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Investment timing, debt structure, and financing constraints

Takashi Shibata† and Michi Nishihara‡

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† Graduate School of Social Sciences, Tokyo Metropolitan University
‡ Graduate School of Economics, Osaka University

Investment timing, debt structure, and financing constraints*

Takashi Shibata[a†] and Michi Nishihara[b‡]

[a]*Graduate School of Social Sciences, Tokyo Metropolitan University*

[b]*Graduate School of Economics, Osaka University*

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Abstract:

We examine optimal investment strategies for a firm financed by bank and market debt with an issuance capacity constraint in which banks have the unique ability to renegotiate outside formal bankruptcy. We show that the firm is more likely to issue market debt than bank debt when debt issuance capacity increases. This theoretical result is consistent with the empirical fact that large/mature corporations are more likely to choose market debt. The choice of debt structure makes corporate investment strategy more complicated, compared with scenarios in which firms have no choice of debt structure. Interestingly, under capacity constraints, the choice of debt structure does not always hasten corporate investment, which is contrary to investment under no choice of debt structure.

Keywords: Real options; debt structure; debt capacity; formal bankruptcy; private workout.

JEL classification: G32; G33, G21.

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†Corresponding author. Tel.: +81 42 677 2310; fax: +81 42 677 2298. Email: tshibata@tmu.ac.jp

‡Email: nishihara@econ.osaka-u.ac.jp

1 Introduction

Modigliani and Miller (1958) examine that investment and financing decisions are completely separable in a perfectly competitive market. Since their seminal study, the corporate finance literature has highlighted the role of market financial frictions between investment and financing decisions.[1] As a result, investment strategies are distorted, compared with the level of investment in a perfectly competitive market.

Brennan and Schwartz (1984), Mauer and Triantis (1994), Mauer and Sarkar (2005), and Sundaresan and Wang (2007) examine the interaction between investment and financing decisions. These models have two major limitations. First, there are no financial frictions. Second, the firm is financed by a single kind of debt, not various debt structures. The drawback of treating corporate debt as uniform is highlighted by the fact that different types of debt instruments have quite different effects on investment strategies (see Hackbarth et al. (2007) and Rauh and Sufi (2010)).

Several recent studies have already begun the task of incorporating either financing frictions or various debt structures separately into the investment timing decision (real options) model. Boyle and Guthrie (2003), Hirth and Uhrig-Homburg (2010), and Nishihara and Shibata (2011) examine investment timing decisions under internal financing constraints. Nishihara and Shibata (2010) and Shibata and Nishihara (2012) investigate the investment timing strategies under debt financing constraints.[2] An interesting result among these earlier papers is that investment strategies are non-monotonic with respect to the financial frictions.[3] Alternatively, most models approximate a firm with only market debt (a single kind of debt), assuming that dispersion of creditors prevents debt reorganization during financial distress. In practice, a leveraged firm in financial distress can try to restructure its outstanding debt into a more affordable form. This model allowing for debt restructuring approximates a firm with bank debt (i.e., non-market debt).[4]

[1] An incomplete list includes Fazzari et al. (1988), Hoshi et al. (1991), Whited (1992), Kaplan and Zingales (1997), Cleary (1999), Gomes (2001), Clementi and Hopenhayn (2006), Hennessy and Whited (2007), and Livdan et al. (2009).

[2] See, e.g., Stiglitz and Weiss (1981), Gale and Hellwig (1985), and Greenwald and Stiglitz (1993) for static models of the investment decision under a financing constraint.

[3] See Boyle and Guthrie (2003), Hirth and Uhrig-Homburg (2010), Nishihara and Shibata (2011), and Shibata and Nishihara (2012) for theoretical analyses. See Cleary et al. (2007) for an empirical analysis.

[4] A partial list of structural models with bank debt includes Mella-Barral and Perraudin (1997), Mella-

Sundaresan and Wang (2007) derive investment strategies under various debt structures by considering debt reorganization strategies for a firm under financial distress. However, these strategies are derived independently of financing frictions.

In this paper, we assume that a firm can issue two classes of debt: bank and market debt. Following Bulow and Shoven (1978), Gertner and Scharfstein (1991), Cantillo and Wright (2000), and Hackbarth et al. (2007), the only difference between bank and market debt is the bankruptcy procedure.[5] In addition, we assume that the firm has limited capacity constraints for issuing debt. The justification for this assumption is that investors are reluctant to lend beyond a certain amount, because issuing debt encourages risk shifting from equity holders to debt holders (see Jensen and Meckling (1976)).

This paper develops a model of financial frictions between investment and financing decisions with various structures of debt and examines its implications for a firm's investment decision strategy. In this model, we have endogenously determined investment timing, coupon payment level, and debt structure (either bank or market debt issuance) under financial frictions. To be more precise, our model is solved as follows. First, given a debt structure, we derive the optimal investment and coupon payment strategies under capacity constraints. Second, we choose the optimal debt structure by comparing the equity values financed by bank debt with those financed by market debt. Finally, under the optimal debt structure, we derive the optimal investment timing and coupon payment level.

Our model builds largely on three papers: McDonald and Siegel (1986), Sundaresan and Wang (2007), and Shibata and Nishihara (2012).[6] Our model becomes an all-equity financing model when the firm cannot issue any kind of debt (i.e., McDonald and Siegel (1986)). Our model is a non-constrained model under various debt structures when the firm can issue bank and market debt without issuance capacity constraints (i.e., Sundaresan and Wang (2007)). Our model becomes a constrained model under a market debt structure when a firm can issue only market debt with an issuance capacity constraint

Barral (1999), Fan and Sundaresan (2000), Broadie et al. (2007), and Hackbarth et al. (2007). None of these papers considers investment strategies.

[5] They assume that payments to market lenders cannot be changed outside the formal bankruptcy process and that the new owners can recapitalize optimally, although costs are incurred.

[6] In a different sense, our model can be regarded as incorporating investment strategies into the structural model for various debt structures developed by Hackbarth et al. (2007).

(i.e., Shibata and Nishihara (2012)). Our model can be regarded as a natural extension of the three models which it is built, and yields several additional important implications.

Our model provides three important results. First, the firm is more likely to issue market debt than bank debt when debt issuance capacity is increased. In particular, whether bank or market debt is issued at the time of investment depends largely on three key parameters (debt capacity, cash-in-flow volatility, and bargaining power in negotiations during financial distress).[7] This finding sheds light on firm decision making regarding investment, financing, and debt choice strategies with respect to debt capacity constraints, and is supported by empirical evidence. In practice, the firms with higher (lower) debt issuance capacity are regarded as large/mature (small/young) corporations. Based on this definition, our results show that large/mature (small/young) corporations are more likely to choose market (bank) debt. Thus, these results are consistent with the empirical findings of Blackwell and Kidwell (1988), Cantillo and Wright (2000), and Denis and Mihov (2003).

The second important result is that, given the debt structure, the investment thresholds have a U-shaped relationship with the debt capacity constraints. These findings are obtained under bank debt financing (Shibata and Nishihara (2012) examine this issue under market debt financing). Under the optimal debt structure, investment thresholds have a discontinuous W-shaped relationship with the debt capacity friction, depending on the other parameters. This discontinuity is caused by the change in the choice of debt structure. Thus, the choice of debt structure makes the corporate investment strategy more complicated, compared with scenarios in which firms have no choice of debt structure. The non-monotonicity between investment and friction is the same as in previous related papers (see, e.g., Boyle and Guthrie (2003), Cleary et al. (2007), Hirth and Uhrig-Homburg (2010), and Shibata and Nishihara (2012)).

The third important result is that the investment thresholds for a firm financed by bank debt with a capacity constraint are not always lower than those for a firm financed by market debt, even when the firm prefers bank debt. This implies that the possibility of choosing the debt structure (the possibility of bank debt issuance) does not always hasten corporate investment. Our result is different to the fact that, without debt issuance con-

[7]This result is similar to that of Hackbarth et al. (2007) who do not consider investment strategies.

straints, the possibility of choosing debt structure always hastens corporate investment. Thus, our result is also contrary to our intuition. The mechanism for investment strategies with debt capacity constraints may be quite different from that without debt capacity constraints. Thus, this result is obtained by incorporating debt capacity constraints.

The remainder of the paper is organized as follows. Section 2 describes the model and derives the value functions. Section 3 provides the solution of our model and considers its properties. Section 4 discusses the model's implications. Section 5 concludes.

2 Model

In this section, we begin with a description of the model. We then provide the value functions for the firms financed by bank debt and by market debt. Then, we formulate our model as an investment optimization problem for the firms financed by bank debt and by market debt with capacity constraints. As a benchmark, we derive the solutions to the two extreme cases in our model. One is the solution for an unlevered (all-equity financed) firm (i.e., the model developed by McDonald and Siegel (1986)). The other is the solution for a firm financed by bank debt and market debt without a capacity constraint (i.e., a model similar to that of Sundaresan and Wang (2007)).

2.1 Setup

A firm possesses an option to invest in a single project at any time. If the investment option is exercised at time t, the firm pays a fixed cost $I > 0$ at time t and receives an instantaneous cash inflow X_t after time t, where X_t is given by the following geometric Brownian motion:

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}z_t^{\mathbb{Q}}, \quad X_0 = x, \tag{1}$$

where $z_t^{\mathbb{Q}}$ denotes a standard Brownian motion defined by a risk-neutral probability space $(\Omega, \mathcal{F}, \mathbb{Q})$, and μ and σ are constant parameters. For convergence, we assume that $r > \mu \geq 0$, where r is a constant risk-free interest rate. It is assumed that the current state variable $X_0 = x$ is sufficiently low that the investment is not undertaken immediately.

In this paper, we assume that the firm issues two classes of perpetual debt: market debt with a promised coupon payment flow c_1 (subscript "1" indicates market debt) and bank debt with a promised coupon payment flow c_2 (subscript "2" indicates bank debt). Following Bulow and Shoven (1978), Gilson et al. (1990), Gertner and Scharfstein (1991), Bolton and Sharfstein (1996), and Hackbarth et al. (2007), we assume that the only difference between bank and market debt is the bankruptcy procedure. Now suppose that the firm in financial distress tries to restructure the debt. Under market debt financing, the coupon payments to the market lender cannot be changed outside of the formal bankruptcy process. Under bank debt financing, the coupon payments to the bank lender are reduced in the course of a costless private workout.

Given a debt structure j ($j \in \{1, 2\}$), let us denote by T_j^{i} and T_j^{d} the investment (indicated by superscript "i") and default (indicated by superscript "d") timings, respectively. Mathematically, the investment and default timings are defined as $T_j^{\mathrm{i}} = \inf\{s \geq 0, X_s \geq x_j^{\mathrm{i}}\}$ and $T_j^{\mathrm{d}} = \inf\{s \geq T_j^{\mathrm{i}}; X_s \leq x_j^{\mathrm{d}}\}$, where x_j^{i} and x_j^{d} denote the associated investment and default thresholds, respectively. In particular, x_1^{d} and x_2^{d} represent the formal bankruptcy threshold for market debt and the negotiation (coupon reduction) threshold for bank debt, respectively. Note that these defaults are defined only after issuing the debt at the time of investment in this model (i.e., the firm is an unlevered firm before investment).

In the next subsection, we derive the value functions for a given debt structure.

2.2 Value functions of the firm financed by market debt

In this subsection, we derive the value functions after the market debt is issued at the time of investment.

Let us denote by $E_1^{\mathrm{a}}(X_t, c_1)$ the equity value at time t after issuing the market debt at the time of investment, where the superscript "a" represents firm value after the market debt issuance. The value, $E_1^{\mathrm{a}}(X_t, c_1)$, is defined as

$$E_1^{\mathrm{a}}(X_t, c_1) = \sup_{T_1^{\mathrm{d}}} \mathbb{E}_t^{\mathbb{Q}}\Big[\int_t^{T_1^{\mathrm{d}}} \mathrm{e}^{-r(u-t)}(1-\tau)(X_u - c_1)\mathrm{d}u\Big], \tag{2}$$

where $\mathbb{E}_t^{\mathbb{Q}}$ denotes the expectation operator at time t under probability measure $\mathbb{Q}$, and τ denotes the tax rate. Note that T_1^{d} is the formal bankruptcy time that the equity holders optimize. As in standard arguments, $E_1^{\mathrm{a}}(X_t, c_1)$ is given by

$$E_1^{\mathrm{a}}(X_t, c_1) = \max_{x_1^{\mathrm{d}}} \Big\{\Pi X_t - (1-\tau)\frac{c_1}{r} - \Big(\Pi x_1^{\mathrm{d}} - (1-\tau)\frac{c_1}{r}\Big)\Big(\frac{X_t}{x_1^{\mathrm{d}}}\Big)^{\gamma}\Big\}, \tag{3}$$

where $\Pi := (1-\tau)/(r-\mu) > 0$ and $\gamma := 1/2 - \mu/\sigma^2 - ((\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2)^{1/2} < 0$. Then, the optimal threshold for formal bankruptcy is obtained by

$$x_1^{\mathrm{d}}(c_1) = \operatorname*{argmax}_{x_1^{\mathrm{d}}} E_1^{\mathrm{a}}(X_t, c_1) = \kappa_1^{-1} c_1, \tag{4}$$

where

$$\kappa_1 := \frac{\gamma - 1}{\gamma} \frac{1}{1-\tau} \Pi r \geq 0. \tag{5}$$

Note that $x_1^{\mathrm{d}}(c_1)$ is given as a linear function of c_1 with $\lim_{c_1 \downarrow 0} x_1^{\mathrm{d}}(c_1) = 0$. The result is obtained by Black and Cox (1976).

The debt value, $D_1^{\mathrm{a}}(X_t, c_1)$, is given as

$$\begin{aligned} D_1^{\mathrm{a}}(X_t, c_1) &= \mathbb{E}_t^{\mathbb{Q}} \Big[\int_t^{T_1^{\mathrm{d}}} \mathrm{e}^{-r(u-t)} c_1 \mathrm{d}u + \mathrm{e}^{-r(T_1^{\mathrm{d}} - t)} (1-\alpha) \Pi x_1^{\mathrm{d}}(c_1) \Big] \\ &= \frac{c_1}{r} \Big(1 - \Big(\frac{X_t}{x_1^{\mathrm{d}}(c_1)} \Big)^{\gamma} \Big) + (1-\alpha) \Pi x_1^{\mathrm{d}}(c_1) \Big(\frac{X_t}{x_1^{\mathrm{d}}(c_1)} \Big)^{\gamma}, \end{aligned} \tag{6}$$

where $\alpha \in (0,1)$ denotes the proportional cost of formal bankruptcy. Because $\lim_{c_1 \downarrow 0} x_1^{\mathrm{d}}(c_1) = 0$ in (4), we have $\lim_{c_1 \downarrow 0} D_1^{\mathrm{a}}(X_t, c_1) = 0$. The total firm value, $V_1^{\mathrm{a}}(X_t, c_1)$, is defined by the sum of the equity and debt values, i.e.,

$$\begin{aligned} V_1^{\mathrm{a}}(X_t, c_1) &= E_1^{\mathrm{a}}(X_t, c_1) + D_1^{\mathrm{a}}(X_t, c_1) \\ &= \Pi X_t + \tau \frac{c_1}{r} \Big(1 - \Big(\frac{X_t}{x_1^{\mathrm{d}}(c_1)} \Big)^{\gamma} \Big) - \alpha \Pi x_1^{\mathrm{d}}(c_1) \Big(\frac{X_t}{x_1^{\mathrm{d}}(c_1)} \Big)^{\gamma}. \end{aligned} \tag{7}$$

The first term on the right-hand side in (7) represents the value of the unlevered firm. The second term represents the value of the tax shield. The third term represents the value of the formal bankruptcy cost. Obviously, we have $\lim_{c_1 \downarrow 0} V_1^{\mathrm{a}}(X_t, c_1) = \Pi X_t$, implying that the total firm value of the levered firm is equal to that of the unlevered firm when the firm does not have any debt.

2.3 Value functions for the firm financed by bank debt

This subsection provides the value functions after the bank debt is issued at the time of investment.

When the bank debt is issued at the time of investment, there are two types of regions: normal and renegotiation regions. On the one hand, let us denote by $E_2^{\mathrm{a}}(X_t, c_2)$,

$D_2^{\mathrm{a}}(X_t, c_2)$, and $V_2^{\mathrm{a}}(X_t, c_2)$ the equity, debt, and total firm values, respectively, in the normal region, where the subscript "a" stands for the normal region after issuing the bank debt. On the other hand, let us denote by $E_2^{\mathrm{b}}(X_t, c_2)$, $D_2^{\mathrm{b}}(X_t, c_2)$, and $V_2^{\mathrm{b}}(X_t, c_2)$ the equity, debt, and total firm values, respectively, in the negotiation region during the period of financial distress. The subscript "b" indicates the negotiation (coupon reduction) region during the period of financial distress after issuing the bank debt. The normal and negotiation regions are divided by the negotiation (coupon reduction) threshold x_2^{d}. That is, the region $\{X_t \geq x_2^{\mathrm{d}}\}$ is the normal region, while the region $\{X_t < x_2^{\mathrm{d}}\}$ is the negotiation region. The firm and bank negotiate and divide the surplus $V_2^{\mathrm{b}}(X_t, c_2) - (1-\alpha)\Pi X_t$ to avoid formal bankruptcy. The division of the surplus between the firm and the bank depends on their relative bargaining powers. Let us denote by η and $1-\eta$ the bargaining powers of the firm and the bank, respectively.

The equity and debt values in the normal region, $E_2^{\mathrm{a}}(X_t, c_2)$ and $D_2^{\mathrm{a}}(X_t, c_2)$, respectively, are given by

$$E_2^{\mathrm{a}}(X_t, c_2) = \sup_{T_2^{\mathrm{d}} \geq t} \mathbb{E}_t^{\mathbb{Q}}\Big[\int_t^{T_2^{\mathrm{d}}} \mathrm{e}^{-r(u-t)}(1-\tau)(X_u - c_2)\mathrm{d}u + \mathrm{e}^{-r(T_2^{\mathrm{d}}-t)} E_2^{\mathrm{b}}(X_{T_2^{\mathrm{d}}}, c_2)\Big], \quad (8)$$

$$D_2^{\mathrm{a}}(X_t, c_2) = \mathbb{E}_t^{\mathbb{Q}}\Big[\int_t^{T_2^{\mathrm{d}}} \mathrm{e}^{-r(u-t)} c_2 \mathrm{d}u + \mathrm{e}^{-r(T_2^{\mathrm{d}}-t)} D_2^{\mathrm{b}}(X_{T_2^{\mathrm{d}}}, c_2)\Big]. \quad (9)$$

The equity and debt values in the negotiation region, $E_2^{\mathrm{b}}(X_t, c_2)$ and $D_2^{\mathrm{b}}(X_t, c_2)$, respectively, are given by

$$E_2^{\mathrm{b}}(X_t, c_2) = \mathbb{E}_t^{\mathbb{Q}}\Big[\int_t^{T_2^{\mathrm{d}}} \mathrm{e}^{-r(u-t)}(1-\tau)(X_u - s(X_u))\mathrm{d}u + \mathrm{e}^{-r(T_2^{\mathrm{d}}-t)} E_2^{\mathrm{a}}(X_{T_2^{\mathrm{d}}}, c_2)\Big], \quad (10)$$

$$D_2^{\mathrm{b}}(X_t, c_2) = \mathbb{E}_t^{\mathbb{Q}}\Big[\int_t^{T_2^{\mathrm{d}}} \mathrm{e}^{-r(u-t)} s(X_u)\mathrm{d}u + \mathrm{e}^{-r(T_2^{\mathrm{d}}-t)} D_2^{\mathrm{a}}(X_{T_2^{\mathrm{d}}}, c_2)\Big], \quad (11)$$

where $s(X_t)$ is the reduced coupon payment in the negotiation region.

First, we begin by solving the reduced coupon payment in the negotiation region. As in the derivation in Sundaresan and Wang (2007) and Shibata and Tian (2012), the reduced coupon payment in the renegotiation region, $s(X_t)$, is given by

$$s(X_t) = (1-\alpha\eta)(1-\tau)X_t, \quad X_t < x_2^{\mathrm{d}}. \quad (12)$$

The reduced coupon payment $s(x)$ is a linear function of x with $\lim_{x\downarrow 0} s(x) = 0$. The lower the cash inflow x, the lower the coupon payment $s(x)$. This means that the firm need not consider formal bankruptcy once it is in the negotiation region.

We then rewrite the equity and debt values in the negotiation region, $E_2^{\mathrm{b}}(X_t, c_2)$ and $D_2^{\mathrm{b}}(X_t, c_2)$, respectively, as

$$E_2^{\mathrm{b}}(X_t, c_2) = \eta\Big[\alpha\Pi X_t - \frac{\tau c_2}{r}\frac{\gamma}{\beta-\gamma}\Big(\frac{X_t}{x_2^{\mathrm{d}}}\Big)^{\beta}\Big], \tag{13}$$

$$D_2^{\mathrm{b}}(X_t, c_2) = (1-\alpha\eta)\Pi X_t - (1-\eta)\frac{\tau c_2}{r}\frac{\gamma}{\beta-\gamma}\Big(\frac{X_t}{x_2^{\mathrm{d}}}\Big)^{\beta}, \tag{14}$$

where $X_t < x_2^{\mathrm{d}}$ and $\beta := 1/2 - \mu/\sigma^2 + ((\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2)^{1/2} > 1$. The values, (13) and (14), are equal to the fraction of the residual value for avoiding formal bankruptcy, depending on the bargaining powers η and $1-\eta$ of the firm and the bank, respectively. The total firm value is defined by the sum of the equity and debt values, i.e.,

$$\begin{aligned} V_2^{\mathrm{b}}(X_t, c_2) &= E_2^{\mathrm{b}}(X_t, c_2) + D_2^{\mathrm{b}}(X_t, c_2) \\ &= \Pi X(t) - \frac{\tau c_2}{r}\frac{\gamma}{\beta-\gamma}\Big(\frac{X_t}{x_2^{\mathrm{d}}}\Big)^{\beta}. \end{aligned} \tag{15}$$

Finally, the value functions in the normal region are derived. Using the standard valuation principle, the equity value in the normal region, $E_2^{\mathrm{a}}(X_t, c_2)$ in (8), is rewritten as

$$E_2^{\mathrm{a}}(X_t, c_2) = \max_{x_2^{\mathrm{d}}}\Big\{\Pi X_t - (1-\tau)\frac{c_2}{r} - \Big\{(1-\alpha\eta)\Pi x_2^{\mathrm{d}} - \frac{c_2}{r}(1-\tau-\tau\frac{\eta\gamma}{\beta-\gamma})\Big\}\Big(\frac{X_t}{x_2^{\mathrm{d}}}\Big)^{\gamma}\Big\}. \tag{16}$$

The optimal negotiation (coupon reduction) threshold is decided to maximize (16) with x_2^{d}, i.e.,

$$x_2^{\mathrm{d}}(c_2) = \operatorname*{argmax}_{x_2^{\mathrm{d}}} E_2^{\mathrm{a}}(X_t, c_2) = \kappa_2^{-1} c_2, \tag{17}$$

where

$$\kappa_2 = \frac{\gamma-1}{\gamma}\frac{1-\alpha\eta}{1-\tau(1-\eta)}\Pi r \geq 0. \tag{18}$$

Note that $x_2^{\mathrm{d}}(c_2)$ is a linear function of c_2 with $\lim_{c_2\downarrow 0} x_2^{\mathrm{d}}(c_2) = 0$.[8] The debt value in the normal region, $D_2^{\mathrm{a}}(X_t, c_2)$ in (9), is given by

$$D_2^{\mathrm{a}}(X_t, c_2) = \frac{c_2}{r} + \Big\{(1-\alpha\eta)\Pi x_2^{\mathrm{d}}(c_2) - \frac{c_2}{r}(1-\tau+\tau\frac{\beta}{\beta-\gamma} - \tau\frac{\eta\gamma}{\beta-\gamma})\Big\}\Big(\frac{X_t}{x_2^{\mathrm{d}}(c_2)}\Big)^{\gamma}. \tag{19}$$

It is straightforward to obtain $\lim_{c_2\downarrow 0} D_2^{\mathrm{a}}(X_t, c_2) = 0$. The total firm value in the normal region, $V_2^{\mathrm{a}}(X_t, c_2)$, is defined by the sum of the equity and debt values, i.e.,

$$\begin{aligned} V_2^{\mathrm{a}}(X_t, c_2) &= E_2^{\mathrm{a}}(X_t, c_2) + D_2^{\mathrm{a}}(X_t, c_2) \\ &= \Pi X_t + \frac{\tau c_2}{r}\Big(1 - \frac{\beta}{\beta-\gamma}\Big(\frac{X_t}{x_2^{\mathrm{d}}(c_2)}\Big)^{\gamma}\Big). \end{aligned} \tag{20}$$

[8] This result is the same as in Sundaresan and Wang (2007) and Shibata and Tian (2012).

Note that there is no bankruptcy cost term in (20). The first term on the right-hand side represents the value of the unlevered firm. The second term represents the value of the tax shield. As in (7), we obtain $\lim_{c_2 \downarrow 0} V_2^{\mathrm{a}}(X_t, c_2) = \Pi X_t$ in (20).

2.4 Investment problem for the constrained levered firm

In this subsection, we formulate the investment decision problem for the firm financed by bank debt and by market debt with issuance capacity constraints.

Suppose that the debt structure j is given ($j \in \{1, 2\}$). Given a debt structure j, the debt capacity constraint is assumed to be

$$\frac{D_j^{\mathrm{a}}(x_j^{\mathrm{i}}, c_j)}{I} \leq q, \tag{21}$$

for some constant $q \geq 0$. The inequality (21) means that the ratio of the debt issuance amount to the investment cost is restricted by the debt issuance capacity ratio q. In other words, the firm has access to debt issuance $D_j^{\mathrm{a}}(x_j^{\mathrm{i}}, c_j)$ up to the amount $qI \geq 0$. For example, the friction might arise because of the risk-shifting problem. Because issuing debt encourages risk shifting from equity holders to debt holders, investors are reluctant to lend beyond a certain amount (see Jensen and Meckling (1976)). Note that we have assumed $q \in [0, +\infty)$, not $q \in [0, 1]$. The reason is that the levered firm has the possibility of issuing the amount $D_j^{\mathrm{a}}(x_j^{\mathrm{i}}, c_j)$, which is more than the amount I, to maximize its value. The case of $D_j^{\mathrm{a}}(x_j^{\mathrm{i}}, c_j) > I$ may lead to the result $q > 1$, which implies that the excess is distributed to equity holders as a dividend.[9]

Let us denote by $E_{j\mathrm{C}}^{\mathrm{o}}(x)$ the equity options value of the constrained firm for a given debt structure j ($j \in \{1, 2\}$), where the superscript "o" represents the options value before investment, and the subscript "C" indicates the constrained firm. Given a debt structure j, the equity options value of the constrained firm is defined as

$$E_{j\mathrm{C}}^{\mathrm{o}}(x) \quad := \sup_{T_{j\mathrm{C}}^{\mathrm{i}} \geq 0, c_{j\mathrm{C}} \geq 0} \mathbb{E}^{\mathbb{Q}} \left[\mathrm{e}^{-rT_{j\mathrm{C}}^{\mathrm{i}}} \left(E_j^{\mathrm{a}}(X_{T_{j\mathrm{C}}^{\mathrm{i}}}, c_{j\mathrm{C}}) - (I - D_j^{\mathrm{a}}(X_{T_{j\mathrm{C}}^{\mathrm{i}}}, c_{j\mathrm{C}}) \right) \right], \tag{22}$$

subject to $D_j^{\mathrm{a}}(X_{T_{j\mathrm{C}}^{\mathrm{d}}}, c_{j\mathrm{C}}) \leq qI$, where $x < x_{j\mathrm{C}}^{\mathrm{i}}$, and $\mathbb{E}^{\mathbb{Q}}$ denotes the expectation operator at time 0. Using standard arguments in Sundaresan and Wang (2007) and Shibata and

[9] Mauer and Sarkar (2005) and Sundaresan and Wang (2007) consider debt issuance without any debt capacity constraint at the time of investment. At the equilibrium, the firm issues the amount $D_j^{\mathrm{a}}(x_j^{\mathrm{i}}, c_j)$, which is more than the amount I, i.e., $D_j^{\mathrm{a}}(x_j^{\mathrm{i}}, c_j) > I$.

Nishihara (2010), the value, $E^{\rm o}_{j{\rm C}}(x)$, is given by

$$E^{\rm o}_{j{\rm C}}(x) = \max_{x^{\rm i}_{j{\rm C}}\geq 0, c_{j{\rm C}}\geq 0} \left(\frac{x}{x^{\rm i}_{j{\rm C}}}\right)^{\beta} \Big\{E^{\rm a}_j(x^{\rm i}_{j{\rm C}}, c_{j{\rm C}}) - \Big(I - D^{\rm a}_j(x^{\rm i}_{j{\rm C}}, c_{j{\rm C}})\Big)\Big\}, \quad (23)$$

subject to $D^{\rm a}_j(x^{\rm i}_{j{\rm C}}, c_{j{\rm C}}) \leq qI$.

We formulate the investment decision problem of the firm financed by bank debt and market debt with issuance capacity constraints. Let us denote by $E^{\rm o}_{\rm C}(x)$ the equity options value of the firm with optimal debt structure subject to capacity constraints. The equity options value $E^{\rm o}_{\rm C}(x)$ is obtained by

$$E^{\rm o}_{\rm C}(x) = \max\{E^{\rm o}_{1{\rm C}}(x), E^{\rm o}_{2{\rm C}}(x)\}, \quad (24)$$

where $E^{\rm o}_{j{\rm C}}(x)$ is given by

$$E^{\rm o}_{j{\rm C}}(x) = \max_{x^{\rm i}_{j{\rm C}}\geq 0, c_{j{\rm C}}\geq 0} \left(\frac{x}{x^{\rm i}_{j{\rm C}}}\right)^{\beta} \{V^{\rm a}_j(x^{\rm i}_{j{\rm C}}, c_{j{\rm C}}) - I\}, \quad (25)$$

$$\text{subject to } D^{\rm a}_j(x_{j{\rm C}}, c_{j{\rm C}}) \leq qI, \quad (26)$$

for any $x < \min\{x^{\rm i}_{1{\rm C}}, x^{\rm i}_{2{\rm C}}\}$ $(j \in \{1, 2\})$.

This problem is related to the issuance of either bank debt or market debt at the time of investment. The reason is that we can remove the mixed issuance of bank debt and market debt (we show that this assumption is correct below).[10]

Before analyzing the optimal investment strategies for a firm with different debt structures subject to a debt issuance capacity constraint, we first review two extreme cases briefly: the optimal investment problems of the unlevered firm and the levered firm with different debt structures without capacity constraints.

2.5 Investment problem for the unlevered firm

In this subsection, we assume that $q = 0$, i.e., $D^{\rm a}_j(x^{\rm i}_j, c_j) = 0$ for all j $(j \in \{1, 2\})$. This problem is equivalent to the investment problem for an all-equity financed firm, which is the simple version of the seminal model by McDonald and Siegel (1986).

Let us denote by $E^{\rm o}_{\rm U}(\cdot)$ the equity options value of the unlevered firm before investment, where the subscript "U" stands for the unlevered firm. When $q = 0$, we have $c_j = 0$ for

[10] Hackbarth et al. (2007) consider the mixed debt policy because they incorporate the priority structures of bank and market debt. Note that we do not introduce them.

all j, which leads to $E^{\rm a}_j(x,c_j)=V^{\rm a}_j(x,c_j)=\Pi x$ and $D^{\rm a}_j(x,c_j)=0$ for all j ($j\in\{1,2\}$). Thus, the value, $E^{\rm o}_{\rm U}(x)$, turns out to be

$$E^{\rm o}_{\rm U}(x)=\max_{x^{\rm i}_{\rm U}\geq 0}\Big\{\Big(\frac{x}{x^{\rm i}_{\rm U}}\Big)^{\beta}(\Pi x^{\rm i}_{\rm U}-I)\Big\}, \tag{27}$$

for any $x<x^{\rm i}_{\rm U}$. The optimal investment strategy and the equity options value are

$$x^{\rm i*}_{\rm U}=\frac{\beta}{\beta-1}\frac{1}{\Pi}I,\quad E^{\rm o}_{\rm U}(x)=\Big(\frac{x}{x^{\rm i*}_{\rm U}}\Big)^{\beta}\frac{1}{\beta-1}I, \tag{28}$$

where the superscript "$*$" represents the optimum.

2.6 Investment problem for the levered firm with different debt structures without capacity constraints

In this subsection, we assume that q is sufficiently large ($q\uparrow+\infty$). Then, the debt issuance capacity constraint becomes immaterial. This problem is similar to that in Sundaresan and Wang (2007).

Let us denote by $E^{\rm o}_{\rm N}(x)$ the equity options value for the firm financed by bank debt and market debt without an issuance capacity constraint. The value, $E^{\rm o}_{\rm N}(x)$, is formulated as

$$E^{\rm o}_{\rm N}(x)=\max\{E^{\rm o}_1(x),E^{\rm o}_2(x)\}, \tag{29}$$

where $E^{\rm o}_j(x)$ is given by

$$E^{\rm o}_j(x)=\max_{x^{\rm i}_j\geq 0,c_j\geq 0}\Big(\frac{x}{x^{\rm i}_j}\Big)^{\beta}\{V^{\rm a}_j(x^{\rm i}_j,c_j)-I\}, \tag{30}$$

for any $x<\min\{x^{\rm i}_1,x^{\rm i}_2\}$ ($j\in\{1,2\}$). The only difference between (24) and (29) is whether a debt financing capacity constraint exists.

The solutions and value for a given debt structure j are[11]

$$x^{\rm i*}_j=\psi_j x^{\rm i*}_{\rm U},\quad c^*_j=\frac{\kappa_j\psi_j}{h_j}x^{\rm i*}_{\rm U},\quad x^{\rm d}_j=\frac{\psi_j}{h_j}x^{\rm i*}_{\rm U},\quad E^{\rm o}_j(x)=\psi_j^{-\beta}E^{\rm o}_{\rm U}(x), \tag{31}$$

where $x<x^{\rm i}_j$ ($j\in\{1,2\}$), and

$$\begin{aligned} h_1&:=\Big(1-\gamma\Big(1+\alpha\frac{1-\tau}{\tau}\Big)\Big)^{-1/\gamma}\geq 1,\quad \psi_1:=\Big(1+\frac{\tau}{1-\tau}\frac{1}{h_1}\Big)^{-1}\leq 1,\\ h_2&:=\Big(\frac{\beta}{\beta-\gamma}(1-\gamma)\Big)^{-1/\gamma}\geq 1,\qquad \psi_2:=\Big(1+\frac{\tau(1-\alpha\eta)}{1-\tau(1-\eta)}\frac{1}{h_2}\Big)^{-1}\leq 1.\end{aligned} \tag{32}$$

[11]See Sundaresan and Wang (2007) and Shibata and Nishihara (2010) for a detailed derivation.

We have two important properties in (31). The first property is that we have $x_j^{\mathrm{i}*} \leq x_{\mathrm{U}}^{\mathrm{i}*}$ and $E_j^{\mathrm{o}}(x) \geq E_{\mathrm{U}}^{\mathrm{o}}(x)$, where $x < x_j^{\mathrm{i}*}$ for any j $(j \in \{1,2\})$.[12] The second property is that the inequality $\psi_j < \psi_k$ implies that $x_j^{\mathrm{i}*} < x_k^{\mathrm{i}*}$ and $E_j^{\mathrm{o}}(x) > E_k^{\mathrm{o}}(x)$ $(j,k \in \{1,2\}, j \neq k)$. We summarize these results as follows.

Lemma 1 *Suppose that the firm does not have a debt financing capacity constraint (i.e., q is sufficiently large). If the investment thresholds for market debt financing are smaller than those for bank debt financing, the non-constrained levered firm prefers market debt financing, i.e.,* $E_{\mathrm{N}}^{\mathrm{o}}(x) = E_1^{\mathrm{o}}(x) = \max\{E_1^{\mathrm{o}}(x), E_2^{\mathrm{o}}(x)\}$. *Otherwise, the non-constrained levered firm prefers bank debt financing, i.e.,* $E_{\mathrm{N}}^{\mathrm{o}}(x) = E_2^{\mathrm{o}}(x) = \max\{E_1^{\mathrm{o}}(x), E_2^{\mathrm{o}}(x)\}$.

3 Model solution

In this section, we derive the solution to the problem formulated in (24). Section 3.1 provides the solution to the investment decision problem for the constrained levered firm with a given debt structure. Section 3.2 examines the optimal debt structure by comparing the equity options values for a firm with bank debt and market debt with issuance capacity constraints. Then, we obtain the solution to the problem (24).

3.1 Optimal investment strategies for a given debt structure

Given a debt structure j $(j \in \{1,2\})$, we define the constant value x_j satisfying

$$D_j^{\mathrm{a}}(x_j, c_j(x_j)) = qI, \tag{33}$$

where $c_j(x)$ is given as

$$c_j(x) := \operatorname*{argmax}_{c_j} V_j^{\mathrm{a}}(x, c_j) = \frac{\kappa_j}{h_j} x, \tag{34}$$

where κ_j and h_j have been defined in (5), (18), and (32). Note that (34) is the optimal coupon payment of the non-constrained levered firm given a debt structure j and is obtained by Leland (1994). We have $x_j^{\mathrm{d}}(c_j(x)) = x/h_j$ by substituting (34) into (4) and

[12] The same result is obtained in the static model of Myers (1977).

(17).[13] The debt values, $D^{\rm a}_1(x, c_1(x))$ and $D^{\rm a}_2(x, c_2(x))$, are obtained as

$$
\begin{aligned}
D^{\rm a}_1(x, c_1(x)) &= \frac{1}{h_1}\Big(\frac{\kappa_1}{r}(1-h_1^{\gamma}) + (1-\alpha)\Pi h_1^{\gamma}\Big)x, &(35)\\
D^{\rm a}_2(x, c_2(x)) &= \frac{1}{h_2}\Big(\frac{\kappa_2}{r}(1-h_2^{\gamma}) + (1-\alpha)\Pi h_2^{\gamma} + \frac{\kappa_2}{r}h_2^{\gamma}\tau(1-\frac{\beta}{\beta-\gamma}+\frac{\eta\gamma}{\beta-\gamma})\Big)x. &(36)
\end{aligned}
$$

Then, $D^{\rm a}_j(x, c_j(x))$ is a strictly monotonically increasing continuous function of x with $\lim_{x\downarrow 0} D^{\rm a}_j(x, c_j(x)) = 0$ and $\lim_{x\uparrow+\infty} D^{\rm a}_j(x, c_j(x)) = +\infty$ for any j. Then, there exists a unique value x_j for any j ($j \in \{1, 2\}$).

Suppose, for example, $x^{\rm i*}_j > x_j$ for any j ($j \in \{1, 2\}$), implying that the investment threshold of the non-constrained levered firm is larger than the critical value. Then, the firm would prefer to issue more debt than the amount qI, to maximize the equity value, while the debt holders restrict the amount to less than qI. Thus, the firm is financially constrained by its debt issuance capacity. Suppose that, instead, $x^{\rm i*}_j \le x_j$. Then, the firm can choose the investment threshold to maximize the total firm value on the condition that the debt issuance is less than the capacity qI. Therefore, the firm is not constrained. We summarize these results as follows.

Lemma 2 *Given a debt structure j ($j \in \{1, 2\}$), there exists a unique x_j satisfying (33). Then, if $x^{\rm i*}_j > x_j$, the firm is financially constrained. Otherwise, it is not.*

Lemma 2 implies that, by comparing the magnitudes of $x^{\rm i*}_j$ and x_j for j, we recognize whether the debt capacity constraint is binding. Using these properties, we have the following results (the proof is given in the Appendix).

Proposition 1 *Suppose that $x^{\rm i*}_j > x_j \ge 0$ for a given debt structure j ($j \in \{1, 2\}$). For $x_j = 0$, because $q = 0$, we have $(x^{\rm i*}_{j\rm C},\ c^*_{j\rm C},\ x^{\rm d*}_{j\rm C}) = (x^{\rm i*}_{\rm U},\ 0,\ 0)$. For $x_j > 0$, because $q > 0$, the solutions $(x^{\rm i*}_{j\rm C},\ c^*_{j\rm C})$ are decided uniquely by two simultaneous equations:*

$$
\begin{aligned}
&\frac{f_{j1}(x^{\rm i*}_{j\rm C}, c^*_{j\rm C})}{f_{j2}(x^{\rm i*}_{j\rm C}, c^*_{j\rm C})} - \frac{f_{j3}(x^{\rm i*}_{j\rm C}, c^*_{j\rm C})}{f_{j4}(x^{\rm i*}_{j\rm C}, c^*_{j\rm C})} = 0, &(37)\\
&D^{\rm a}_j(x^{\rm i*}_{j\rm C}, c^*_{j\rm C}) - qI = 0, &(38)
\end{aligned}
$$

where f_{j1}, f_{j2}, f_{j3}, and f_{j4} are given as

$$
f_{11} \ := \ (\beta-1)\Pi x^{\rm i*}_{1\rm C} + \Big(\beta\frac{\tau}{r} - (\beta-\gamma)\Big(\frac{x^{\rm i*}_{1\rm C}}{\kappa^{-1}c^*_{1\rm C}}\Big)^{\gamma}(\frac{\tau}{r} + \alpha\Pi\kappa_1^{-1})\Big)c^*_{1\rm C} - \beta I, \qquad (39)
$$

[13] See Shibata and Nishihara (2010) for the derivation of $c_j(x)$ and $x^{\rm d}(c_j(x))$ in detail.

$$f_{12} := \Big(\frac{x^{\mathrm{i}*}_{1\mathrm{C}}}{\kappa_1^{-1}c^*_{1\mathrm{C}}}\Big)^{\gamma}\gamma\Big(\frac{1}{r}-(1-\alpha)\Pi\kappa_1^{-1}\Big)c^*_{1\mathrm{C}}, \tag{40}$$

$$f_{13} := \frac{\tau}{r}-\Big(\frac{x^{\mathrm{i}*}_{1\mathrm{C}}}{\kappa_1^{-1}c^*_{1\mathrm{C}}}\Big)^{\gamma}(1-\gamma)\Big(\frac{\tau}{r}+\alpha\Pi\kappa_1^{-1}\Big), \tag{41}$$

$$f_{14} := \frac{1}{r}-\Big(\frac{x^{\mathrm{i}*}_{1\mathrm{C}}}{\kappa_1^{-1}c^*_{1\mathrm{C}}}\Big)^{\gamma}(1-\gamma)\Big(\frac{1}{r}-(1-\alpha)\Pi\kappa_1^{-1}\Big), \tag{42}$$

and

$$f_{21} := (\beta-1)\Pi x^{\mathrm{i}*}_{2\mathrm{C}}+\beta\frac{\tau}{r}\Big(1-\Big(\frac{x^{\mathrm{i}*}_{2\mathrm{C}}}{\kappa_2^{-1}c^*_{2\mathrm{C}}}\Big)^{\gamma}\Big)c^*_{2\mathrm{C}}-\beta I, \tag{43}$$

$$f_{22} := \Big(\frac{x^{\mathrm{i}*}_{2\mathrm{C}}}{\kappa_2^{-1}c^*_{2\mathrm{C}}}\Big)^{\gamma}\gamma\Big(\frac{1}{r}(1-\tau+\tau\frac{\beta}{\beta-\gamma}-\tau\frac{\eta\gamma}{\beta-\gamma})-(1-\alpha\eta)\Pi\kappa_2^{-1}\Big)c^*_{2\mathrm{C}}, \tag{44}$$

$$f_{23} := \frac{\tau}{r}\Big(1-\Big(\frac{x^{\mathrm{i}*}_{2\mathrm{C}}}{\kappa_2^{-1}c^*_{2\mathrm{C}}}\Big)^{\gamma}(1-\gamma)\frac{\beta}{\beta-\gamma}\Big), \tag{45}$$

$$f_{24} := \frac{1}{r}-\Big(\frac{x^{\mathrm{i}*}_{2\mathrm{C}}}{\kappa_2^{-1}c^*_{2\mathrm{C}}}\Big)^{\gamma}(1-\gamma)\Big(\frac{1}{r}(1-\tau+\tau\frac{\beta}{\beta-\gamma}-\tau\frac{\eta\gamma}{\beta-\gamma})-(1-\alpha\eta)\Pi\kappa_2^{-1}\Big). \tag{46}$$

Moreover, $x^{\mathrm{d}*}_{j\mathrm{C}} = x^{\mathrm{d}}_j(c^*_{j\mathrm{C}})$ is obtained by substituting $c^*_{j\mathrm{C}}$ into (4) and (17). Suppose, on the other hand, that $x^{\mathrm{i}*}_j \le x_j$ for a given debt structure j (because q is sufficiently large). We obtain ($x^{\mathrm{i}*}_{j\mathrm{C}}$, $c^*_{j\mathrm{C}}$, $x^{\mathrm{d}*}_{j\mathrm{C}}$) = ($x^{\mathrm{i}*}_j$, c^*_j, $x^{\mathrm{d}*}_j$). Substituting the solutions into the equity value before investment yields

$$E^{\mathrm{o}}_{j\mathrm{C}}(x) = \Big(\frac{x}{x^{\mathrm{i}*}_{j\mathrm{C}}}\Big)^{\beta}(V^{\mathrm{a}}_j(x^{\mathrm{i}*}_{j\mathrm{C}}, c^*_{j\mathrm{C}})-I). \tag{47}$$

Proposition 1 implies that the solutions (investment thresholds and coupon payments) for the constrained levered firm are determined by solving two simultaneous equations. Given a debt structure j, the solutions include those for the unlevered firm in the extreme case satisfying $x^{\mathrm{i}*}_{j\mathrm{C}} > x_j = 0$ because of $q = 0$, and the non-constrained levered firm in the extreme case satisfying $x_j \ge x^{\mathrm{i}*}_j$ because q is sufficiently large.

Given a debt structure j, the equity options value of the constrained levered firm is monotonically increasing with respect to q. It is easy to show the proof. For any q', q'' with $q'' \ge q' \ge 0$, we suppose that the respective optimal values are given as $(x/x^{\mathrm{i}\prime})^{\beta}(V^{\mathrm{a}}_j(x^{\mathrm{i}\prime}_{j\mathrm{C}}, c'_{j\mathrm{C}})-I)$, $(x/x^{\mathrm{i}\prime\prime})^{\beta}(V^{\mathrm{a}}_j(x^{\mathrm{i}\prime\prime}_{j\mathrm{C}}, c''_{j\mathrm{C}})-I)$, and $(x/x^{\mathrm{i}\prime})^{\beta}(V^{\mathrm{a}}_j(x^{\mathrm{i}\prime}_{j\mathrm{C}}, c'_{j\mathrm{C}})-I) > (x/x^{\mathrm{i}\prime\prime})^{\beta}(V^{\mathrm{a}}_j(x^{\mathrm{i}\prime\prime}_{j\mathrm{C}}, c''_{j\mathrm{C}})-I)$. Then, because the firm in state q'' can increase its value by choosing $(x^{\mathrm{i}\prime}_{j\mathrm{C}}, c'_{j\mathrm{C}})$, there is a contradiction. Thus, we summarize this result as follows.

Corollary 1 *Given a debt structure j ($j \in \{1, 2\}$), the equity options value of the constrained levered firm $E^{\mathrm{o}}_{j\mathrm{C}}(x)$ is monotonically increasing with respect to q. In particular,* *we have*

$$E^{\mathrm{o}}_j(x) \geq E^{\mathrm{o}}_{j\mathrm{C}}(x) \geq E^{\mathrm{o}}_{\mathrm{U}}(x), \tag{48}$$

for any $q \geq 0$ *with* $\lim_{q\downarrow 0} E^{\mathrm{o}}_{j\mathrm{C}} = E^{\mathrm{o}}_{\mathrm{U}}(x)$ *and* $\lim_{q\uparrow +\infty} E^{\mathrm{o}}_{j\mathrm{C}} = E^{\mathrm{o}}_j(x)$.

The result obtained in Corollary 1 fits well with empirical studies by Whited (1992). Note that Corollary 1 does not state which is larger, $E^{\mathrm{o}}_{1\mathrm{C}}(x)$ or $E^{\mathrm{o}}_{2\mathrm{C}}(x)$ for any $q \geq 0$. In the next subsection, we examine their magnitudes by using numerical examples.

3.2 Optimal debt structure strategies

In the previous subsection, we considered the solutions and their values for a given debt structure j. In this subsection, we examine the solutions to the problem (24). Recall that we cannot obtain the analytical solution of $E^{\mathrm{o}}_{j\mathrm{C}}(x)$ for $j \in \{1,2\}$. By comparing the magnitudes of $E^{\mathrm{o}}_{1\mathrm{C}}(x)$ and $E^{\mathrm{o}}_{2\mathrm{C}}(x)$ for given parameters, we obtain the solutions to the problem defined by (24) numerically.

Suppose that the basic parameters are $r = 0.09$, $\mu = 0.01$, $I = 5$, $\tau = 0.15$, $\alpha = 0.4$, and $x = 0.4$.[14] Whether bank or market debt is issued depends on the combination of the three key parameters: q (debt issuance friction), η (bargaining power), and σ (cash-in-flow volatility).

Figure 1 depicts the regions of $E^{\mathrm{o}}_{j\mathrm{C}}(x) > E^{\mathrm{o}}_{k\mathrm{C}}(x)$ in (η, σ) space $(j, k \in \{1,2\}, j \neq k)$. Three lines indicate the boundaries of $E^{\mathrm{o}}_{1\mathrm{C}}(x) = E^{\mathrm{o}}_{2\mathrm{C}}(x)$ for $q = 1$, $q = 0.8$, and $q = 0.6$. Under the basic parameters, the boundary for $q = 1$ is the same as the one for $q \uparrow +\infty$ (i.e., the one for the non-constrained levered firm). For the regions of smaller σ and larger η, we see $E^{\mathrm{o}}_{1\mathrm{C}}(x) \geq E^{\mathrm{o}}_{2\mathrm{C}}(x)$, i.e., the firm prefers market debt financing. For the regions of larger σ and smaller η, in contrast, we have $E^{\mathrm{o}}_{2\mathrm{C}}(x) \geq E^{\mathrm{o}}_{1\mathrm{C}}(x)$, i.e., the firm prefers bank debt financing. Importantly, an increase in q enlarges the regions of $E^{\mathrm{o}}_{1\mathrm{C}}(x) > E^{\mathrm{o}}_{2\mathrm{C}}(x)$. We then have the following.

[Insert Figure 1 about here]

Observation 1 *Consider the optimization problem for the constrained levered firm defined by (24). As debt issuance capacity increases, the firm becomes more likely to issue market debt.*

[14] Under the basic parameters, $x = 0.4$ always satisfies $x < \min\{x^{\mathrm{i}*}_{\mathrm{U}}, x^{\mathrm{i}*}_{j\mathrm{C}}, x^{\mathrm{i}*}_j\}$ for all j $(j \in \{1,2\})$.

We consider in detail the effects of the debt issuance constraints. Figure 2 illustrates the investment thresholds and equity options values with the debt capacity ratio q for $\eta = 1$ and $\sigma \in \{0.1, 0.15, 0.2\}$ (the top, middle, and bottom panels represent $\sigma = 0.10$, $\sigma = 0.15$, and $\sigma = 0.20$, respectively). The three left-hand side panels depict the investment thresholds with respect to q. Recall from Lemma 2 that the firm is constrained by debt issuance capacity if $x_j < x_j^{\mathrm{i}*}$ ($j \in \{1, 2\}$), while it is not otherwise. In the top left-hand side panel, for example, if $q < 0.9449$ ($q < 0.7045$) because $x_1 < x_1^{\mathrm{i}*}$ ($x_2 < x_2^{\mathrm{i}*}$), the firm is constrained by market (bank) debt issuance capacity (the two other left-hand side panels follow similarly). We have $\lim_{q \downarrow 0} x_{j\mathrm{C}}^{\mathrm{i}*} = x_{\mathrm{U}}^{\mathrm{i}*}$ and $\lim_{q \uparrow +\infty} x_{j\mathrm{C}}^{\mathrm{i}*} = x_j^{\mathrm{i}*}$ ($j \in \{1, 2\}$). Interestingly, $x_{j\mathrm{C}}^{\mathrm{i}*}$ is not always between $x_j^{\mathrm{i}*}$ and $x_{\mathrm{U}}^{\mathrm{i}*}$ for $q \in (0, +\infty)$. That is, the investment thresholds $x_{j\mathrm{C}}^{\mathrm{i}*}$ are non-monotonic with respect to q.[15]

[Insert Figure 2 about here]

The three right-hand side panels of Figure 2 demonstrate the equity options values with respect to q. There are three difference cases. First, in the top right-hand side panel, we have $E_{1\mathrm{C}}^{\mathrm{o}}(x) \geq E_{2\mathrm{C}}^{\mathrm{o}}(x)$ for all regions of q. Second, in the middle right-hand side panel, we see that $E_{2\mathrm{C}}^{\mathrm{o}}(x) > E_{1\mathrm{C}}^{\mathrm{o}}(x)$ for $q < \hat{q} = 0.71$ and $E_{1\mathrm{C}}^{\mathrm{o}}(x) \geq E_{2\mathrm{C}}^{\mathrm{o}}(x)$ for $q \geq \hat{q}$. Finally, in the bottom right-hand side panel, $E_{2\mathrm{C}}^{\mathrm{o}}(x) \geq E_{1\mathrm{C}}^{\mathrm{o}}(x)$ for all regions of q.

Figure 3 depicts the investment thresholds and equity options values with respect to q for $\eta = 0.5$ and $\sigma \in \{0.05, 0.075, 0.1\}$ (the top, middle, and bottom panels represent $\sigma = 0.05$, $\sigma = 0.075$, and $\sigma = 0.1$, respectively). In the three left-hand side panels, the investment thresholds have a U-shaped relationship with q. In the three right-hand side panels, there are three cases at the equilibrium. The properties of the three different cases under $\eta = 0.5$ are exactly the same as those under $\eta = 1$ in Figure 2. As a result, we conclude that there are three different cases for $\eta > 0$ because we choose $\eta = 1$ and $\eta = 0.5$ arbitrarily.

[Insert Figure 3 about here]

Figure 4 demonstrates the investment thresholds and equity options values with respect to q for $\eta = 0$ and $\sigma \in \{0.1, 0.15, 0.2\}$ (the top, middle, and bottom panels are

[15] See Section 4.1 for non-monotonicity in detail.

assumed to be $\sigma = 0.1$, $\sigma = 0.15$, and $\sigma = 0.2$, respectively). The three left-hand side panels demonstrate the non-monotonic relationship between x^{i*}_{jC} and q. In the three right-hand side panels, we have $E^{o}_{2C}(x) > E^{o}_{1C}(x)$ for all regions of q and for all regions of σ. Consequently, under $\eta = 0$, the firm prefers bank debt financing at the time of investment.

[Insert Figure 4 about here]

In Figures 2 to 4, we have considered the investment thresholds and equity options values with respect to q for η ($\eta \in \{0, 0.5, 1\}$). We summarize the results obtained in these figures as follows.

Observation 2 *Consider the optimization problem for the constrained levered firm defined by (24). For $q = 0$, the solution and the value are the same as those in (28). For $q \uparrow +\infty$, the solution and the value are the same as those in Lemma 1. We assume that $q \in (0, +\infty)$. For $\eta \in (0, 1]$, there are three optimal cases, depending on the parameters.*

(i) There exists $E^{o}_{2C}(x) \geq E^{o}_{1C}(x)$ for all regions of q. Then, the constrained levered firm chooses bank debt issuance at the time of investment.

(ii) There exists a unique $\hat{q}$ such that $E^{o}_{2C}(x) \geq E^{o}_{1C}(x)$ for $q \leq \hat{q}$ and $E^{o}_{1C}(x) > E^{o}_{2C}(x)$ for $q > \hat{q}$. Then, the firm prefers bank debt issuance for $q \leq \hat{q}$, while it prefers market debt issuance for $q > \hat{q}$.

(iii) There exists $E^{o}_{1C}(x) \geq E^{o}_{2C}(x)$ for all regions of q. Then, the firm chooses market debt issuance.

For $\eta = 0$, on the other hand, there exists only one case. The firm chooses bank debt issuance at the time of investment.

These results are obtained by incorporating the debt capacity constraints. They shed light on models of bank and market debt financing across debt issuance capacity. In particular, in case (ii) for the firm with $\eta > 0$, increasing q prompts a change in the choice of debt structure. These results are similar to those in Hackbarth et al. (2007) who do not consider the endogenous timing of investment.[16]

[16] Hackbarth et al. (2007) focus only on the firms with $\eta = 0$ and $\eta = 1$. The firms with $\eta = 0$ and $\eta = 1$ are defined as the weak and strong firms, respectively.

Observations 1 and 2 are most closely related to empirical studies on debt composition. According to the definition by Rajan (1992) and Hackbarth et al. (2007), the firms with larger q, larger η, and smaller σ (smaller q, smaller η, and larger σ) best approximate large/mature (small/young) corporations. Then, based on the above definition, small/young firms are more likely to issue bank debt, whereas large/mature firms are more likely to issue market debt. These implied results in Observations 1 and 2 fit well with the empirical findings of Blackwell and Kidwell (1988), Cantillo and Wright (2000), and Denis and Mihov (2003).

4 Model implications

In this section, we consider the more important implications of our model. Section 4.1 investigates the effects of debt issuance constraints on the investment thresholds. Section 4.2 examines the relationship between the investment thresholds and the corresponding values. Section 4.3 considers the effects of optimal debt structures. Section 4.4 analyzes the comparative statics with the other key parameters (cash-in-flow volatility and bargaining power in private workouts) when the firm is financially constrained. Section 4.5 discusses the effects of a debt issuance constraint on the other solutions.

4.1 Non-monotonicity between investment thresholds and debt issuance constraints

This subsection examines the effects of a debt issuance constraint on investment thresholds. As demonstrated in Figures 2 to 4, to the extent that we have numerically solved for the investment thresholds for various parameters, we could not find any example of a monotonic relationship between $x_{j\mathrm{C}}^{\mathrm{i}*}$ and q for any j ($j \in \{1, 2\}$). Consequently, we have the following result.

Observation 3 *Given a debt structure, the investment thresholds have a U-shaped relationship with the debt capacity friction q in almost all cases.*

Shibata and Nishihara (2012) have already shown a non-monotonic relationship between $x_{1\mathrm{C}}^{\mathrm{i}*}$ and q. The new result obtained in Observation 3 is that we find a non-monotonic relationship between x_{2C}^{i*} and q. The non-monotonicity property is consistent with theoretical studies by Boyle and Guthrie (2003) and Hirth and Uhrig-Homburg (2010) and empirical study by Cleary et al. (2007).

In order to consider the most interesting case, we assume that $\eta = 1$ and $\sigma = 0.1$.[17] Under these parameters, as shown in the middle right-hand side panel of Figure 2, we have already shown that the firm chooses bank debt issuance at the time of investment for $0 < q < \hat{q} = 0.71$, and market debt issuance for $q \geq \hat{q} = 0.71$. Figure 5 provides a close-up of important points in the middle left-hand side panel of Figure 2. The large dotted line shows the investment thresholds at the equilibrium. Then, an increase in q leads to a jump in the investment thresholds, because the debt structure strategy has changed from bank debt to market debt. We see that, at $q = 0.71$, the investment threshold jumps from $x_{2C}^{i*} = 0.6612$ to $x_{1C}^{i*} = 0.6585$. We summarize the result as follows.

[Insert Figure 5 about here]

Observation 4 *Consider the optimization problem for the constrained levered firm defined by (24). Then, at the equilibrium, the investment thresholds have a discontinuous W-shaped relationship with the friction parameter q, depending on the two other key parameters, η and σ.*

With the optimal choice of debt structure, the relationship between the investment thresholds and debt capacity is a more complicated non-monotonicity relationship, compared with the scenario in which firms have no choice of debt structure.

4.2 Relationship between investment thresholds and values

In this subsection, we consider the relationship between the investment thresholds and the corresponding values. Recall in Lemma 1 that $x_1^{i*} \leq x_2^{i*}$ if and only if $E_1^{o}(x) \geq E_2^{o}(x)$, where $x < \min\{x_1^{i*}, x_2^{i*}\}$, when q is sufficiently large.

[Insert Figure 6 about here]

[17] Recall that the firm has three different debt financing cases for $\eta \in (0, 1]$ but only one debt financing case for $\eta = 0$. The three cases for $\eta \in (0, 1]$ include the one case for $\eta = 0$. Furthermore, the three cases for $\eta = 1$ are the same as those for $\eta \in (0, 1)$. From now on, we assume $\eta = 1$.

The two top panels of Figure 6 show the effects of cash-in-flow volatility, σ, on investment thresholds and equity options values, respectively. We now suppose $q = 0.7$ and $\eta = 1$ for $\sigma \in [0.1, 0.2]$.[18] In the top left-hand side panel, we have $x^{\mathrm{i}*}_{1\mathrm{C}} < x^{\mathrm{i}*}_{2\mathrm{C}}$ for $\sigma < 0.1579$. In the top right-hand side panel, we see that $E^{\mathrm{o}}_{1\mathrm{C}}(x) \geq E^{\mathrm{o}}_{2\mathrm{C}}(x)$ for $\sigma \leq 0.1495$.[19] From these two results, we obtain $x^{\mathrm{i}*}_{1\mathrm{C}} < x^{\mathrm{i}*}_{2\mathrm{C}}$ and $E^{\mathrm{o}}_{1\mathrm{C}}(x) < E^{\mathrm{o}}_{2\mathrm{C}}(x)$ for $\sigma \in (0.1495, 0.1579)$.

The two middle panels of Figure 6 demonstrate the impact of firm bargaining power, η, on the investment thresholds and equity options values, respectively. We assume $q = 0.7$ $\sigma = 0.15$ for $\eta \in [0, 1]$.[20] In the middle left-hand side panel, we have $x^{\mathrm{i}*}_{1\mathrm{C}} > x^{\mathrm{i}*}_{2\mathrm{C}}$ for $\eta < 0.8198$. In the middle right-hand side panel, we see that $E^{\mathrm{o}}_{1\mathrm{C}}(x) \leq E^{\mathrm{o}}_{2\mathrm{C}}(x)$ for $\eta \leq 0.7930$. From these two results, we obtain $x^{\mathrm{i}*}_{1\mathrm{C}} > x^{\mathrm{i}*}_{2\mathrm{C}}$ and $E^{\mathrm{o}}_{1\mathrm{C}}(x) > E^{\mathrm{o}}_{2\mathrm{C}}(x)$ for $\eta \in (0.7930, 0.8198)$. We summarize the two above results as follows.

Observation 5 *Consider the optimization problem for the constrained levered firm defined by (24). Then, the investment thresholds with bank (market) debt financing are not always lower than those with market (bank) debt financing when the firm prefers bank (market) debt financing to market (bank) debt financing.*

Observation 5 implies that the relationship between the investment thresholds and values for the constrained levered firm is not necessarily consistent with that for the non-constrained levered firm in Lemma 1. Thus, these findings provide important new insights by extending the investment decision problem for the non-constrained levered firm to the problem for the constrained levered firm.

4.3 Effects of optimal debt structures

This subsection examines how the possibility of bank debt issuance influences corporate investment. Recall that our model becomes that of Shibata and Nishihara (2012) after removing the possibility of bank debt issuance at the time of investment. Thus, this

[18] For all the regions of $\sigma \in [0.1, 0.2]$ with $q = 0.7$ and $\eta = 1$, we have $x^{\mathrm{i}*}_j > x_j$ $(j \in \{1, 2\})$, implying that the firm is financially constrained by its debt issuance capacity.

[19] We confirm one of the results obtained in Figure 1, namely, that an increase in σ changes the firm's debt capital strategies from market debt financing to bank debt financing.

[20] For all the regions of $\eta \in [0, 1]$ with $q = 0.7$ and $\sigma = 0.15$, we have $x^{\mathrm{i}*}_j > x_j$ $(j \in \{1, 2\})$, implying that the firm is financially constrained by its debt issuance capacity.

subsection makes clear the difference between our model and the previous model developed by Shibata and Nishihara (2012).

Suppose, as a benchmark, that q is sufficiently large, and that the firm can issue bank debt in addition to market debt at the time of investment. Then, the possibility of bank debt issuance increases the equity options values and hastens corporate investment. The reason is that, as shown in Lemma 1, $x_1^{\mathrm{i}*} \geq x_2^{\mathrm{i}*}$ if and only if $E_1^{\mathrm{o}}(x) \leq E_2^{\mathrm{o}}(x)$, where $x < \min\{x_1^{\mathrm{i}*}, x_2^{\mathrm{i}*}\}$.

Suppose that the firm comes up against the debt capacity constraint, and that the firm can issue bank debt in addition to market debt. Consider, for example, $q \in (0.63, 0.71)$, $\eta = 1$, and $\sigma = 0.15$ (see Figure 5, which provides a close-up of the middle left-hand side panel of Figure 2). Then, interestingly, the firm's access to bank debt delays corporate investment, although it always increases the equity options value (see the middle right-hand side panel of Figure 2). Thus, we summarize the result as follows.

Observation 6 *Suppose that the firm faces the debt issuance constraint at the time investment. Then, the choice of debt structures does not always hasten corporate investment.*

Our model differs from that of Shibata and Nishihara (2012) in that the firm can choose the optimal debt structure (i.e., the firm has the possibility of issuing bank debt). We can predict intuitively that choosing the optimal debt structure enables the firm to hasten its corporate investment. However, interestingly, this intuition is not always correct when the firm comes up against financial constraints.

4.4 Comparative statics

This subsection considers the effects of the cash-in-flow volatility parameter, σ, and the bargaining power parameter, η, on the investment thresholds, equity options values, and credit spreads. The credit spreads are defined as $cs_j(x_{j\mathrm{C}}^{\mathrm{i}*}, c_{j\mathrm{C}}^*) = c_{j\mathrm{C}}^*/D_j^{\mathrm{a}}(x_{j\mathrm{C}}^{\mathrm{i}*}, c_{j\mathrm{C}}^*) - r$.

From the two top and the bottom left-hand side panels of Figure 6, we summarize the effects of σ as follows.

Observation 7 *Consider the optimization problem for the constrained levered firm defined by (24). When volatility is greater, the firm is more likely to prefer bank debt.*

Denis and Mihov (2003) and Rauh and Sufi (2010) find that low-credit-quality firms are more likely to have bank debt. Firms with low-credit-quality are regarded as having high cash-in-flow volatility. Furthermore, Blackwell and Kidwell (1988) observe that small and risky firms sell debt privately. Moreover, the bottom left-hand side panel demonstrates that the credit spread is increasing with σ, which is the same as in the standard contingent claims models (see, e.g., Leland (1994)).

From the two middle and the bottom right-hand side panels, we summarize the effects of η as follows.

Observation 8 *Consider the optimization problem for the constrained levered firm defined by (24). When the bargaining power of the firm is higher (in other words, when the bargaining power of the bank is lower), the firm is more likely to choose market debt.*

The bottom right-hand side panel of Figure 6 depicts the credit spread with η. The higher η, the higher $cs_2(x_{2\mathrm{C}}^{\mathrm{i}*}, c_2^*)$, and the lower $E_{2\mathrm{C}}^{\mathrm{o}}(x)$. Davydenko and Strebulaev (2007) demonstrate empirically that higher bargaining power of equity holders results in higher spread and lower equity values. Our theoretical finding fits well with the empirical finding.

4.5 Effects of debt capacity constraints on the other solutions

In this subsection, we consider the effects of debt capacity constraints on the coupon, default, and values at the time of investment. As in the previous subsection, we assume $\eta = 1$.

Figures 7 and 8 demonstrate the effects of the debt issuance constraint on the other solutions and values (the left- and right-hand side panels are assumed to be $\sigma = 0.1$ and $\sigma = 0.2$). In the left-hand side panels of Figures 7 and 8, the firm chooses market debt issuance (see the top right-hand side panel of Figure 2). In the right-hand side panels of Figures 7 and 8, the firm chooses bank debt issuance (see the bottom right-hand side panel of Figure 2).

The two top panels of Figure 7 show the coupon payments with respect to q. We see that $c_{j\mathrm{C}}^*$ is monotonically increasing with respect to q for j ($j \in \{1, 2\}$). The next observation summarizes this property of the coupon payments.

Observation 9 *Given a debt structure, the coupon payments for the constrained levered firm are between those for the unlevered firm and those for the non-constrained levered firm.*

Observation 9 contrasts with Observation 3, where $x^{\mathrm{i}*}_{j\mathrm{C}}$ is not always in the regions $[x^{\mathrm{i}*}_j, x^{\mathrm{i}*}_{\mathrm{U}}]$.[21] Thus, it is less costly to distort $x^{\mathrm{i}*}_{j\mathrm{C}}$ away from the regions $[x^{\mathrm{i}*}_j, x^{\mathrm{i}*}_{\mathrm{U}}]$ than to distort $c^*_{j\mathrm{C}}$ away from the regions $[0, c^*_j]$.

[Insert Figure 7 about here]

The two middle panels of Figure 7 display the default probability defined by $\mathbb{E}^{\mathbb{Q}}[\mathrm{e}^{-r(T^{\mathrm{i}}_j - T^{\mathrm{d}}_j)}] = (x^{\mathrm{i}*}_{j\mathrm{C}}/x^{\mathrm{d}*}_{j\mathrm{C}})^{\gamma}$ with respect to q. We see that the default probabilities are monotonically increasing with respect to q. Comparing two probabilities, we have $(x^{\mathrm{i}*}_{2\mathrm{C}}/x^{\mathrm{d}*}_{2\mathrm{C}})^{\gamma} > (x^{\mathrm{i}*}_{1\mathrm{C}}/x^{\mathrm{d}*}_{1\mathrm{C}})^{\gamma}$, which implies that the default (coupon reduction) for the firm financed by bank debt is more likely to occur than the default (formal bankruptcy) for the firm financed by market debt. The reason for this finding is that the distance between $x^{\mathrm{i}*}_{2\mathrm{C}}$ and $x^{\mathrm{d}*}_{2\mathrm{C}}$ is smaller than the distance between $x^{\mathrm{i}*}_{1\mathrm{C}}$ and $x^{\mathrm{d}*}_{1\mathrm{C}}$ because there is no default cost for the firm financed by bank debt. Moreover, even though the firm chooses bank debt financing, the default probability for bank debt financing is larger than that for market debt financing.

The two bottom panels of Figure 7 depict the credit spreads. The credit spreads are also monotonically increasing with respect to q. We have $cs_2(x^{\mathrm{i}*}_{2\mathrm{C}}, c^*_2) > cs_1(x^{\mathrm{i}*}_{1\mathrm{C}}, c^*_1)$. These results correspond to the fact that the default probabilities for bank debt are larger than for market debt. Davydenko and Strebulaev (2007) find empirically that the possibility of strategic debt service increases corporate credit spreads. Thus, our theoretical findings are consistent with empirical findings.

The two top panels of Figure 8 depict the equity and debt values at the time of investment with respect to q. We see that the equity values are monotonically decreasing with respect to q. On the other hand, the debt values are monotonically increasing with respect to q. We confirm that the debt value is equal to the debt issuance capacity for the regions in which the firm is constrained, and that it is constant for the regions in which the firm is not constrained.

[Insert Figure 8 about here]

[21] Recall that there are two control variables, x^{i}_j and c_j, for the optimization problem defined by (25).

The two middle panels of Figure 8 illustrate the total firm values with respect to q. We see that the total firm values have a U-shaped relationship with q, which corresponds to the U-shaped relationship between the investment threshold and q. In the left-hand side panel ($\sigma = 0.1$), the firm prefers market debt financing for all regions of q. The total firm values for the firms financed by market debt are lower than those for the firms financed by bank debt for all regions of q. In the right-hand side panel ($\sigma = 0.2$), the firm prefers bank debt financing for all regions of q. The total firm values for the firms financed by bank debt are not always lower than those for the firms financed by market debt.[22] These results are similar to those obtained in Observation 3.

The two bottom panels of Figure 8 illustrate the leverages defined as $D_j^{\mathrm{a}}(x_j^{\mathrm{i}}, c_j)/V_j^{\mathrm{a}}(x_j^{\mathrm{i}}, c_j)$ with respect to q. We see that the leverages are monotonically increasing with respect to q.

5 Concluding remarks

We have proposed a model that analyzes optimal investment strategies under an optimal structure of bank debt and market debt with issuance constraints. By extending previous studies to consider various debt structures with debt issuance capacity constraints, we shed light on decisions regarding a firm's investment, financing, and debt choice strategies with issuance constraints. As a result, we can discuss the availability and effects of various debt structures under issuance capacity constraints.

We have three important results with respect to financial frictions. The first result is that increasing debt issuance capacity makes a firm more likely to issue market debt than bank debt. In particular, which type of debt is issued at the time of investment depends largely on three parameters (debt issuance capacity, cash-in-flow volatility, and bargaining power in the renegotiation during financial distress). These insights from our model are supported by empirical evidence.

The second result is that, for a given debt structure, the relationship between investment thresholds and debt capacity is non-monotonic. This finding is obtained under

[22]For $\eta = 1$ and $\sigma = 0.15$, at the equilibrium, the total firm values have a discontinuous W-shaped relationship with the friction. The reason is that their shape corresponds to that of the investment thresholds, which have a discontinuous W-shaped relationship with the friction.

market debt financing as well as bank debt financing. We could not find any example of a monotonic relationship, to the extent that we have solved numerically for various parameters. Moreover, under the optimal debt structure, the investment thresholds have a discontinuous W-shaped relationship with the debt capacity constraint, depending on the parameters. This discontinuity is caused by the change in the choice of debt structure. Thus, the optimal debt structure makes the corporate investment strategy more complicated, compared with the scenario in which firms have no choice of debt structure.

The third result is that the investment thresholds for the firm financed by bank debt with a capacity constraint are not always lower than those for the firm financed by market debt, even when the firm prefers bank debt. This implies that the choice of debt structure (the possibility of bank debt issuance) does not always hasten corporate investment. This result is contrary to our intuition. The mechanism for investment strategies with financial frictions may be quite different from that without financial frictions. Thus, it is important that we consider the impact of financial frictions on the investment strategies.

Appendix

In this subsection, we derive the functions f_{j1}, f_{j2}, f_{j3}, and f_{j4} in Proposition 1. Note that f_{jk} is a function of $(x_{j\mathrm{C}}^{\mathrm{i}*}, c_{j\mathrm{C}}^{*})$ for all j and k ($j \in \{1,2\}, k \in \{1,2,3,4\}$). Because the derivations of f_{1k} and f_{2k} are the same ($k \in \{1,2,3,4\}$), we show only the derivation of f_{2k}. In the subgame to the investment decision problem for the firm with constraint bank debt financing, the Lagrangian is formulated as

$$\begin{aligned}\mathcal{L} &= {x_{2\mathrm{C}}^{\mathrm{i}}}^{-\beta}\Big(\Pi x_{2\mathrm{C}}^{\mathrm{i}} + \frac{\tau c_{2\mathrm{C}}}{r} - \frac{\tau c_{2\mathrm{C}}}{r}\frac{\beta}{\beta-\gamma}\Big(\frac{x_{2\mathrm{C}}^{\mathrm{i}}}{\kappa_2^{-1}c_{2\mathrm{C}}}\Big)^{\gamma} - I\Big) \\ &\quad +\lambda\Big(qI - \frac{c_{2\mathrm{C}}}{r} - \Big((1-\alpha\eta)\Pi\kappa_2^{-1}c_{2\mathrm{C}} - \frac{c_{2\mathrm{C}}}{r}(1-\tau+\tau\frac{\beta}{\beta-\gamma} - \tau\frac{\eta\gamma}{\beta-\gamma})\Big)\Big(\frac{x_{2\mathrm{C}}^{\mathrm{i}}}{\kappa_2^{-1}c_{2\mathrm{C}}}\Big)^{\gamma}\Big)\Big),\end{aligned} \tag{A.1}$$

where $\lambda \geq 0$ denotes the multiplier on the constraint. The Karush-Kuhn-Tucker conditions are given by

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial x_{2\mathrm{C}}^{\mathrm{i}}} &= (-\beta){x_{2\mathrm{C}}^{\mathrm{i}*}}^{-\beta-1}\Big(\Pi x_{2\mathrm{C}}^{\mathrm{i}} + \frac{\tau c_{2\mathrm{C}}}{r} - \frac{\tau c_{2\mathrm{C}}}{r}\frac{\beta}{\beta-\gamma}\Big(\frac{x_{2\mathrm{C}}^{\mathrm{i}}}{\kappa_2^{-1}c_{2\mathrm{C}}}\Big)^{\gamma} - I\Big) \\ &\quad +{x_{2\mathrm{C}}^{\mathrm{i}*}}^{-\beta}\Big(\Pi - \frac{\tau c_{2\mathrm{C}}^{*}}{r}\frac{\beta}{\beta-\gamma}\Big(\frac{x_{2\mathrm{C}}^{\mathrm{i}*}}{\kappa_2^{-1}c_{2\mathrm{C}}^{*}}\Big)^{\gamma}\gamma {x_{2\mathrm{C}}^{\mathrm{i}*}}^{-1}\Big) \\ &\quad -\lambda\Big((1-\alpha\eta)\Pi\kappa_2^{-1}c_{2\mathrm{C}}^{*} - \frac{c_{2\mathrm{C}}^{*}}{r}(1-\tau+\tau\frac{\beta}{\beta-\gamma} - \tau\frac{\eta\gamma}{\beta-\gamma})\Big)\Big(\frac{x_{2\mathrm{C}}^{\mathrm{i}*}}{\kappa_2^{-1}c_{2\mathrm{C}}^{*}}\Big)^{\gamma}\gamma {x_{2\mathrm{C}}^{\mathrm{i}*}}^{-1} = 0,\end{aligned} \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial c_{2\mathrm{C}}} = x_{2\mathrm{C}}^{\mathrm{i}*\,-\beta}\Big(\frac{\tau}{r} - \frac{\tau}{r}\frac{\beta}{\beta-\gamma}\Big(\frac{x_{2\mathrm{C}}^{\mathrm{i}*}}{\kappa_2^{-1}c_{2\mathrm{C}}^*}\Big)^{\gamma}(1-\gamma)\Big) \tag{A.3}$$

$$-\lambda\Big(\frac{1}{r} + \Big((1-\alpha\eta)\Pi\kappa_2^{-1} - \frac{1}{r}(1-\tau+\tau\frac{\beta}{\beta-\gamma} - \tau\frac{\eta\gamma}{\beta-\gamma})\Big)(1-\gamma)\Big(\frac{x_{2\mathrm{C}}^{\mathrm{i}*}}{\kappa_2^{-1}c_{2\mathrm{C}}^*}\Big)^{\gamma}\Big) = 0,$$

and

$$\lambda\Big(qI - \frac{c_{2\mathrm{C}}^*}{r} - \Big((1-\alpha\eta)\Pi\kappa_2^{-1}c_{2\mathrm{C}}^* - \frac{c_{2\mathrm{C}}^*}{r}(1-\tau+\tau\frac{\beta}{\beta-\gamma} - \tau\frac{\eta\gamma}{\beta-\gamma})\Big)\Big(\frac{x_{2\mathrm{C}}^{\mathrm{i}*}}{\kappa_2^{-1}c_{2\mathrm{C}}^*}\Big)^{\gamma}\Big)\Big) = 0. \tag{A.4}$$

Assume that $\lambda > 0$, which is equivalent to $x_{2\mathrm{C}}^{\mathrm{i}*} > x_1$ from Lemma 2. For $q = 0$, because $c_{2\mathrm{C}}^* = 0$ from (A.4), we have $x_{2\mathrm{C}}^{\mathrm{i}*} = x_{\mathrm{U}}^{\mathrm{i}*}$ and $x_{2\mathrm{C}}^{\mathrm{d}*} = 0$. For $q > 0$, we have $c_{2\mathrm{C}}^* > 0$. Then $(x_{2\mathrm{C}}^{\mathrm{i}*}, c_{2\mathrm{C}}^*)$ are obtained uniquely by two simultaneous equations:

$$\frac{f_{21}(x_{2\mathrm{C}}^{\mathrm{i}*}, c_{2\mathrm{C}}^*)}{f_{22}(x_{2\mathrm{C}}^{\mathrm{i}*}, c_{2\mathrm{C}}^*)} - \frac{f_{23}(x_{2\mathrm{C}}^{\mathrm{i}*}, c_{2\mathrm{C}}^*)}{f_{24}(x_{2\mathrm{C}}^{\mathrm{i}*}, c_{2\mathrm{C}}^*)} = 0, \quad D_2^{\mathrm{a}}(x_{2\mathrm{C}}^{\mathrm{i}*}, c_{2\mathrm{C}}^*) - qI = 0. \tag{A.5}$$

The first equation is given by rearranging (A.2) and (A.3), while the second equation is given by (A.4). Thus, f_{21}, f_{22}, f_{23}, and f_{24} are obtained as (43)-(46). Similarly, we can derive f_{11}, f_{12}, f_{13}, and f_{14}.

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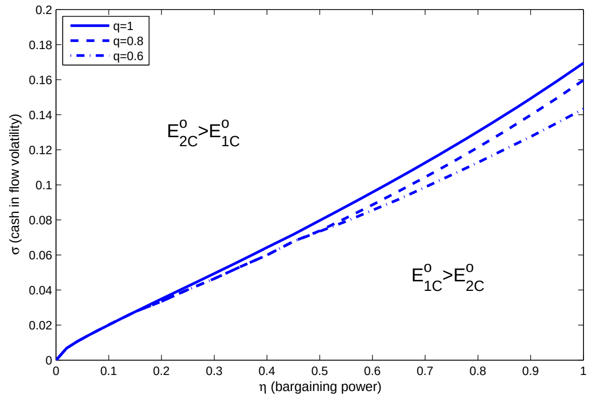

Figure 1: The regions of $E^{\mathrm{o}}_{1\mathrm{C}} > E^{\mathrm{o}}_{2\mathrm{C}}$ ($E^{\mathrm{o}}_{1\mathrm{C}} < E^{\mathrm{o}}_{2\mathrm{C}}$) for a given q

These lines indicate the boundaries of $E^{\mathrm{o}}_{1\mathrm{C}} = E^{\mathrm{o}}_{2\mathrm{C}}$. For the left-hand side of the boundary, we have the regions of $E^{\mathrm{o}}_{2\mathrm{C}} > E^{\mathrm{o}}_{1\mathrm{C}}$. For the right-hand side of the boundary, we obtain $E^{\mathrm{o}}_{1\mathrm{C}} > E^{\mathrm{o}}_{2\mathrm{C}}$. An increase in q enlarges the regions of $E^{\mathrm{o}}_{1\mathrm{C}} > E^{\mathrm{o}}_{2\mathrm{C}}$. The boundary line for $q = 1$ is the same as that for $q \uparrow +\infty$, which corresponds to that for the non-constrained firm developed by Sundaresan and Wang (2007).

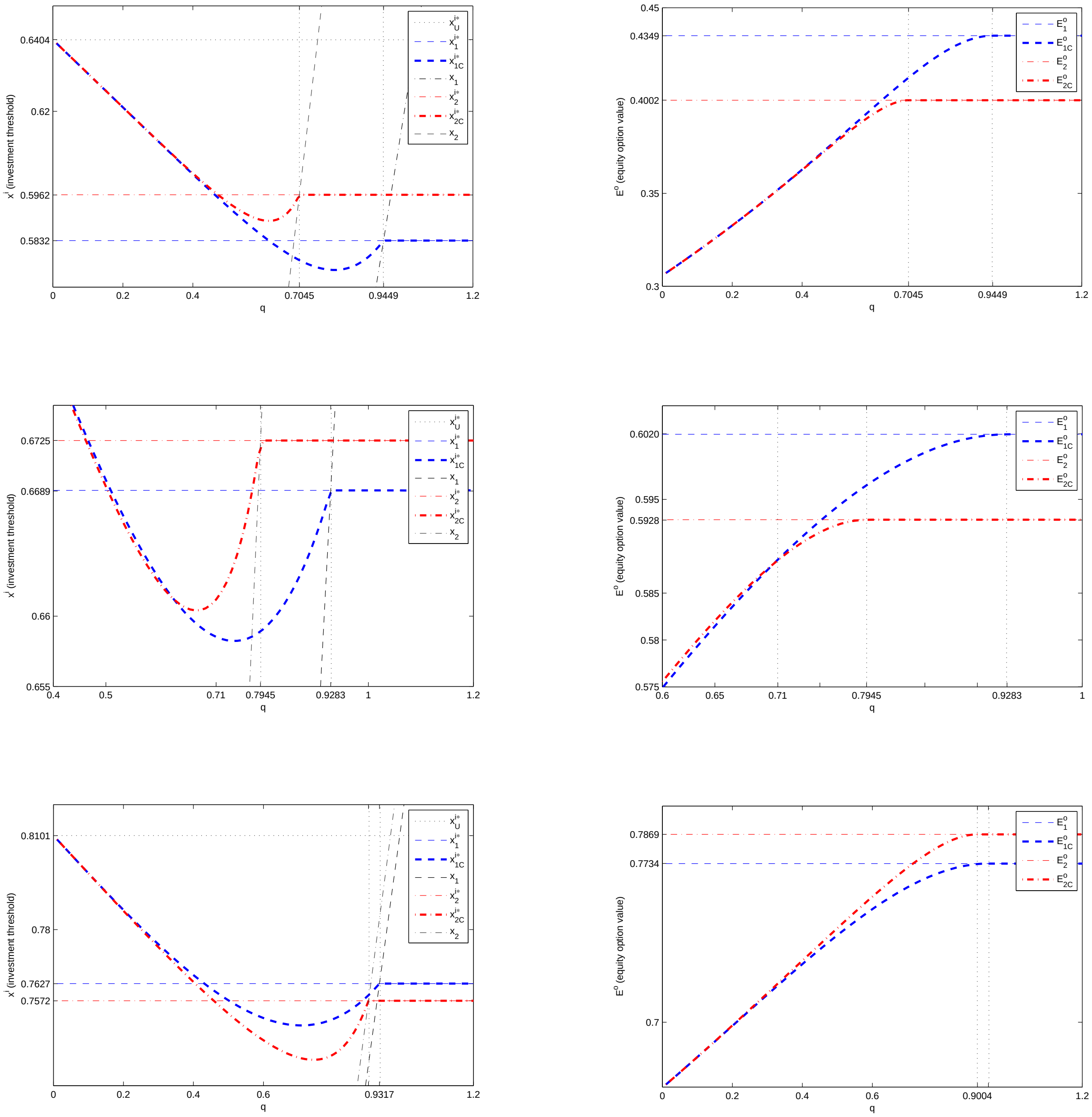

Figure 2: Investment thresholds and equity values with respect to q for $\eta = 1$

The top, middle, and bottom panels are assumed to be $\sigma = 0.10$, $\sigma = 0.15$, and $\sigma = 0.20$, respectively.

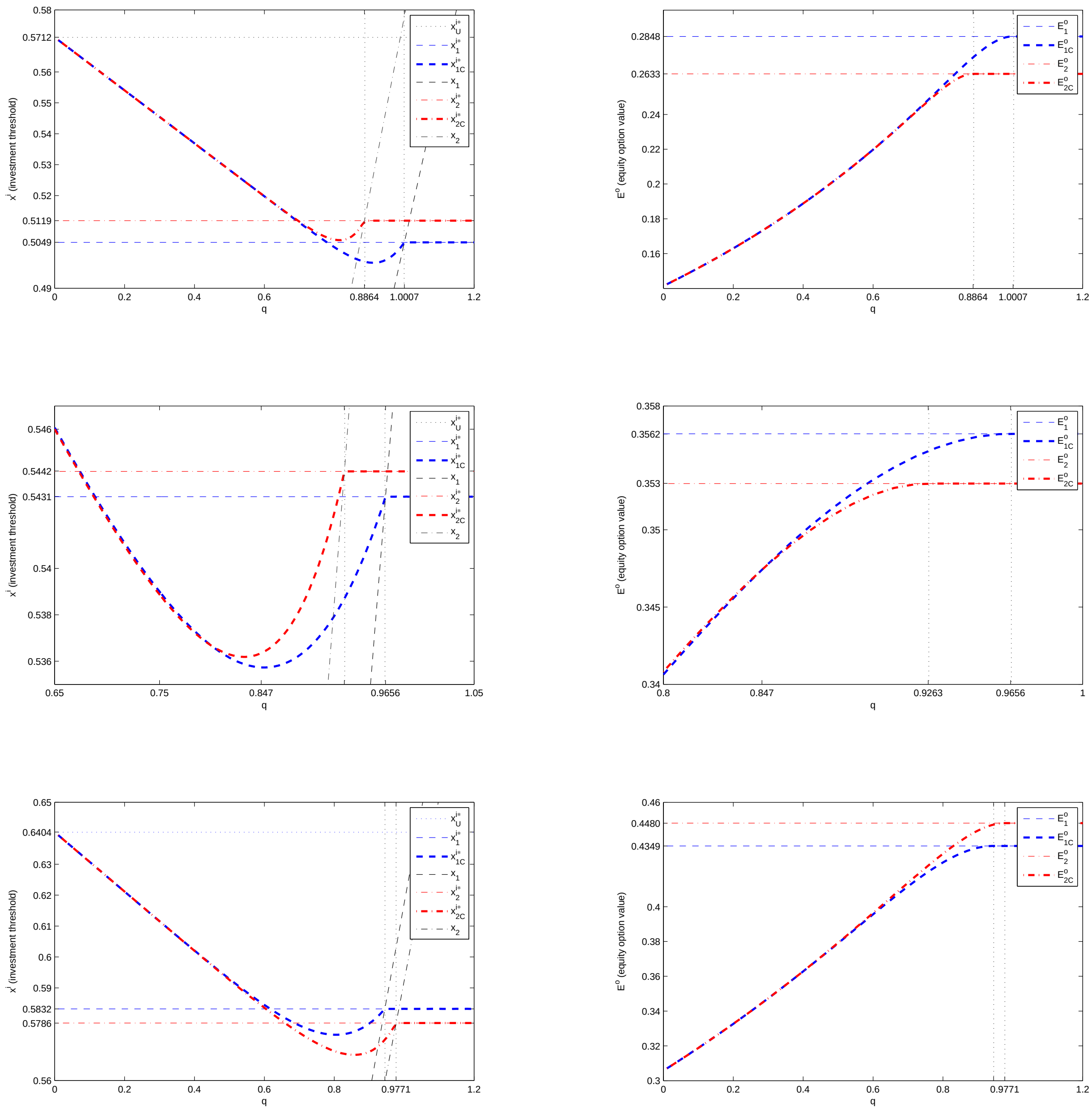

Figure 3: Investment thresholds and equity values with respect to q for $\eta = 0.5$

The top, middle, and bottom panels are assumed to be $\sigma = 0.05$, $\sigma = 0.075$, and $\sigma = 0.1$, respectively.

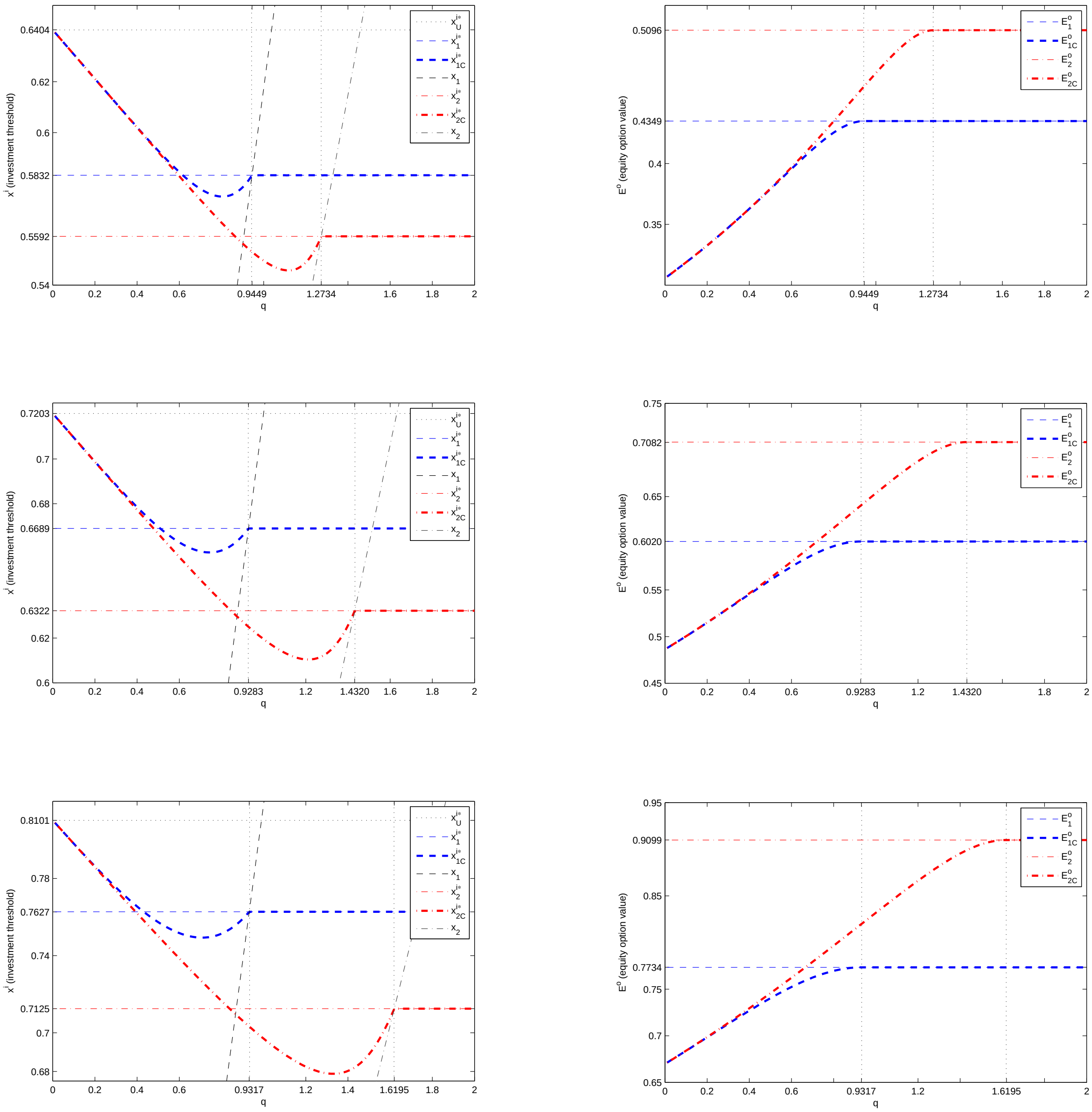

Figure 4: Investment thresholds and equity options values with respect to q for $\eta = 0$

The top, middle, and bottom panels are assumed to be $\sigma = 0.10$, $\sigma = 0.15$, and $\sigma = 0.20$, respectively.

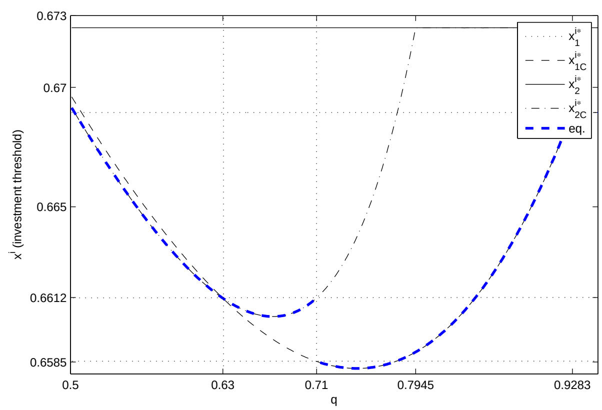

Figure 5: Optimal investment thresholds with respect to q under $\eta = 1$ and $\sigma = 0.15$ The firm prefers bank debt financing for $0 < q < 0.71$ while it prefers market debt financing for $q \geq 0.71$. Investment thresholds jump downward at $q = 0.71$ by increasing q.

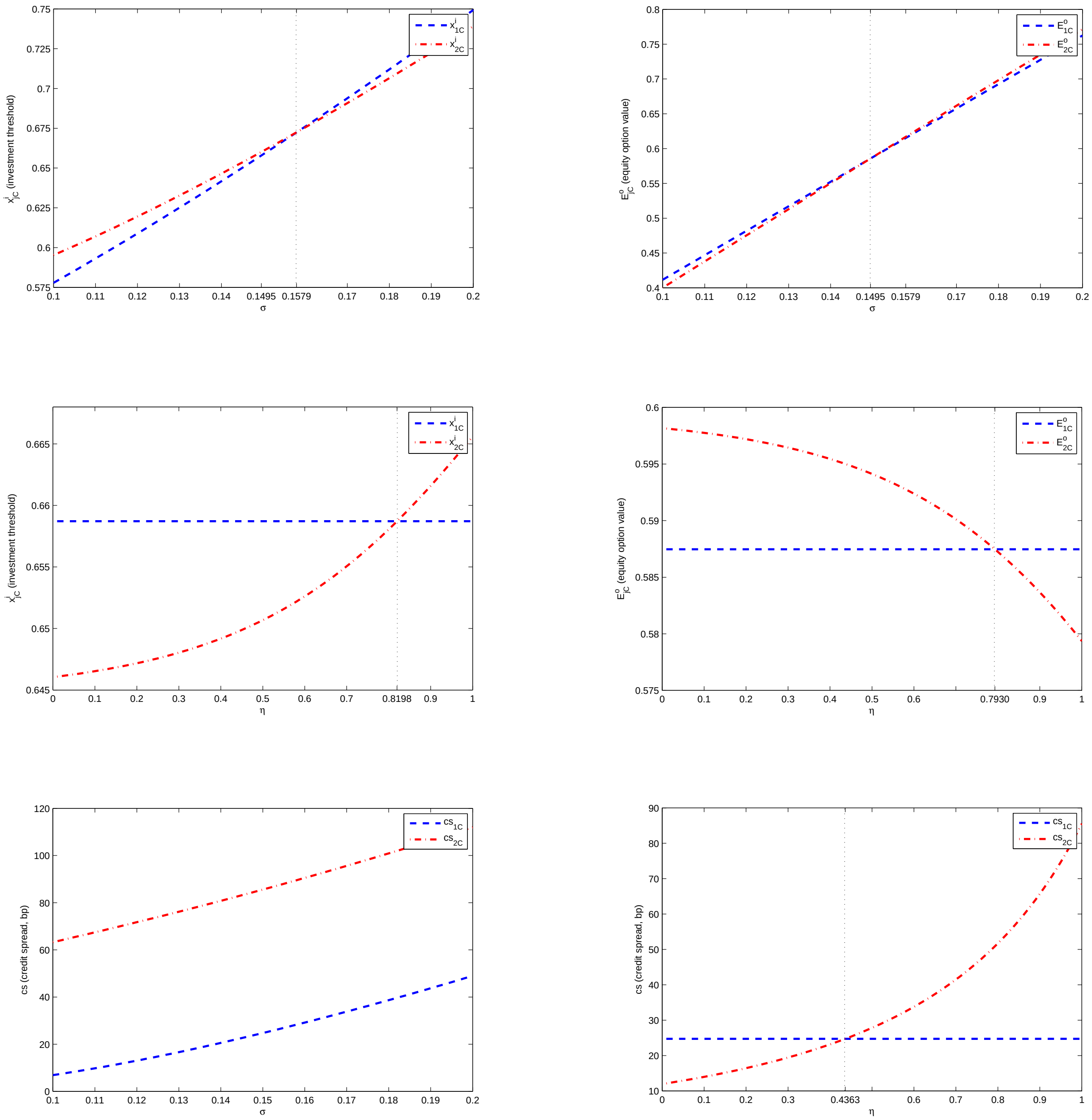

Figure 6: Investment thresholds and values with σ for $\eta = 1$

In the two top panels, we assume $q = 0.7$ and $\eta = 1$. Then, we have $x_{1C}^{i*} < x_{2C}^{i*}$ and $E_{1C}^{o}(x) < E_{2C}^{o}(x)$ for $\sigma \in (0.1495, 0.1579)$. In the two middle panels, we assume $q = 0.7$ and $\sigma = 0.15$. Then we see $x_{1C}^{i*} > x_{2C}^{i*}$ and $E_{1C}^{o}(x) > E_{2C}^{o}(x)$ for $\eta \in (0.7930, 0.8198)$. The bottom panels depict the credit spreads.

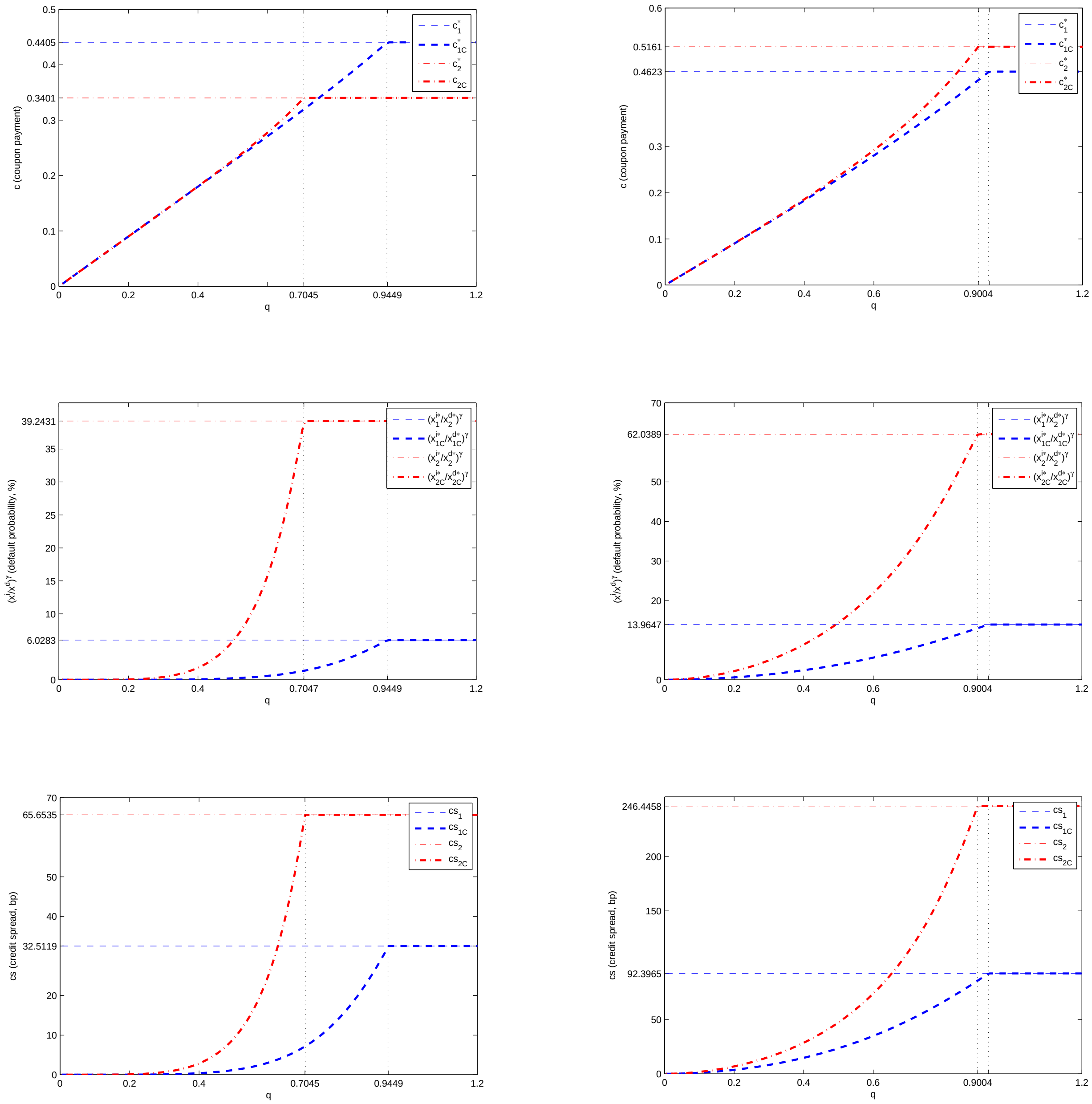

Figure 7: The effects of financial constraints for $\eta = 1$

This figure illustrates the effects of q on the coupon payment (top panels), the default probability (middle panels), and the credit spread (bottom panels). In this figure, we simply write $cs_j(x_j^{\mathrm{i}*}, c_j^*)$ and $cs_j(x_{j\mathrm{C}}^{\mathrm{i}*}, c_{j\mathrm{C}}^*)$ as cs_j and $cs_{j\mathrm{C}}$, respectively ($j \in \{1, 2\}$).

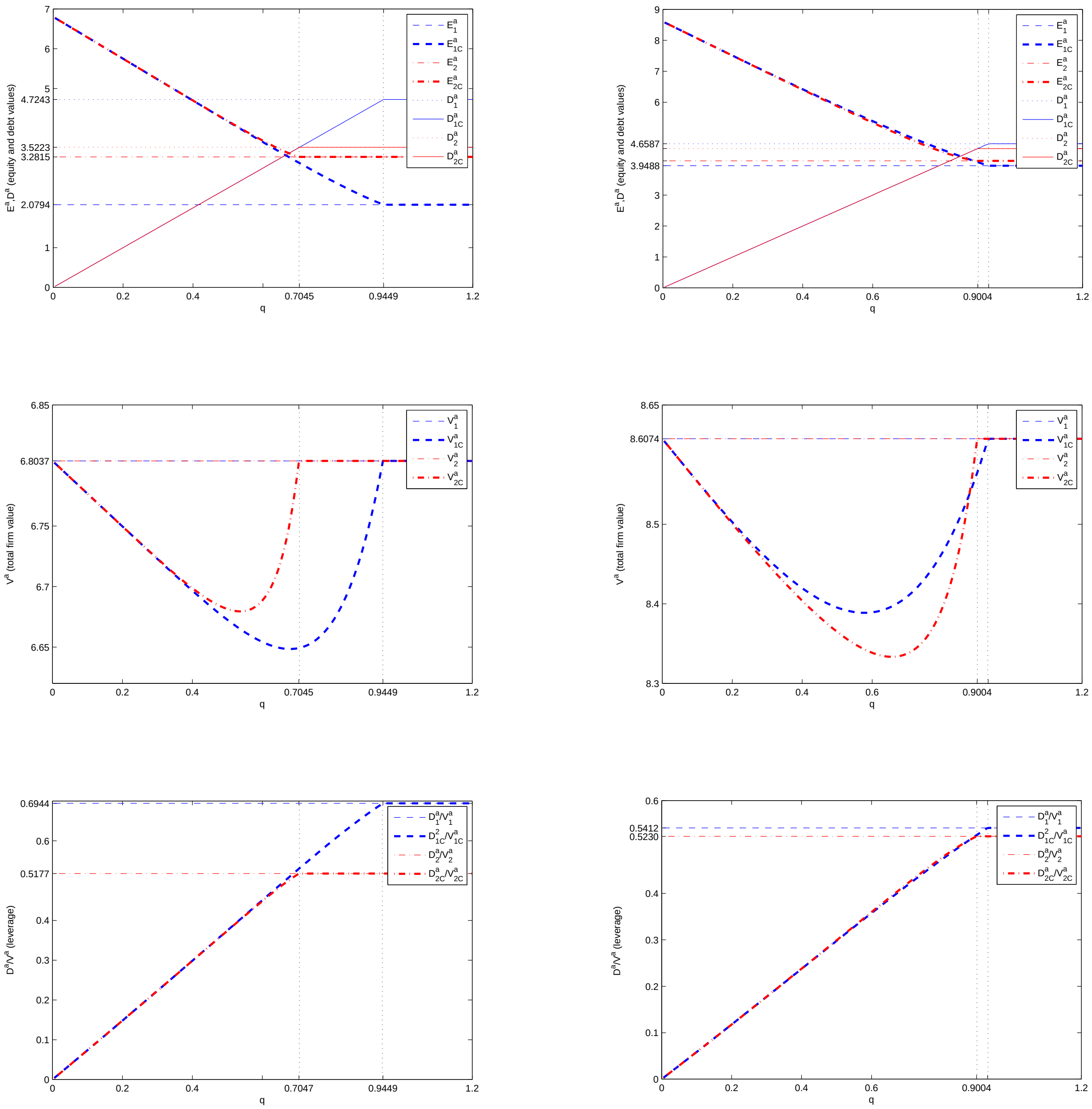

Figure 8: The effects of financial constraints for $\eta = 1$

This figure illustrates the effects of q on the equity and debt values (top panels), the total firm value (middle panels), and the leverage (bottom panels). In this figure, we simply write $F_j^{\mathrm{a}}(x_j^{\mathrm{i}*}, c_j^*)$, $F_j^{\mathrm{a}}(x_{j\mathrm{C}}^{\mathrm{i}*}, c_{j\mathrm{C}}^*)$, as F_j^{a}, $F_{j\mathrm{C}}^{\mathrm{a}}$, respectively ($F \in \{E, D, V\}, j \in \{1, 2\}$).