectively.

The left-hand side panel of Figure 1 depicts the scenario for debt financing. Recall that the initial value $X_0 = x > 0$ is sufficiently small. Once X_t increases and arrives at the investment threshold (higher boundary) x_1^{i}, equity holders exercise the investment by issuing debt. After the investment, if X_t is decreased and reaches the coupon switching threshold (lower boundary) x_1^{d}, the coupon payments are reduced through negotiations between equity and debt holders. Moreover, if X_t arrives at x_1^{d} from below, the reduced coupon payments are changed to normal coupon payments.

[Insert Figure 1 about here]

For any time $t > T_1^{\mathrm{i}}$, the equity an debt values in the normal region are respectively defined by

$$
\begin{aligned}
&E_1^{\mathrm{a}}(X_t, c_1, \delta_1) \\
&= \sup_{T_1^{\mathrm{d}} \geq t} \mathbb{E}_t^{\mathbb{Q}} \Big[\int_t^{T_1^{\mathrm{d}}} \mathrm{e}^{-r(u-t)} (1-\tau)(\delta_1 X_u - c_1) \mathrm{d}u + \mathrm{e}^{-r(T_1^{\mathrm{d}} - t)} E_1^{\mathrm{b}}(X_{T_1^{\mathrm{d}}}, c_1, \delta_1) \Big],
\end{aligned} \tag{3}
$$

$$
D_1^{\mathrm{a}}(X_t, c_1, \delta_1) = \mathbb{E}_t^{\mathbb{Q}} \Big[\int_t^{T_1^{\mathrm{d}}} \mathrm{e}^{-r(u-t)} c_1 \mathrm{d}u + \mathrm{e}^{-r(T_1^{\mathrm{d}} - t)} D_1^{\mathrm{b}}(X_{T_1^{\mathrm{d}}}, c_1, \delta_1) \Big]. \tag{4}
$$

The equity and debt values in the bankruptcy region are respectively given by

$$
\begin{aligned}
&E_1^{\mathrm{b}}(X_t, c_1, \delta_1) \\
&= \mathbb{E}_t^{\mathbb{Q}} \Big[\int_t^{T_1^{\mathrm{d}}} \mathrm{e}^{-r(u-t)} ((1-\tau)\delta_1 X_u - s(X_u, \delta_1)) \mathrm{d}u + \mathrm{e}^{-r(T_1^{\mathrm{d}} - t)} E_1^{\mathrm{a}}(X_{T_1^{\mathrm{d}}}, c_1, \delta_1) \Big],
\end{aligned} \tag{5}
$$

$$
D_1^{\mathrm{b}}(X_t, c_1, \delta_1) = \mathbb{E}_t^{\mathbb{Q}} \Big[\int_t^{T_1^{\mathrm{d}}} \mathrm{e}^{-r(u-t)} s(X_u, \delta_1) \mathrm{d}u + \mathrm{e}^{-r(T_1^{\mathrm{d}} - t)} D_1^{\mathrm{a}}(X_{T_1^{\mathrm{d}}}, c_1, \delta_1) \Big], \tag{6}
$$

where $s(X_t, \delta_1)$ is the reduced coupon payment in the bankruptcy region. The total firm value is defined by $V_1^j(X_t, c_1, \delta_1) = E_1^j(X_t, c_1, \delta_1) + D_1^j(X_t, c_1, \delta_1)$ ($j \in \{\mathrm{a}, \mathrm{b}\}$).

We begin by calculating the total firm values. Through the standard variation principle, we obtain the following differential equations:

$$r V_1^{\mathrm{a}}(x, c_1, \delta_1) \tag{7}$$
$$= (1-\tau)\delta_1 x + \tau c_1 + \mu x \frac{\partial V_1^{\mathrm{a}}(x, c_1, \delta_1)}{\partial x} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 V_1^{\mathrm{a}}(x, c_1, \delta_1)}{\partial x^2}, \quad X_t \geq x^{\mathrm{d}},$$
$$r V_1^{\mathrm{b}}(x, c_1, \delta_1) \tag{8}$$
$$= (1-\tau)\delta_1 x + \mu x \frac{\partial V_1^{\mathrm{b}}(x, c_1, \delta_1)}{\partial x} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 V_1^{\mathrm{b}}(x, c_1, \delta_1)}{\partial x^2}, \quad X_t < x^{\mathrm{d}},$$

subject to the boundary conditions:

$$V_1^{\mathrm{a}}(x_1^{\mathrm{d}}, c_1, \delta_1) = V_1^{\mathrm{b}}(x_1^{\mathrm{d}}, c_1, \delta_1), \quad \frac{\partial V_1^{\mathrm{a}}(x, c_1, \delta_1)}{\partial x}\bigg|_{x=x_1^{\mathrm{d}}} = \frac{\partial V_1^{\mathrm{b}}(x, c_1, \delta_1)}{\partial x}\bigg|_{x=x_1^{\mathrm{d}}}. \tag{9}$$

Thus, $V_1^{\mathrm{a}}(x, c_1, \delta_1)$ and $V_1^{\mathrm{b}}(x, c_1, \delta_1)$ are obtained through

$$V_1^{\mathrm{a}}(X_t, c_1, \delta_1) = \Pi\delta_1 X_t + \frac{\tau c_1}{r}\left(1 - \frac{\beta}{\beta-\gamma}\left(\frac{X_t}{x_1^{\mathrm{d}}}\right)^{\gamma}\right), \quad X_t \geq x_1^{\mathrm{d}}, \tag{10}$$
$$V_1^{\mathrm{b}}(X_t, c_1, \delta_1) = \Pi\delta_1 X_t - \frac{\tau c_1}{r}\frac{\gamma}{\beta-\gamma}\left(\frac{X_t}{x_1^{\mathrm{d}}}\right)^{\beta}, \quad X_t < x_1^{\mathrm{d}}, \tag{11}$$

where $\beta := 1/2 - \mu/\sigma^2 + ((\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2)^{1/2} > 1$ and $\gamma := 1/2 - \mu/\sigma^2 - ((\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2)^{1/2} < 0$.

We then derive the reduced coupon payment in the bankruptcy region. Let us denote by η and $1-\eta$ the bargaining powers of equity and debt holders, respectively. As in the derivations in Fan and Sundaresan (2000) and Sundaresan and Wang (2007), the reduced coupon payment in the bankruptcy region, $s(X_t, \delta_1)$, is given by

$$s(X_t, \delta_1) = (1-\alpha\eta)(1-\tau)\delta_1 X_t, \quad X_t < x_1^{\mathrm{d}}. \tag{12}$$

The reduced coupon payment $s(x, \delta_1)$ is a linear function of x with $\lim_{x\downarrow 0} s(x, \delta_1) = 0$. The lower the state variable x, the lower the reduced coupon payment $s(x, \delta_1)$. This means that equity and debt holders do not need to consider liquidation once it is in the negotiation region.

Equity and debt holders negotiate and divide the surplus to prevent wasteful liquidation. The incremental value for equity holders is $wV_1^{\mathrm{b}}(X_t, c_1, \delta_1)$ while that for debt holders is $(1-w)V_1^{\mathrm{b}}(X_t, c_1, \delta_1) - (1-\alpha)\Pi\delta_1 X_t$. The division of the surplus between equity and debt holders depends on their relative bargaining powers. Using Nash bargaining, the fraction of total firm value that equity holders receive from renegotiation satisfies

$$\begin{aligned} w(X_t, c_1, \delta_1) &= \underset{w}{\operatorname{argmax}} \Big(wV_1^{\mathrm{b}}(X_t, c_1, \delta_1)\Big)^{\eta}\Big((1-w)V_1^{\mathrm{b}}(X_t, c_1, \delta_1) - (1-\alpha)\Pi\delta_1 X_t\Big)^{1-\eta} \\ &= \eta - \frac{\eta(1-\alpha)\Pi\delta_1 X_t}{V_1^{\mathrm{b}}(X_t, c_1, \delta_1)}. \end{aligned} \tag{13}$$

Thus, the sharing rules are

$$\begin{aligned} E_1^{\mathrm{b}}(X_t, c_1, \delta_1) &= \eta V_1^{\mathrm{b}}(X_t, c_1, \delta_1) - \eta(1-\alpha)\Pi\delta_1 X_t \\ &= \eta\Big(\alpha\Pi\delta_1 X_t - \frac{\tau c_1}{r}\frac{\gamma}{\beta-\gamma}\Big(\frac{X_t}{x_1^{\mathrm{d}}}\Big)^{\beta}\Big), \end{aligned} \tag{14}$$

$$\begin{aligned} D_1^{\mathrm{b}}(X_t, c_1, \delta_1) &= (1-\eta)V_1^{\mathrm{b}}(X_t, c_1, \delta_1) + \eta(1-\alpha)\Pi\delta_1 X_t \\ &= (1-\alpha\eta)\Pi\delta_1 X_t - (1-\eta)\frac{\tau c_1}{r}\frac{\gamma}{\beta-\gamma}\Big(\frac{X_t}{x_1^{\mathrm{d}}}\Big)^{\beta}, \end{aligned} \tag{15}$$

where $X_t < x_1^{\mathrm{d}}$. Moreover, the equity value in the normal region, $E_1^{\mathrm{a}}(X_t, c_1, \delta_1)$ in (3), is rewritten as

$$\begin{aligned} &E_1^{\mathrm{a}}(X_t, c_1, \delta_1) \\ &= \max_{x_1^{\mathrm{d}}} \Pi\delta_1 X_t - (1-\tau)\frac{c_1}{r} - \Big((1-\alpha\eta)\Pi\delta_1 x_1^{\mathrm{d}} - \frac{c_1}{r}(1-\tau-\tau\frac{\eta\gamma}{\beta-\gamma})\Big)\Big(\frac{X_t}{x_1^{\mathrm{d}}}\Big)^{\gamma}. \end{aligned} \tag{16}$$

The optimal coupon switching threshold is chosen to satisfy $\partial E_1^{\mathrm{a}}/\partial x_1^{\mathrm{d}} = \partial E_1^{\mathrm{b}}/\partial x_1^{\mathrm{d}}$, i.e.,

$$x_1^{\mathrm{d}}(c_1, \delta_1) = \frac{c_1}{\kappa_1\delta_1}, \tag{17}$$

where $\kappa_1 := (\gamma-1)(1-\alpha\eta)\Pi r/(\gamma(1-\tau(1-\eta))) \geq 0$. Note that $x_1^{\mathrm{d}}(c_1, \delta_1)$ is a linear function of c_1 with $\lim_{c_1\downarrow 0} x_1^{\mathrm{d}}(c_1, \delta_1) = 0$, which implies that $\lim_{c_1\downarrow 0} E_1^{\mathrm{a}}(X_t, c_1, \delta_1) = \Pi\delta_1 X_t$.

The debt value in the normal region, $D_1^{\mathrm{a}}(X_t, c_1, \delta_1)$ in (4), is given by

$$D_1^{\mathrm{a}}(X_t, c_1, \delta_1) = \frac{c_1}{r} + \Big((1-\alpha\eta)\Pi\delta_1 x_1^{\mathrm{d}}(c_1, \delta_1) - \frac{c_1}{r}(1-\tau+\tau\frac{\beta}{\beta-\gamma} - \tau\frac{\eta\gamma}{\beta-\gamma})\Big)\Big(\frac{X_t}{x_1^{\mathrm{d}}(c_1, \delta_1)}\Big)^{\gamma}. \tag{18}$$

It is straightforward to obtain $\lim_{c_1\downarrow 0} D_1^{\mathrm{a}}(X_t, c_1, \delta_1) = 0$.

2.4 Investment problem for a constrained levered firm

In this subsection, we formulate the optimization problem for a constrained levered (debt-financed) firm.

We assume that the firm's debt issuance limit is constrained, i.e.,

$$\frac{D_1^{\mathrm{a}}(x_1^{\mathrm{i}}, c_1, \delta_1)}{I(\delta_1)} \leq q, \tag{19}$$

for some constant $q \geq 0$. The left-hand side of inequality (19) represents the ratio of debt value to the cost at the time of investment. Thus, inequality (19) implies that the debt issuance upper limit is restricted. Constraint (19) might arise because of the risk-shifting problem. Because issuing debt encourages risk to shift from equity to debt holders, debt holders are reluctant to lend more than a certain amount (see Jensen and Meckling (1976)). The parameter q imposes restrictions on the firm's debt issuance limit. The lower the ratio q, the more severe the debt issuance constraint. In the extreme case of $q = 0$, the firm is not allowed to issue debt. Conversely, when q is sufficiently large, the debt issuance constraint is immaterial. Note that we have assumed that $q \in [0, +\infty)$, not $q \in [0, 1]$. The reason for this is that the levered firm has the possibility of issuing the amount $D_1^{\mathrm{a}}(x_1^{\mathrm{i}}, c_1, \delta_1)$, which is greater than the amount $I(\delta_1)$, to maximize its value. The case of $D_1^{\mathrm{a}}(x_1^{\mathrm{i}}, c_1, \delta_1)/I(\delta_1) > 1$ implies that the excess is distributed to equity holders as a dividend.

Let us denote by $E_1^{**}(x)$ the equity option value of the constrained levered firm before investment, where the superscript "$**$" represents the optimum for the *constrained* problem. The equity option value is defined as

$$E_1^{**}(x) := \sup_{T_1^{\mathrm{i}}, c_1, \delta_1} \mathbb{E}_0^{\mathbb{Q}} \left[\mathrm{e}^{-rT_1^{\mathrm{i}}} \left\{ E_1^{\mathrm{a}}(X_{T_1^{\mathrm{i}}}, c_1, \delta_1) - \Big(I(\delta_1) - D_1^{\mathrm{a}}(X_{T_1^{\mathrm{i}}}, c_1, \delta_1) \Big) \right\} \right], \tag{20}$$

subject to (19), where we assume that debt is fairly priced. Using the standard arguments in Dixit and Pindyck (1994), we have $\mathbb{E}_0^{\mathbb{Q}}[\mathrm{e}^{-rT_1^{\mathrm{i}}}] = (x/x_1^{\mathrm{i}})^{\beta}$, where $x_1^{\mathrm{i}} > x > 0$.

The optimization problem for the constrained levered firm is formulated as

$$E_1^{**}(x) = \max_{x_1^{\mathrm{i}}, c_1, \delta_1} \left(\frac{x}{x_1^{\mathrm{i}}} \right)^{\beta} \left\{ V_1^{\mathrm{a}}(x_1^{\mathrm{i}}, c_1, \delta_1) - I(\delta_1) \right\}, \tag{21}$$

$$\text{subject to } \frac{D_1^{\mathrm{a}}(x_1^{\mathrm{i}}, c_1, \delta_1)}{I(\delta_1)} \leq q, \tag{22}$$

where $x < x_1^{\mathrm{i}}$. Note that the triple $(x_1^{\mathrm{i}}, c_1, \delta_1)$ is endogenously determined.

When $q = 0$, our problem is the investment decision problem for an unlevered (all-equity financed) firm. When q is sufficiently large, our problem is the investment decision problem for a nonconstrained levered firm.

Before solving our problem, we first briefly review two extreme cases.

2.5 Investment problem for an unlevered firm

In this subsection, we assume that $q = 0$, i.e., $D_1^{\mathrm{a}}(x_1^{\mathrm{i}}, c_1, \delta_1) = 0$. This problem is the extended version of the seminal model of McDonald and Siegel (1986), in that the investment quantity is endogenously determined.

Let us denote by $E_0^*(x)$ the equity option value of the unlevered firm before investment, where the superscript "$*$" represents the optimum for the *nonconstrained* problem. The optimization problem for the all-equity financed firm is given by

$$E_0^*(x) = \max_{x_0^{\mathrm{i}}, \delta_0} \left(\frac{x}{x_0^{\mathrm{i}}}\right)^{\beta} \left\{\Pi \delta_0 x_0^{\mathrm{i}} - I(\delta_0)\right\}, \tag{23}$$

where $x < x_0^{\mathrm{i}}$. Note that we have used the fact that $E_1^{\mathrm{a}}(x, 0, \delta_0) = \Pi \delta_0 x$. We then obtain the following.

Proposition 1 *Consider the optimization problem for an unlevered (all-equity financed) firm. The solutions and value are*

$$\delta_0^* = \delta^*, \quad x_0^{\mathrm{i}*} = \frac{\theta}{\Pi} \frac{I(\delta^*)}{\delta^*}, \quad E_0^*(x) = \left(\frac{x}{x_0^{\mathrm{i}*}}\right)^{\beta} (\theta - 1) I(\delta^*), \tag{24}$$

where $x < x_0^{\mathrm{i}*}$ and δ^* is implicitly defined by satisfying

$$\theta := \frac{\beta}{\beta - 1} = \frac{\delta^* I'(\delta^*)}{I(\delta^*)}. \tag{25}$$

Note that $x_0^{\mathrm{i}*}$ is obtained through δ^*. We use these solutions and values as a benchmark.

2.6 Investment problem for a nonconstrained levered firm

In this subsection, we assume that q is sufficiently large ($q \uparrow +\infty$). This problem is similar to the extended model of Sundaresan and Wang (2007) in that the investment quantity is endogenously determined. Let us denote by $E_1^*(x)$ the equity option value of the nonconstrained levered firm before investment. The problem for the nonconstrained levered firm is formulated as the only one equation of (21) by removing inequity (22), where the superscript "$**$" is changed to "$*$." The solutions and value are as follows.

Proposition 2 *Consider the optimization problem for a nonconstrained levered (debt-financed) firm. The solutions are obtained through*

$$\delta_1^* = \delta^*, \quad x_1^{\mathrm{i}*} = \psi_1 x_0^{\mathrm{i}*}, \quad c_1^* = \frac{\kappa_1}{h_1} \delta^* x_1^{\mathrm{i}*}, \tag{26}$$

where

$$h_1 := \Big(\frac{\beta}{\beta-\gamma}(1-\gamma)\Big)^{-1/\gamma} \geq 1, \quad \psi_1 := \Big(1 + \frac{\tau(1-\alpha\eta)}{1-\tau(1-\eta)}\frac{1}{h_1}\Big)^{-1} \leq 1. \tag{27}$$

The equity option value is given by

$$E_1^*(x) = \psi_1^{-\beta} E_0^*(x) = \Big(\frac{x}{x_1^{\mathrm{i}*}}\Big)^{\beta}(\theta-1)I(\delta^*), \tag{28}$$

where $x < x_1^{\mathrm{i}*}$.

Note that $\delta_1^* = \delta_0^* = \delta^*$ and $x_1^{\mathrm{i}*} \leq x_0^{\mathrm{i}*}$. In addition, we find that $E_1^*(x) \geq E_0^*(x)$ because $\psi_1 \leq 1$ and $\beta > 1$. We refer to this property as the "leverage effect." See Myers (1977) for more detail.

The right-hand side panel of Figure 1 depicts the equity option value $E_k^*(x)$ with the state variable x ($k \in \{0,1\}$). We see that $x_1^{\mathrm{i}*} < x_0^{\mathrm{i}*}$, $E_1^*(x_1^{\mathrm{i}*}) = E_0^*(x_0^{\mathrm{i}*}) = (\theta-1)I(\delta^*)$ and $E_1^*(x) > E_0^*(x)$, where $x < x_1^{\mathrm{i}*}$.

3 Model solution

In this section, we provide the solutions to our problem, which was described in the previous section. We then discuss some properties of the solutions.

Recall that there are three control variables $(x_1^{\mathrm{i}**}, c_1^{**}, \delta_1^{**})$ in our problem. We can intuitively determine the properties of our solution. Our intuitive conjecture is that $x_1^{\mathrm{i}**} \in [x_1^{\mathrm{i}*}, x_0^{\mathrm{i}*}]$, $c_1^{**} \in [0, c_1^*]$, and $\delta_1^{**} \geq \delta^*$. The first and second conjectures signify that the investment thresholds and coupon payments are monotonic with the debt issuance upper limit q. The final conjecture indicates that the investment quantity for a constrained levered firm is larger than that for a nonconstrained levered firm. The reason for this is that an increase in δ_1 enables the constrained levered firm to raise the debt issuance amount $qI(\delta_1)$. However, these conjectures are not always correct.

The following proposition provides one component δ_1^{**} of the solutions (the proof is given in Appendix A).

Proposition 3 *Consider the optimization problem for a constrained levered firm. In this case,* $\delta_1^{**} = \delta^*$. *There exists a unique* x_1^{p} *satisfying*

$$\frac{D_1^{\mathrm{a}}(x_1^{\mathrm{p}}, c_1(x_1^{\mathrm{p}}, \delta^*), \delta^*)}{I(\delta^*)} = q, \tag{29}$$

where $c_1(x, \delta^*) = \kappa_1 \delta^* x / h_1 = argmax_{c_1} V_1^{\mathrm{a}}(X_t, c_1, \delta^*)$.[3] *Then, if* $x_1^{\mathrm{i}*} > x_1^{\mathrm{p}}$, *the firm is financially constrained. Otherwise, it is not.*

Interestingly, contrary to our intuition, the statement $\delta_1^{**} = \delta^*$ implies an invariance of δ_1^{**} with q. However, this result is similar to that of Wong (2010), in which debt renegotiation (strategic debt service) is not permitted during periods of financial distress. Thus, we demonstrate that the investment quantity does not vary with the debt issuance limit even under debt renegotiation. The second statement implies that, by comparing the magnitudes of $x_1^{\mathrm{i}*}$ and x_1^{p}, we recognize whether the debt issuance constraint is binding. Using these properties, the following proposition yields two other components of the solutions, $x_1^{\mathrm{i}**}$ and c_1^{**} (the proof is given in Appendix A).

Proposition 4 *Consider the optimization problem for a constrained levered firm. Suppose that* $x_1^{\mathrm{i}*} > x_1^{\mathrm{p}} \geq 0$. *For* $x_1^{\mathrm{p}} = 0$, *we have that* $x_1^{\mathrm{i}**} = x_0^{\mathrm{i}*}$ *and* $c_1^{**} = 0$. *For* $x_1^{\mathrm{p}} > 0$, *the solutions* $x_1^{\mathrm{i}**}$ *and* c_1^{**} *are determined uniquely by solving two simultaneous equations:*

$$\frac{f_{11}(x_1^{\mathrm{i}**}, c_1^{**}, \delta^*)}{f_{12}(x_1^{\mathrm{i}**}, c_1^{**}, \delta^*)} = \frac{f_{13}(x_1^{\mathrm{i}**}, c_1^{**}, \delta^*)}{f_{14}(x_1^{\mathrm{i}**}, c_1^{**}, \delta^*)}, \quad c_1^{**}\Big(\frac{1}{r} - \Phi_1\Big(\frac{\kappa_1 \delta^* x_1^{\mathrm{i}**}}{c_1^{**}}\Big)^{\gamma}\Big) = qI(\delta^*), \tag{30}$$

where f_{11}, f_{12}, f_{13}, f_{14}, *and* Φ_1 *are given by*

$$f_{11} := (\beta - 1)\Pi \delta^* x_1^{\mathrm{i}**} + \beta \frac{\tau}{r}\Big(1 - \Big(\frac{\kappa_1 \delta^* x_1^{\mathrm{i}**}}{c_1^{**}}\Big)^{\gamma}\Big) c_1^{**} - \beta I(\delta^*), \tag{31}$$

$$f_{12} := \Big(\frac{\kappa_1 \delta^* x_1^{\mathrm{i}**}}{c_1^{**}}\Big)^{\gamma} \gamma c_1^{**} \Phi_1, \tag{32}$$

$$f_{13} := \frac{\tau}{r}\Big(1 - \Big(\frac{\kappa_1 \delta^* x_1^{\mathrm{i}**}}{c_1^{**}}\Big)^{\gamma} (1-\gamma) \frac{\beta}{\beta - \gamma}\Big), \tag{33}$$

$$f_{14} := \frac{1}{r} - \Big(\frac{\kappa_1 \delta^* x_1^{\mathrm{i}**}}{c_1^{**}}\Big)^{\gamma} (1-\gamma)\Phi_1, \tag{34}$$

$$\Phi_1 := \frac{1}{r}\Big(1 - \tau + \tau\frac{\beta}{\beta-\gamma} - \tau\frac{\eta\gamma}{\beta - \gamma}\Big) - (1 - \alpha\eta)\frac{\Pi}{\kappa_1}. \tag{35}$$

Suppose, on the other hand, that $x_1^{\mathrm{i}*} \leq x_1^{\mathrm{p}}$. *We obtain* $x_1^{\mathrm{i}**} = x_1^{\mathrm{i}*}$ *and* $c_1^{**} = c_1^*$. *Substituting the solutions into the equity value before investment yields*

$$E_1^{**}(x) = \Big(\frac{x}{x_1^{\mathrm{i}**}}\Big)^{\beta}\Big\{V_1^{\mathrm{a}}(x_1^{\mathrm{i}**}, c_1^{**}, \delta^*) - I(\delta^*)\Big\}, \tag{36}$$

where $x < x_1^{\mathrm{i}**}$. *In particular,*

$$E_1^*(x) = \lim_{q \uparrow +\infty} E_1^{**}(x) \geq E_1^{**}(x) \geq \lim_{q \downarrow 0} E_1^{**}(x) = E_0^*(x). \tag{37}$$

[3] This result is obtained by Leland (1994). Note that $c_1^* = c_1(x_1^{\mathrm{i}*}, \delta^*)$ in (26).

To consider the properties of the solutions, we consider numerical examples. In order to do so, the investment cost function is assumed to be

$$I(\delta) = I_0 + \delta^2. \tag{38}$$

Suppose that the basic parameters are $r = 0.09$, $\mu = 0.02$, $I_0 = 4$, $\tau = 0.15$, $\alpha = 0.4$, $\sigma = 0.2$, $\eta = 1$, and $x = 0.4$. Under the basic parameters, $\delta^* = 8.3631$ and $x < x_1^{\mathrm{i}**}$ for any q.

[Insert Figure 2 about here]

The top left-hand side panel of Figure 2 depicts the investment thresholds with the debt issuance upper limit q. According to Proposition 3, a firm is financially constrained if $q < 1.0040$ (i.e., $x_1^{\mathrm{i}*} < x_1^{\mathrm{p}}$) and is not otherwise. For $q = 0$, $x_1^{\mathrm{i}**} = x_0^{\mathrm{i}*}$. When q is sufficiently large, $x_1^{\mathrm{i}**} = x_1^{\mathrm{i}*}$. Interestingly, $x_1^{\mathrm{i}**}$ has a U-shaped relationship with q, i.e., $x_1^{\mathrm{i}**}$ is decreasing with q for $q \in [0, 0.8205]$ while it is increasing with q for $q \in (0.8205, 1.0040]$. This result is contrary to our intuition because $x_1^{\mathrm{i}**}$ is not always between $x_1^{\mathrm{i}*}$ and $x_0^{\mathrm{i}*}$, i.e., $x_1^{\mathrm{i}**} < x_1^{\mathrm{i}*}$ for $q \in (0.4683, 1.0040)$, implying that the investment is accelerated (the investment threshold is decreased) when the firm is financially constrained.

The U-shaped relationship of investment with friction is the same as that found in Wong (2010) and Shibata and Nishihara (2012). However, since these studies do not consider debt renegotiation during periods of financial distress, firms are liquidated when the state variable arrives at the bankruptcy threshold. Thus, we demonstrate that the U-shaped relationship is obtained even under debt renegotiation. The intuition behind the U-shaped relationship with q is as follows. An increase in q increases the debt value $D_1^{\mathrm{a}}(x_1^{\mathrm{i}}, c_1, \delta_1)$. Then, two conflicting effects influence the shape of the investment threshold. First, an increase in $D_1^{\mathrm{a}}(x_1^{\mathrm{i}}, c_1, \delta_1)$ increases x_1^{i} because $D_1^{\mathrm{a}}(x, c_1, \delta_1)$ is increasing with x. We call this effect the "financing constraint effect." Second, an increase in $D_1^{\mathrm{a}}(x_1^{\mathrm{i}}, c_1, \delta_1)$ decreases x_1^{i} via $(x/x_1^{\mathrm{i}})^{\beta}\{E_1^{\mathrm{a}}(x_1^{\mathrm{i}}, c_1, \delta_1) - (I(\delta_1) - D_1^{\mathrm{a}}(x_1^{\mathrm{i}}, c_1, \delta_1))\}$. This effect corresponds to the previously defined "leverage effect." Thus, whether an increase in $D_1^{\mathrm{a}}(x_1^{\mathrm{i}}, c_1, \delta_1)$ increases or decreases x_1^{i} depends on the magnitude of these two conflicting effects. For low values of q, an increase in q decreases x_1^{i} because the latter effect dominates the former. For high values of q, by contrast, an increase in q increases x_1^{i} because the former effect dominates the latter. Consequently, x_1^{i} has a U-shaped relationship with q.

The top right-hand side panel of Figure 2 demonstrates coupon payments c_1 with q. We observe that c_1^{**} is monotonically increasing with q. The middle left-hand side panel shows the invariance of investment quantities $\delta_1^{**} = \delta^*$ with q. We confirm the first statement of Proposition 3.

The next observation summarizes one property of the solutions to our model.

Observation 1 *First, $x_1^{\mathrm{i}**}$ has a nonmonotonic relationship with q. Second, c_1^{**} is monotonically increasing in q. Finally, $\delta_1^{**} = \delta^*$.*

From the first and second statements, $x_1^{\mathrm{i}**}$ is not always in the regions of $[x_1^{\mathrm{i}*}, x_0^{\mathrm{i}*}]$, although c_1^{**} is always in the regions of $[0, c_1^*]$. Consequently, it is less costly to distort $x_1^{\mathrm{i}**}$ away from the regions of $[x_1^{\mathrm{i}*}, x_0^{\mathrm{i}*}]$ than to distort c_1^{**} away from the regions of $[0, c_1^*]$.

The middle right-hand side panel depicts the equity option value before investment with q. We see that $E_1^{**}(x)$ is monotonically increasing with q with $\lim_{q\downarrow 0} E_1^{**}(x) = E_0^*(x)$ and $\lim_{q\uparrow +\infty} E_1^{**}(x) = E_1^*(x)$. This numerical simulation confirms the second statement in Proposition 4.

The credit spread and default probability are defined as

$$cs_1^j := \frac{c_1^j}{D_1^{\mathrm{a}}(x_1^{\mathrm{i}j}, c_1^j, \delta^*)} - r, \quad p_1^j = \left(\frac{x_1^{\mathrm{i}j}}{x_1^{\mathrm{d}}(c_1^j, \delta^*)}\right)^{\gamma}, \quad j \in \{*, **\} \tag{39}$$

respectively. Note that these two measures are regarded as measures of return and risk for debt holders. The bottom left- and right-hand side panels of Figure 2 illustrate the credit spreads and default probabilities with q. We find that the debt issuance limit constraint causes a decrease in credit spreads and default probabilities, i.e.,

$$cs_1^{**} \leq cs_1^*, \quad p_1^{**} \leq p_1^*. \tag{40}$$

Regarding the first inequality, the debt issuance constraint causes a lower decrease in the coupon c_1^{**} than in debt value $D_1^{\mathrm{a}}(x_1^{\mathrm{i}**}, c_1^{**}, \delta^*)$. The implication of the second inequality is that, because $\gamma < 0$, the distance between $x_1^{\mathrm{i}**}$ and $x_1^{\mathrm{d}}(c_1^{**}, \delta^*)$ is larger than the distance between $x_1^{\mathrm{i}*}$ and $x_1^{\mathrm{d}}(c_1^*, \delta^*)$. This implies that a constrained levered firm is less likely to default than a nonconstrained levered firm. Thus, the more severe the debt issuance bounds, the lower the credit spreads and default probabilities are. These links between risk and return are the same as those traditionally suggested in the literature and also match the results of Gomes and Schmid (2010). The following characterizes the properties of financing constraints.

Observation 2 *Debt financing constraints lead to a low-risk (low-default probabilities) and low-return (low-credit spreads) situation for debt holders.*

Finally, to understand the effects of investment quantity δ, we compare the solutions for optimal $\delta^* > 1$ and fixed $\delta = 1$. We write the solutions as a function of δ, e.g., $x_1^{i**}(\delta^*)$ and $x_1^{i**}(1)$. In this case, the former is the investment threshold for an optimal δ^* while the latter is the investment threshold for a fixed $\delta = 1$. The left-hand side panel of Figure 3 represents $x_1^{i**}(\delta^*)$ and $x_1^{i**}(1)$ with q. We observe that $x_1^{i**}(\delta^*) > x_1^{i**}(1)$. However, note that the firm is financially constrained if $q < 1.0040$ for both an optimal $\delta^* > 1$ and a fixed $\delta = 1$. These results imply that an increase in δ does not change the firm's financially constrained region of q. The right-hand side panel presents $cs_1(\delta^*)$, $cs_1(1)$, $p_1(\delta^*)$, and $p_1(1)$. We observe that $cs_1(\delta^*) = cs_1(1)$ and $p_1(\delta^*) = p_1(1)$. Thus, cs_1 and p_1 are invariant with δ.

[Insert Figure 3 about here]

4 Effects of debt renegotiation

In this section, we consider the effects of debt renegotiation. As shown in Sundaresan and Wang (2007), interestingly, while renegotiation increases *ex post* total firm value by preventing costly liquidation, renegotiation also induces a cost on *ex ante* equity option value. It is therefore possible for equity holders to prefer to issue debt without renegotiation, depending on key parameters such as volatility σ, bargaining power η, and financing friction q.

To understand the effect of debt renegotiation, we compare our solutions with those of Wong (2010) and Shibata and Nishihara (2012), which consider the investment problem for a firm financed by debt without renegotiation. We denote by $(x_2^{i**}, c_2^{**}, \delta_2^{**})$ and $E_2^{**}(x)$ the solutions and value for the firm financed by debt without renegotiation (indicated by the subscript "2"). See Appendix B for the solutions and values.

Subsection 4.1 considers whether equity holders prefer to issue debt with or without renegotiation. Subsection 4.2 investigates the distortion of debt financing limit constraints caused by the presence or absence of debt renegotiation. Subsection 4.3 examines credit spreads and default probabilities for debt having the option for renegotiation. Subsection 4.4 demonstrates the invariance of investment quantity in relation to debt structure choices.

4.1 Preference for debt renegotiation

This subsection considers whether debt with or without renegotiation is preferred to issue by equity holders. We cannot analytically determine which is larger, $E_1^{**}(x)$ or $E_2^{**}(x)$. We will examine their magnitudes by using numerical examples.

The upper left-hand side panel of Figure 4 depicts the regions of $E_1^{**}(x) > E_2^{**}(x)$ in (η, σ) space. The three lines indicate the boundaries of $E_1^{**}(x) = E_2^{**}(x)$ for $q = 2$, $q = 0.8$, and $q = 0.6$. Under the basic parameters, the boundary for $q = 2$ is the same as that for $q \uparrow +\infty$ (i.e., that for a nonconstrained levered firm). For the regions of larger σ and smaller η, in contrast, $E_1^{**}(x) > E_2^{**}(x)$, i.e., equity holders prefer to issue debt with renegotiation. For the regions of smaller σ and larger η, $E_2^{**}(x) > E_1^{**}(x)$, i.e., equity holders choose debt without renegotiation. Importantly, a decrease in q enlarges the regions in which $E_1^{**}(x) > E_2^{**}(x)$. We summarize these results as follows.

[Insert Figure 4 about here]

Observation 3 *Whether debt with or without renegotiation is issued depends on the combination of three key parameters: q (friction), η (bargaining power), and σ (volatility). As the debt issuance limit decreases, equity holders become more likely to issue debt with renegotiation.*

The other three panels of Figure 4 show the equity option values with q. The parameters of the upper-right, lower-left, and lower-right panels correspond to those of Points A, B, and C of the upper-left panel, respectively. In the upper-right panel, equity holders prefer debt without the option for renegotiation for all q. In the lower-left panel, equity holders prefer debt with the option for negotiation for $q < \hat{q} = 0.7357$, but that without the option for renegotiation for $q \geq \hat{q}$. In the lower-right panel, equity holders prefer debt with the option for renegotiation for all q.

These results are mostly related to the stylized facts. Following Bulow and Shoven (1978), Gilson et al. (1990), Gertner and Scharfstein (1991), Hart and Moore (1995), Bolton and Scharfstein (1996), and Hackbarth et al. (2007), debt with renegotiation is regarded as bank debt, while debt without renegotiation is thought of as market debt. According to the definition of Rajan (1992), firms with larger q and smaller σ approximate large/mature corporations. Based on this definition, our results imply that large/mature (small/young) firms are more likely to issue market (bank) debt. Thus, our results fit well with the empirical findings of Blackwell and Kidwell (1988), Denis and Mihov (2003), and Rauh and Sufi (2010).

4.2 Distortion of symmetric relationship

In this subsection, we consider the symmetric relationship between investment and financing decisions.

As a benchmark, suppose that q is sufficiently large ($q \uparrow +\infty$). Then, we have

$$x_j^{\mathrm{i}*} = \psi_j x_0^{\mathrm{i}*} \leq x_0^{\mathrm{i}*}, \quad E_j^*(x) = \psi_j^{-\beta} E_0^*(x) \geq E_0^*(x), \quad j \in \{1, 2\}, \tag{41}$$

where $\psi_1 \leq 1$ and $\psi_2 \leq 1$ are given by (27) and (B.7), respectively. Because $\beta > 1$, we obtain the following result.

Lemma 1 *Suppose that equity holders do not have a debt issuance limit constraint. Then, there is a symmetric relationship, i.e.,*

$$x_1^{\mathrm{i}*} \leq x_2^{\mathrm{i}*} \quad \text{if and only if} \quad E_1^*(x) \geq E_2^*(x). \tag{42}$$

Lemma 1 implies that having the choice of debt structure (i.e., the choice between debt with and without renegotiation) always increases the equity option value and speeds up the investment (decreases the investment threshold) compared with having no choice of debt structure. Thus, under no financial constraints, there is such an intuitive symmetric relationship. However, under financial constraints, this relationship does not necessarily hold.

The two top panels of Figure 5 show the effects of volatility, σ, on the equity option values and investment thresholds. The key parameters are assumed to be $q = 0.95$ and $\eta = 1$ for $\sigma \in [0.1, 0.2]$. Then, we have $x_j^{\mathrm{i}*} > x_j^{\mathrm{p}}$ ($j \in \{1, 2\}$), implying that the firm is financially constrained by debt issuance bounds. In the top left-hand side panel, we see that $E_1^{**}(x) \leq E_2^{**}(x)$ for $\sigma \leq 0.1642$. In the top right-hand side panel, $x_1^{\mathrm{i}**} > x_2^{\mathrm{i}**}$

for $\sigma < 0.1914$. From these two results, we obtain $E_1^{**}(x) > E_2^{**}(x)$ and $x_1^{i**} > x_2^{i**}$ for $\sigma \in (0.1642, 0.1914)$. Our finding is as follows.

[Insert Figure 5 about here]

Observation 4 *When debt financing constraints are effective, there is not always a symmetric relationship between equity option values and investment thresholds.*

Note that the finding in Observation 4 provides important new insights into the choice of debt structure. Interestingly, when equity holders encounter a debt issuance limit constraint, having the choice of debt structure does not necessarily hasten corporate investment (i.e., does not necessarily decrease the investment threshold), although it always increases the equity option value. As a consequence, we may obtain results that are contrary to our intuition when financing constraints are binding.

In addition, when the debt issuance constraint is binding, it is possible for equity holders to prefer to issue debt without renegotiation even though the investment threshold under debt financing with renegotiation is smaller than that under debt financing without renegotiation. That is, financial frictions distort the symmetric relationship between investment and financing decisions.

4.3 Credit spreads and default probabilities

This subsection examines how the debt issuance limit influences credit spreads and default probabilities.

The two middle panels of Figure 5 depict credit spreads and default probabilities. The parameters of the middle left- and right-hand side panels correspond to those of Points A and C in the upper left-hand side panel of Figure 4, in which the firm prefers debt with and without renegotiation, respectively. In the two middle panels, we have

$$cs_1^{**} \geq cs_2^{**}, \quad p_1^{**} \geq p_2^{**}. \tag{43}$$

These inequalities imply that debt with renegotiation represents higher risks (high default probability) and higher returns (high credit spread) compared with debt without renegotiation. Importantly, irrespective of the equity holders' preferred debt structure, the credit spread and default probabilities for debt with renegotiation are larger than those for that without renegotiation. Regarding the first inequality, the ratio of c_1^{**} to $D_1^{\mathrm{a}}(x_1^{\mathrm{i}**}, c_1^{**}, \delta^*)$ is higher than the ratio of c_2^{**} to $D_2^{\mathrm{a}}(x_2^{\mathrm{i}**}, c_2^{**}, \delta^*)$, implying that the normal coupon of the debt with renegotiation, c_1^*, is relatively high compared to that of debt without renegotiation c_2^{**}. The implication of the second inequality is that because $\gamma < 0$, the distance between $x_1^{\mathrm{i}**}$ and $x_1^{\mathrm{d}**}$ is smaller than that between $x_2^{\mathrm{i}**}$ and $x_2^{\mathrm{d}**}$, implying that firms financed by debt with renegotiation are more likely to default than firms financed by debt without renegotiation. These numerical simulations suggest the following observation.

Observation 5 *The credit spreads and default probabilities for debt that can be renegotiated are larger than those for debt that cannot be renegotiated.*

These findings are consistent with the empirical results of Davydenko and Strebulaev (2007), who find that the possibility of strategic debt service increases corporate credit spreads and default probabilities.

4.4 Invariance of project quantity in relation to chosen debt structure

This subsection examines the effects of project quantity on the choice between debt with and without renegotiation.

As a benchmark, suppose that $q \uparrow +\infty$. We have already obtained $E_j^*(x;\delta) = \psi_j^{-\beta} E_0^*(x;\delta)$ $(\delta \in \{1, \delta^*\})$. Note that ψ_j does not depend on δ. Under no financial constraints,

$$E_j^*(x;\delta^*) \geq E_k^*(x;\delta^*) \quad \text{if and only if} \quad E_j^*(x;1) \geq E_k^*(x;1), \tag{44}$$

for j and k $(j, k \in \{1,2\}, j \neq k)$.

Suppose that $q < +\infty$. Since we cannot compare these values analytically, we examine them through numerical examples. The bottom panel of Figure 5 shows $E_j^{**}(x;\delta)$ and $E_j^{**}(x;\delta)$ with σ $(j \in \{1,2\})$. For $\sigma = 0.1545$, we see that $E_1^{**}(x;\delta^*) = E_2^{**}(x;\delta^*)$ and $E_1^{**}(x;1) = E_2^{**}(x;1)$. We have $E_1^{**}(x;\delta^*) < E_2^{**}(x;\delta^*)$ and $E_1^{**}(x;1) < E_2^{**}(x;1)$ for $\sigma < 0.1545$, while $E_1^{**}(x;\delta) > E_2^{**}(x;\delta)$ for $\sigma > 0.1545$. Thus,

$$E_j^{**}(x;\delta^*) \geq E_k^{**}(x;\delta^*) \quad \text{if and only if} \quad E_j^{**}(x;1) \geq E_k^{**}(x;1), \tag{45}$$

for j and k ($j, k \in \{1, 2\}, j \neq k$). This result indicates the invariance of the investment quantity in relation to the chosen debt structure.

5 Concluding remarks

This paper examines the effects of financing friction, debt renegotiation, and endogenous investment quantity in the investment timing decision problem for constrained levered firms. In particular, we extend Sundaresan and Wang (2007) to incorporate debt financing limit constraints and endogenous investment quantity as well as Wong (2010) to introduce debt renegotiation during financial distress.

We obtain four new results. First, the investment thresholds have a U-shaped relationship with the upper limit of debt issuance. Interestingly, contrary to our intuition, this result implies that the relaxation of financing constraints does not always accelerate investment. However, the investment quantity does not vary according to the debt financing upper limit. Consequently, debt issuance constraints distort investment timing. Nevertheless, they do not lead to an efficient investment quantity.

Second, to consider the effects of debt renegotiation, suppose that equity holders have the option of choosing between two types of debts: debt with and without renegotiation. Then, as the debt financing limit becomes more severe, equity holders become more likely to issue debt with renegotiation. Thus, a decrease in the debt financing limit significantly increases the possibility that equity holders will issue debt with renegotiation. In practice, firms with lower debt financing limits tend to be small/young corporations, and debt with (without) renegotiation is regarded as bank (market) debt. Based on this definition, our result implies that small/young corporations are more likely to issue bank debt. This segmentation is broadly consistent with the stylized facts.

Third, financing constraints distort the symmetric relationship between the equity option value and the investment threshold. Under no financing friction, there is a symmetric relationship: if the investment threshold is smaller under debt financing renegotiation than under that without renegotiation, equity holders prefer debt with renegotiation to that without renegotiation. However, under financing friction, even though the investment threshold is smaller under debt with renegotiation than under that without renegotiation, equity holders may prefer debt without renegotiation. Debt issuance limit constraints therefore have the possibility of breaking down the symmetric relationship between investment and financing decisions.

Finally, financial constraints lead to low risk (low default probability) and low returns (low credit spread). Thus, we show that severe financing constraints cause credit spreads and default probabilities to decrease. In addition, credit spreads and default probabilities for debt with renegotiation are higher than those for that without renegotiation. Thus, debt with renegotiation is relatively higher risk and generates higher returns. These results fit well with the empirical results of Davydenko and Strebulaev (2007).

Appendix A. Proof of the proposition

Proof of Proposition 3

First, we show that $\delta_1^{**} = \delta^*$. For problem (21) subject to (22), the Lagrangian is formulated as

$$\mathcal{L} \;=\; \left(\frac{x}{x_1^{\mathrm{i}}}\right)^{\beta}\Big\{V_1^{\mathrm{a}}(x_1^{\mathrm{i}}, c_1, \delta_1) - I(\delta_1)\Big\} + \lambda\Big\{qI(\delta_1) - D_1^{\mathrm{a}}(x_1^{\mathrm{i}}, c_1, \delta_1)\Big\}, \tag{A.1}$$

where $\lambda \geq 0$ denotes the multiplier on the constraint. The Karush-Kuhn-Tucker conditions are given by $\partial\mathcal{L}/\partial x_1^{\mathrm{i}} = 0$, $\partial\mathcal{L}/\partial c_1 = 0$, $\partial\mathcal{L}/\partial\delta_1 = 0$, and

$$\lambda\Big\{qI(\delta_1^{**}) - D_1^{\mathrm{a}}(x_1^{\mathrm{i}**}, c_1^{**}, \delta_1^{**})\Big\} = 0. \tag{A.2}$$

If debt issuance constraints are binding, $\lambda > 0$. By substituting $(\partial\mathcal{L}/\partial c_1)c_1 = 0$ and $qI(\delta_1^{**}) = D_1^{\mathrm{a}}(x_1^{\mathrm{i}**}, c_1^{**}, \delta_1^{**})$ into $(\partial\mathcal{L}/\partial x_1^{\mathrm{i}})x_1^{\mathrm{i}} = 0$, we obtain

$$\left(\frac{x}{x_1^{\mathrm{i}**}}\right)^{\beta}\Big\{(1-\beta)V_1^{\mathrm{a}}(x_1^{\mathrm{i}**}, c_1^{**}, \delta_1^{**}) + \beta I(\delta_1^{**})\Big\} - \lambda qI(\delta_1^{**}) = 0. \tag{A.3}$$

Similarly, substituting $(\partial\mathcal{L}/\partial c_1)c_1 = 0$ and $qI(\delta_1^{**}) = D_1^{\mathrm{a}}(x_1^{\mathrm{i}**}, c_1^{**}, \delta_1^{**})$ into $(\partial\mathcal{L}/\partial\delta_1)\delta_1 = 0$ produces

$$\left(\frac{x}{x_1^{\mathrm{i}**}}\right)^{\beta}\Big\{V_1^{\mathrm{a}}(x_1^{\mathrm{i}**}, c_1^{**}, \delta_1^{**}) - \delta_1^{**}I'(\delta_1^{**})\Big\} - \lambda q\Big\{\delta_1^{**}I'(\delta_1^{**}) - I(\delta_1^{**})\Big\} = 0. \tag{A.4}$$

By removing λq from (A.3) and (A.4), we obtain

$$\Big\{V_1^{\mathrm{a}}(x_1^{\mathrm{i}**}, c_1^{**}, \delta_1^{**}) - I(\delta_1^{**})\Big\}\Big\{\beta I(\delta_1^{**}) - (\beta-1)\delta_1^{**}I'(\delta_1^{**})\Big\} = 0. \tag{A.5}$$

Because $V_1^{\mathrm{a}}(x_1^{\mathrm{i}**}, c_1^{**}, \delta_1^{**}) > I(\delta_1^{**})$, we have $\beta/(\beta-1) = \delta_1^{**} I'(\delta_1^{**})/I(\delta_1^{**})$, implying that $\delta_1^{**} = \delta^*$.

Next, we demonstrate the existence of a unique x_1^{p}. The left-hand side of equation (29), $F(x) := D_1^{\mathrm{a}}(x, c_1(x, \delta^*), \delta^*)/I(\delta^*)$, is a strictly monotonically increasing continuous function of x with $\lim_{x\downarrow 0} F(x) = 0$ and $\lim_{x\uparrow +\infty} F(x) = +\infty$. It is straightforward to find that there exists a unique value x_1^{p}. Suppose that $x_1^{\mathrm{i}*} > x_1^{\mathrm{p}}$. Then the firm would prefer to borrow more than the amount $qI(\delta^*)$ to maximize equity value. However, the maximum borrowing amount is $qI(\delta^*)$. Thus, the firm is financially constrained by its debt borrowing limit. Suppose instead that $x_1^{\mathrm{i}*} \le x_1^{\mathrm{p}}$. Then, the firm is not constrained. We complete the proof.

Proof of Proposition 4

We have already found that $\delta_1^{**} = \delta^*$. Substituting δ^* into (A.1), we formulate the Lagrangian. We have $\partial\mathcal{L}/\partial x_1^{\mathrm{i}} = 0$, $\partial\mathcal{L}/\partial c_1 = 0$, and (A.2) as the Karush-Kuhn-Tucker conditions. Then, removing λ from the first two conditions produces the first equation of (30). Because $\lambda > 0$ in (A.2), the final condition is the second equation of (30).

We show the proof of the result that E_1^{**} is monotonically increasing with q. For any q', q'' with $q'' \ge q' \ge 0$, we suppose that the respective optimal values are given by

$$\left(\frac{x}{x_1^{\mathrm{i}\prime}}\right)^{\beta} \{V_1^{\mathrm{a}}(x_1^{\mathrm{i}\prime}, c_1', \delta^*) - I(\delta^*)\}, \quad \left(\frac{x}{x_1^{\mathrm{i}\prime\prime}}\right)^{\beta} \{V_1^{\mathrm{a}}(x_1^{\mathrm{i}\prime\prime}, c_1'', \delta^*) - I(\delta^*)\}, \tag{A.6}$$

and

$$(x/x_1^{\mathrm{i}\prime})^{\beta}\{V_1^{\mathrm{a}}(x_1^{\mathrm{i}\prime}, c_1', \delta^*) - I(\delta^*)\} > (x/x_1^{\mathrm{i}\prime\prime})^{\beta}(V_1^{\mathrm{a}}(x_1^{\mathrm{i}\prime\prime}, c_1'', \delta^*) - I(\delta^*). \tag{A.7}$$

Then, because a firm in state q'' can increase its value by choosing $(x_1^{\mathrm{i}\prime}, c_1')$, there is a contradiction.

Appendix B. Related paper

In this appendix, we define the investment optimization problem for firms financed by debt without renegotiation developed by Wong (2010) and Shibata and Nishihara (2012). We then derive the solutions and its value.

The optimization problem is formulated as

$$E_2^{**}(x) = \max_{x_2^{\rm i}, c_2, \delta_2} \left(\frac{x}{x_2^{\rm i}}\right)^{\beta} \Big\{ E_2^{\rm a}(x_2^{\rm i}, c_2, \delta_2) - \Big(I(\delta_2) - D_2^{\rm a}(x_2^{\rm i}, c_2, \delta_2) \Big) \Big\}, \tag{B.1}$$

$$\text{subject to } \frac{D_2^{\rm a}(x_2^{\rm i}, c_2, \delta_2)}{I(\delta_2)} \leq q, \tag{B.2}$$

where $x < x_1^{\rm i}$, $E_2^{\rm a}(X_t, c_2, \delta_2)$ and $D_2^{\rm a}(X_t, c_2, \delta_2)$ denote the equity and debt values after investment of a firm financed by debt without renegotiation, respectively. The equity value $E_2^{\rm a}(X_t, c_2, \delta_2)$ is given by

$$\begin{aligned} E_2^{\rm a}(X_t, c_2, \delta_2) &= \sup_{T_2^{\rm d}} \mathbb{E}_t^{\mathbb{Q}} \Big[\int_t^{T_2^{\rm d}} {\rm e}^{-r(u-t)} (1-\tau)(\delta_2 X_u - c_2) {\rm d}u \Big] \\ &= \max_{x_2^{\rm d}} \Big\{ \delta_2 \Pi X_t - (1-\tau)\frac{c_2}{r} - \Big(\delta_2 \Pi x_2^{\rm d} - (1-\tau)\frac{c_2}{r} \Big) \Big(\frac{X_t}{x_2^{\rm d}} \Big)^{\gamma} \Big\}, \end{aligned} \tag{B.3}$$

for any time $t > T_2^{\rm i}$. Then, the optimal liquidation threshold, $x_2^{\rm d}(\delta_2, c_2)$ is given by

$$x_2^{\rm d}(c_2, \delta_2) = \operatorname*{argmax}_{x_2^{\rm d}} E_2^{\rm a}(X_t, c_2, \delta_2) = \frac{c_2}{\kappa_2 \delta_2}, \tag{B.4}$$

where $\kappa_2 := (\gamma - 1)\Pi r / (\gamma(1-\tau)) \geq 0$. Note that $\lim_{c_2 \downarrow 0} x_2^{\rm d}(c_2, \delta_2) = 0$, as shown in Black and Cox (1976). The debt value, $D_2^{\rm a}(X_t, c_2, \delta_2)$, is defined by

$$\begin{aligned} D_2^{\rm a}(X_t, c_2, \delta_2) &= \mathbb{E}_t^{\mathbb{Q}} \Big[\int_t^{T_2^{\rm d}} {\rm e}^{-r(u-t)} c_2 {\rm d}u + {\rm e}^{-r(T_2^{\rm d}-t)} (1-\alpha)\delta_2 \Pi x_2^{\rm d}(\delta_2, c_2) \Big] \\ &= \frac{c_2}{r}\Big(1 - \Big(\frac{\kappa_2 \delta_2 X_t}{c_2} \Big)^{\gamma} \Big) + (1-\alpha)\Pi \kappa_2^{-1} c_2 \Big(\frac{\kappa_2 \delta_2 X_t}{c_2} \Big)^{\gamma}. \end{aligned} \tag{B.5}$$

The total firm value is defined by the sum of these two values, i.e., $V_2^{\rm a}(X_t, c_2, \delta_2) := E_2^{\rm a}(X_t, c_2, \delta_2) + D_2^{\rm a}(X_t, c_1, \delta_2)$. Note that there does not exist a "region b" under debt financing without renegotiation because the firm is liquidated once the state variable X_t reaches $x_2^{\rm d}(\delta_2, c_2)$.

We derive the solutions of the investment problem for firms financed by debt without renegotiation. First, suppose $q = 0$, implying that $D_2^{\rm a}(X_t, c_2, \delta_2) = 0$. Then the problem is the same as in (23). The solutions are given in Proposition 1. Second, suppose that q is sufficiently large. The solutions are

$$\delta_2^* = \delta^*, \quad x_2^{\rm i*} = \psi_2 x_0^{\rm i*}, \quad c_2^* = \frac{\kappa_2}{h_2}\delta^* x_2^{\rm i*}, \tag{B.6}$$

where

$$h_2 = \Big(1 - \gamma\Big(1 + \alpha\frac{1-\tau}{\tau} \Big) \Big)^{-1/\gamma} \geq 1, \quad \psi_2 := \Big(1 + \frac{\tau}{1-\tau}\frac{1}{h_2} \Big)^{-1} \leq 1. \tag{B.7}$$

The equity option value before investment is

$$E_2^*(x) = \psi_2^{-\beta} E_0^*(x) = \left(\frac{x}{x_2^{\mathrm{i}*}}\right)^{\beta} (\theta - 1) I(\delta^*). \tag{B.8}$$

Third, suppose that $q \in (0, +\infty)$. Then, $\delta_2^{**} = \delta^*$ (the proof is the same as in Proposition 3). The other solutions, $x_2^{\mathrm{i}**}$ and c_2^{**}, are obtained by solving two simultaneous equations:

$$\frac{f_{21}(x_2^{\mathrm{i}**}, c_2^{**}, \delta^*)}{f_{22}(x_2^{\mathrm{i}**}, c_2^{**}, \delta^*)} = \frac{f_{23}(x_2^{\mathrm{i}**}, c_2^{**}, \delta^*)}{f_{24}(x_2^{\mathrm{i}**}, c_2^{**}, \delta^*)}, \quad c_2^{**}\left(\frac{1}{r} - \Phi_2\left(\frac{\kappa_2 \delta^* x_2^{\mathrm{i}**}}{c_2^{**}}\right)^{\gamma}\right) = qI(\delta^*), \tag{B.9}$$

where f_{21}, f_{22}, f_{23}, and f_{24} are given as

$$f_{21} := (\beta - 1)\Pi\delta^* x_2^{\mathrm{i}**} + \left(\beta\frac{\tau}{r} - (\beta - \gamma)\left(\frac{\kappa_2 \delta^* x_2^{\mathrm{i}**}}{c_2^{**}}\right)^{\gamma} \left(\frac{\tau}{r} + \frac{\alpha\Pi}{\kappa_2}\right)\right) c_2^{**} - \beta I(\delta^*), \tag{B.10}$$

$$f_{22} := \left(\frac{\kappa_2 \delta^* x_2^{\mathrm{i}**}}{c_2^{**}}\right)^{\gamma} \gamma c_2^{**} \Phi_2, \tag{B.11}$$

$$f_{23} := \frac{\tau}{r} - \left(\frac{\kappa_2 \delta^* x_2^{\mathrm{i}**}}{c_2^{**}}\right)^{\gamma} (1 - \gamma)\left(\frac{\tau}{r} + \frac{\alpha\Pi}{\kappa_2}\right), \tag{B.12}$$

$$f_{24} := \frac{1}{r} - \left(\frac{\kappa_2 \delta^* x_2^{\mathrm{i}**}}{c_2^{**}}\right)^{\gamma} (1 - \gamma)\Phi_2, \tag{B.13}$$

$$\Phi_2 = \frac{1}{r} - (1 - \alpha)\frac{\Pi}{\kappa_2}. \tag{B.14}$$

The equity value before investment is

$$E_2^{**}(x) = \left(\frac{x}{x_2^{\mathrm{i}**}}\right)^{\beta} \left\{V_2^{\mathrm{a}}(x_2^{\mathrm{i}**}, c_2^{**}, \delta^*) - I(\delta^*)\right\}. \tag{B.15}$$

Finally, as in (37), for any q,

$$E_2^*(x) = \lim_{q \uparrow +\infty} E_2^{**}(x) \geq E_2^{**}(x) \geq \lim_{q \downarrow 0} E_2^{**}(x) = E_0^*(x). \tag{B.16}$$

References

Black, F., Cox, J. C., 1976. Valuing corporate securities: some effects of bond indenture provisions. Journal of Finance, 31(2), 351–367.

Blackwell, D. W., Kidwell, D. S., 1988. An investigation of cost differences between public sales and private placements of debt. Journal of Financial Economics, 22(2), 253–278.

Bolton, P., Scharfstein, D. S., 1996. Optimal debt structure and the number of creditors. Journal of Political Economy, 104(1), 1–25.

Boyle, G. W., Guthrie, G. A., 2003. Investment, uncertainty, and liquidity. Journal of Finance, 63(5), 2143–2166.

Brennan, A., Schwartz, E., 1984. Optimal financial policy and firm valuation. Journal of Finance, 39(3), 593–607.

Bulow, J. I., Shoven, J. B., 1978. The bankruptcy decision. Bell Journal of Economics, 9(2), 436–445.

Chava, S., Roberts, M. R., 2008. How does financing impact investment?: The role of debt covenants. Journal of Finance, 63(5), 2085–2121.

Cleary, S., Povel, P., Raith, M., 2007. The U-shaped investment curve: Theory and evidence. Journal of Financial and Quantitative Analysis, 42(1), 1–40.

Clementi, G. L., Hopenhayn, H. A., 2006. A theory of financing constraints and firm dynamics. Quarterly Journal of Economics, 121, 229–265.

Davydenko, S. A., Strebulaev, I. A., 2007. Strategic actions and credit spreads: An empirical investigation. Journal of Finance, 62(6), 2633–2671.

Denis, D., Mihov, V., 2003. The choice among bank debt, non-bank private debt, and public debt: Evidence from new corporate borrowings. Journal of Financial Economics, 70(1), 3-28.

Fan, H., Sundaresan, S., 2000. Debt valuation, strategic debt service and optimal dividend policy. Review of Financial Studies, 13(4), 1057–1099.

Gertner, R. H., Scharfstein, D. S., 1991. A theory of workouts and the effect of reorganization law. Journal of Finance, 46(4), 1189–1222.

Gilson, S. C., John, K., Lang, L., 1990. Troubled debt restructurings: An empirical study of private reorganization of firms in default. Journal of Financial Economics, 27(2), 315–353.

Gomes, J. F., 2001. Financing investment. American Economic Review, 90, 1263–1285.

Gomes, J. F., Schmid, L., 2010. Levered returns. Journal of Finance, 65(2), 467–494.

Hackbarth, D., Hennessy, C. A., Leland, H. E., 2007. Can the trade-off theory explain debt structure? Review of Financial Studies, 20(5), 1389–1428.

Hart, O., Moore, J., 1995. Debt and seniority: An analysis of the role of hard claims in constraining management. American Economic Review, 85(3), 567–585.

Hennessy, C. A., Whited, T. M., 2007. How costly is external financing? Evidence from a structural estimation. Journal of Finance, 62(4), 1705–1745.

Jensen, M. C., Meckling, W. H., 1976. Theory of the firm: Managerial behavior, agency costs and ownership structure. Journal of Financial Economics, 3(4), 305–360.

Leland, H. E., 1994. Corporate debt value, bond covenants, and optimal capital structure. Journal of Finance, 49(4), 1213–1259.

Mauer, D. C., Triantis, A., 1994. Interactions of corporate financing and investment decisions. Journal of Finance, 49(4), 1253–1277.

McDonald, R., Siegel, D. R., 1986. The value of waiting to invest. Quarterly Journal of Economics, 101(4), 707–727.

Modigliani, F., Miller, M. H., 1958. The cost of capital, corporation finance, and the theory of investment. American Economic Review, 48(3), 261–297.

Nishihara, M., Shibata, T., 2013. The effects of external financing costs on investment timing and sizing decisions. Journal of Banking and Finance, 37(4), 1160-1175.

Rajan, R., 1992. Insiders and outsiders: The choice between informed and arm's length debt. Journal of Finance, 47(4), 1367–1400.

Rauh, J., Sufi, A., 2010. Capital structure and debt structure. Review of Financial Studies, 23(12), 4242–4280.

Shibata, T., Nishihara, M., 2012. Investment timing under debt issuance constraint. Journal of Banking and Finance, 36(4), 981–991.

Sundaresan, S., Wang, N., 2007. Investment under uncertainty with strategic debt service. American Economic Review Paper and Proceedings, 97(2), 256–261.

Wong, K. P., 2010. On the neutrality of debt in investment intensity. Annal of Finance, 6(6), 335–356.

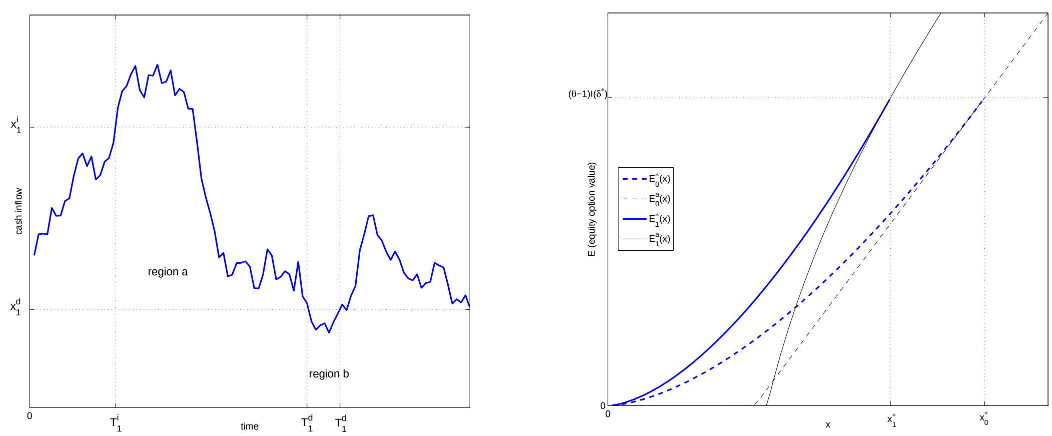

Figure 1: Scenarios of debt financing and equity values

The left-hand side panel illustrates the scenarios of debt financing. Regions a ($x \geq x_1^{\mathrm{d}}$) and b ($x < x_1^{\mathrm{d}}$) correspond to the normal and bankruptcy regions, respectively. The right-hand side panel depicts the equity value with the initial state variable. Note that we have $E_1(x_1^{\mathrm{i}*}) = E_0(x_0^{\mathrm{i}*}) = (\theta - 1)I(\delta^*)$, implying that $E_1^*(x) \geq E_0^*(x)$ where $x < x_1^{\mathrm{i}*}$.

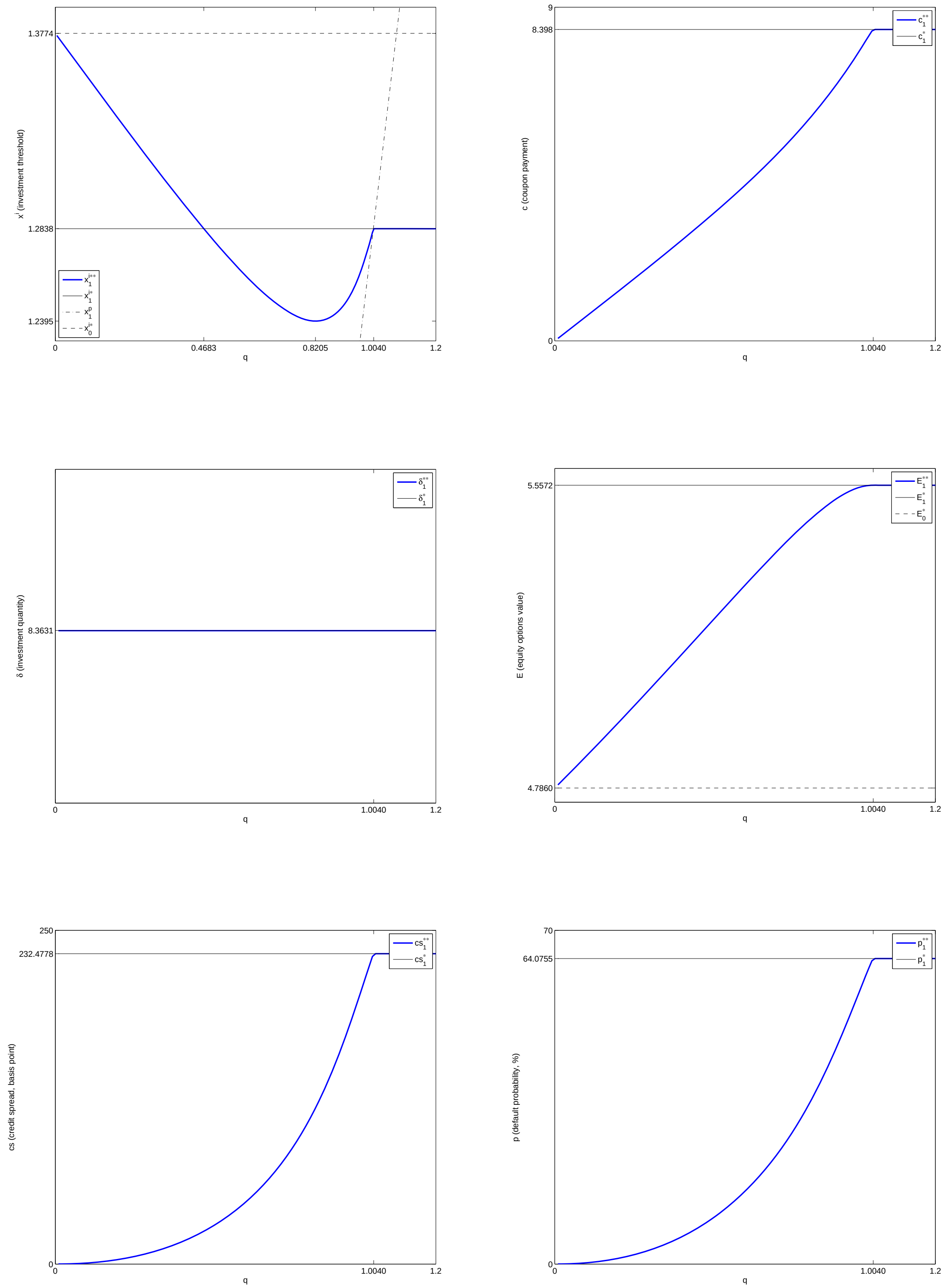

Figure 2: Solutions and value with q

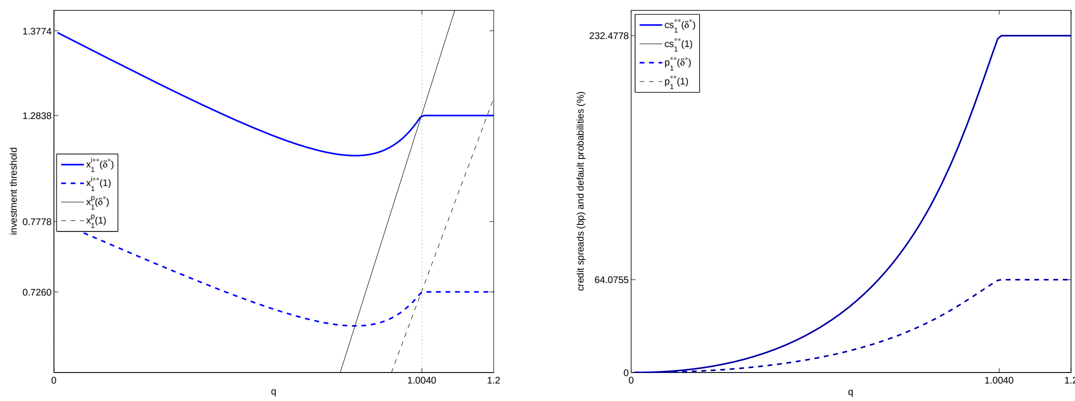

Figure 3: Effects of investment quantities

The left-hand side panel depicts the neutrality of investment quantities on financially constrained critical points. The right-hand side panel shows the neutrality on credit spreads and default probabilities.

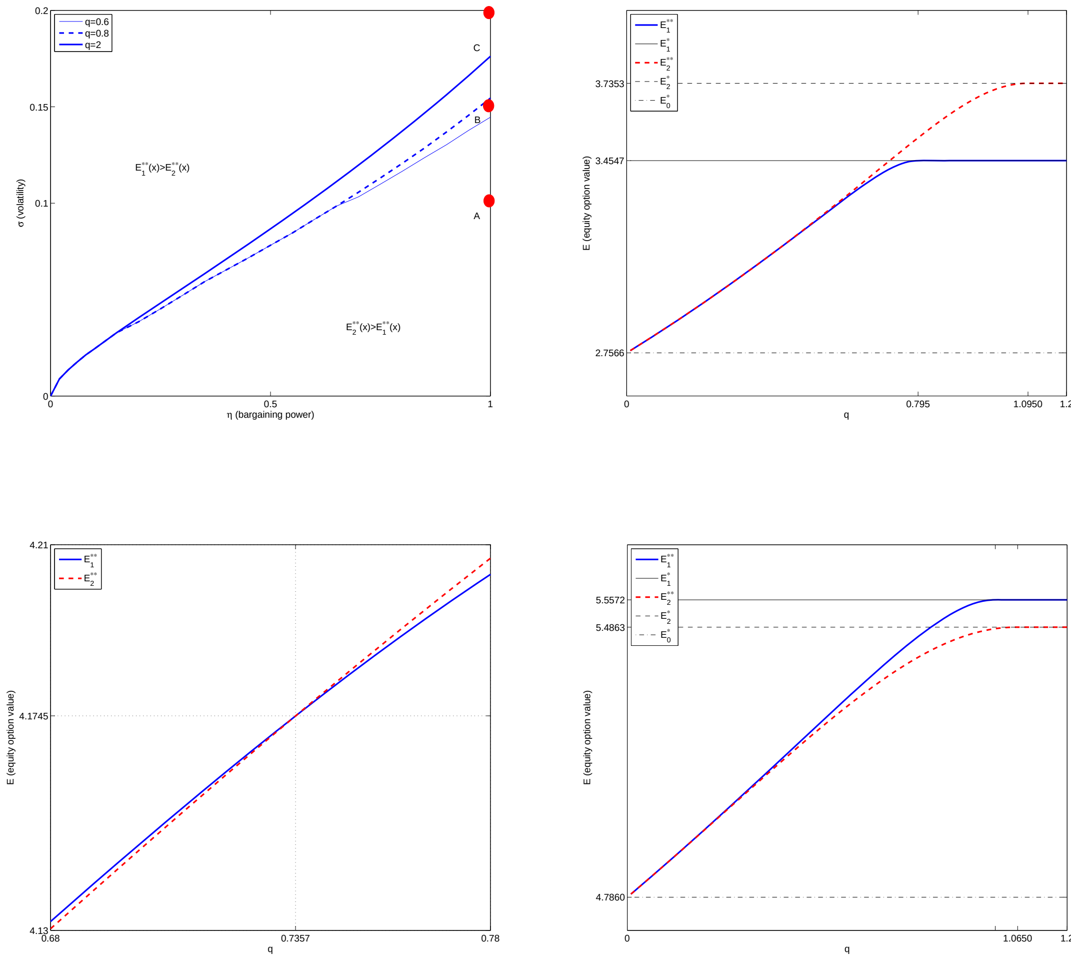

Figure 4: Equity option values before investment

The parameters of the upper-right ($\sigma = 0.1$), lower-left ($\sigma = 0.15$), and lower-right ($\sigma = 0.2$) panels correspond to those of Points A, B, and C of the upper-left panel, respectively.

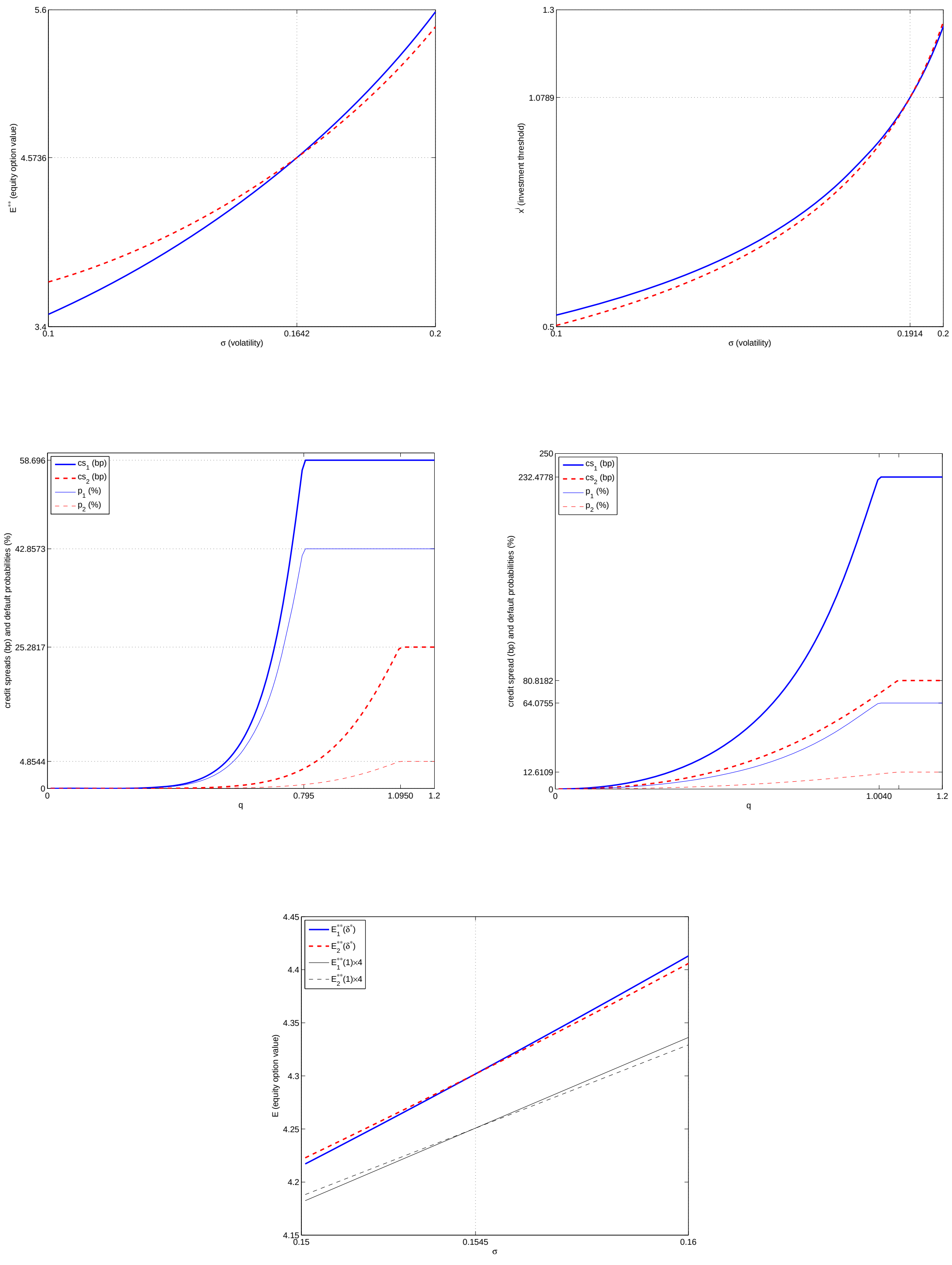

Figure 5: Effects of debt renegotiation