Research Paper Series

No. 74

Investment and Capital Structure Decisions of Foreign Subsidiary with International Debt Shifting and Exchange Rate Uncertainty

Masaaki Kijima† and Yuan Tian‡

May, 2010

† Graduate School of Social Sciences, Tokyo Metropolitan University
‡ Graduate School of Social Sciences, Tokyo Metropolitan University

Investment and Capital Structure Decisions of Foreign Subsidiary with International Debt Shifting and Exchange Rate Uncertainty

Masaaki Kijima and Yuan Tian

Graduate School of Social Sciences, Tokyo Metropolitan University

(This Version: May 30, 2010)

Abstract This paper examines the impact of international debt shifting and exchange rate uncertainty on investment and capital structure decisions of foreign subsidiary. We find that debt shifting induces earlier investment, earlier default, higher leverage, and larger *ex ante* firm value of foreign subsidiary. When debt shifting is not so costly, the optimal leverage of foreign subsidiary increases as the tax rate differential increases. Moreover, when the correlation between exchange rate and foreign cash flow uncertainties is positive (negative, respectively), foreign investment advances as exchange rate uncertainty decreases (increases) as well as the correlation increases. These results reveal that the impact of debt shifting and exchange rate uncertainty on investment and capital structure policies cannot be ignored, supporting existing empirical findings.

Keywords: Multinational companies; Foreign direct investment; Debt shifting; Real options

JEL classification: F31; F34; G11; G33

1 Introduction

With the advance of globalization process, it becomes more profitable for multinational companies (MNCs) to access foreign markets through foreign direct investment (FDI). FDI has been the subject of considerable research in the past decades. Besides the internal firm-specific factors (e.g., immobility of intangible human resources and technologies), there are several external factors (e.g., tax rate differential, exchange rate, trade barrier, investment financial subsidies) that determine the location and magnitude of FDI. In particular, the tax rate differential and the exchange rate uncertainty have received increasing attention from both researchers and policymakers.

Table 1 shows corporate tax rates in different countries in 2008. Hines (1999) and

Table 1: Corporate tax rates in some countries.

Country	Japan	US	Hong Kong	Ireland	Global average
Corporate tax rate	40.7%	40%	16.5%	12.5%	25.9%

Source: KPMG's corporate tax rate survey 2008.

Grubert and Mutti (2000) document that the international differential in corporate taxation significantly influences the location of FDI.[1] To increase after-tax profits, MNCs have incentives to shift profits from low- to high-tax countries. Since debt payments are deductible (tax benefits of debt), it is well known that international tax rate differential creates opportunities for debt shifting from high- to low-tax countries.[2] On the basis of a large sample of European firms over the 1994–2003 period, Huizinga *et al.* (2008) find that a foreign subsidiary's capital structure (leverage) at its establishment is positively related to domestic corporate tax rate as well as the difference between the domestic and foreign tax rates. Moreover, they report that ignoring international debt shifting arising from tax rate differential would understate the impact of tax rate on debt policy by about 25%.

Besides the tax rate differential, exchange rate uncertainty seems another important factor that determines the location and magnitude of FDI. There are generous empirical evidences that support this legend, although the literature has not yet come to a consensus on the exact relationship between FDI and exchange rate uncertainty. The most cited articles on this subject are Cushman (1985, 1988) and Goldberg and Kolstad (1995).[3]

[1]Dharmapala and Hines (2009) find that small and better-governed countries are more likely to become tax havens.

[2]See Hines (1999), Mills and Newberry (2004), and Mintz and Smart (2004) for details.

[3]Cushman (1985) investigates FDI from the US to Canada, France, Germany, Japan and the UK over the period 1963–1978, while Cushman (1988) analyzes the opposite FDI over the period 1963–1986.

While they find positive impact of exchange rate volatility on FDI, the negative impact of exchange rate volatility is reported by Urata and Kawai (2000) and Servén (2003).[4]

There are two possible reasons for the ambiguous results. One is the different measurement of exchange rate volatility, as Amuedo-Dorantes and Pozo (2001) point out. While some papers use the standard deviation of the observed quarterly exchange rate data (as in Goldberg and Kolstad, 1995), some papers use a GARCH construction for the data (as in Servén, 2003). Another possibility is the aggregation problem. Most previous studies use aggregated national-level data. However, exchange rate uncertainty on FDI may be different among industries, as Kiyota and Urata (2004) point out. In particular, Goldberg and Kolstad (1995) suggest the importance of accounting for the correlation between exchange rate and foreign demand uncertainties, and show that an increase in the correlation leads to an increase in FDI. The purpose of this paper is to develop a theoretic model to examine the impact of debt shifting and exchange rate uncertainty on investment and capital structure decisions of foreign subsidiary. It is hoped that our model provides a starting block to resolve the ambiguity of the effect of exchange rate uncertainty on FDI.

The most related theoretical models to our paper seem Panteghini (2009) and Yu *et al.* (2007). While Panteghini (2009) provides a model that analyzes how debt shifting affects the capital structure decision of foreign subsidiary without considering investment, Yu *et al.* (2007) examine the investment decision without considering the capital structure decision. Moreover, for the sake of model simplicity, they omit the exchange rate uncertainty, which is an important factor for FDI as mentioned above. Panteghini (2009) demonstrates that the foreign subsidiary's coupon level is positively affected by both the foreign tax rate and the tax rate differential. However, since the investment decision is not considered, Panteghini (2009) cannot examine the impact of debt shifting on foreign subsidiary's leverage upon investment. On the other hand, Yu *et al.* (2007) compare the effects of two policies on FDI timing: entry cost subsidy and tax rate reduction, and find that a host goverment should adopt entry cost subsidy rather than tax rate reduction in order to accelerate FDI. As we shall see in Section 4.4, this result also holds even in our framework where debt shifting and exchange rate uncertainty are considered.

The main contribution of this paper is to extend Panteghini (2009) and Yu *et al.* (2007) by introducing the exchange rate uncertainty and examining the impact of debt shifting on both investment and capital structure decisions of foreign subsidiary.[5] By employing

---

Goldberg and Kolstad (1995) use the data on quarterly bilateral flows between the US, Canada, Japan, and the UK over the period 1978–1991.

[4]Urata and Kawai (2000) study Japanese firms' decisions on the location of their FDI. By analyzing a firm-level panel data covering the period 1980–94 for 117 countries from four manufacturing industries in Japan, they find that high exchange rate volatility discourages FDI. Servén (2003) builds a GARCH-based measure of real exchange rate volatility and finds that it has a strong negative effect on FDI.

[5]If we ignore the exchange rate uncertainty and tax rate differential, our model is reduced to Sundaresan

the change-of-measure technique, we are able to provide an analytically tractable framework that incorporates exchange rate uncertainty and thus better interpret the empirical findings on international debt shifting and FDI.

Suppose that a parent firm, which is located in domestic country with high tax rate, considers establishment of a subsidiary in foreign country with low tax rate. By investigating both the investment and capital structure decisions of the foreign subsidiary, we find that debt shifting induces earlier investment, earlier default, higher leverage, and larger *ex ante* firm value of foreign subsidiary. When debt shifting is not so costly, the optimal leverage of foreign subsidiary at its establishment increases as the tax rate differential increases. Inefficiency in capital structure policy due to ignoring the debt shifting cannot be ignored. This is consistent with the empirical findings reported in Huizinga *et al.* (2008). When the correlation between exchange rate and foreign cash flow uncertainties is positive, FDI advances as the exchange rate uncertainty as well as the correlation increases, which echoes the result found in Goldberg and Kolstad (1995). Moreover, our model predicts the opposite case as well. Namely, when the correlation is negative, FDI advances as the exchange rate uncertainty decreases.

The rest of this paper is organized as follows. Section 2 describes the setup of our model. In Section 3, we examine the investment and capital structure decisions of foreign subsidiary by employing the change-of-measure technique. Section 4 analyzes the characteristics of the model solutions and provides several model predictions. More precisely, we first examine the impact of debt shifting on investment and capital structure policies and inefficiency due to ignoring the debt shifting. Then, the impact of uncertainties and their correlation on FDI and inefficiency due to ignoring the exchange rate uncertainty are investigated. Finally, Section 5 concludes this paper. Some detailed proofs are found in Appendix A.

2 The Model setup

Suppose that a parent firm, which is located in country d (domestic country), considers establishment of a subsidiary in country f (foreign country). The parent firm and the foreign subsidiary together form a multinational company (MNC), which is assumed to be risk neutral. The MNC can finance the investment cost by issuing both equity and a perpetual debt with continuous coupon payments.[6] It is assumed that the investment cost is constant and *irreversible*. The investment cost is denoted by I and the instantaneous coupon payment by c in the foreign currency.

and Wang (2007), who examine investment and capital structure decisions of a domestic firm.

[6]As in Leland and Toft (1996), we can consider debt with finite average maturity by assuming that debt is continuously retired at par at a constant rate.

Soon after establishing the foreign subsidiary, the MNC spontaneously receives an EBIT (earnings before interest and taxes) $X(t)$ in foreign currency and pays coupon c to debtholders. We assume that the EBIT $X(t)$ in foreign market follows a geometric Brownian motion (GBM) under the domestic risk-neutral measure $\mathbb{P}$:[7]

$$\frac{\mathrm{d}X(t)}{X(t)} = \mu_x \mathrm{d}t + \sigma_x \mathrm{d}w_1(t), \tag{1}$$

where μ_x is the instantaneous expected growth rate of $X(t)$,[8] σ_x is the associated volatility, and $w_1(t)$ is a standard Brownian motion. On the other hand, due to the no-arbitrage condition, the exchange rate $Q(t)$ (the domestic currency price of one unit of foreign currency) also follows a GBM

$$\frac{\mathrm{d}Q(t)}{Q(t)} = (r_d - r_f)\mathrm{d}t + \sigma_q \mathrm{d}w_2(t), \tag{2}$$

where r_d and r_f are the risk-free interest rates in countries d and f, respectively, σ_q is the associated volatility, and $w_2(t)$ is another standard Brownian motion correlated to $w_1(t)$ with $\mathrm{d}w_1(t)\mathrm{d}w_2(t) = \rho\mathrm{d}t$. Hence, the EBIT $X(t)$ is correlated to the exchange rate $Q(t)$ with correlation ρ.[9] The initial value $X(0)$ is assumed to be sufficiently low; i.e., the EBIT in foreign market has not yet been favorable enough to cover the investment cost.

For the sake of simplicity, we follow Panteghini (2009) and assume that the parent firm produces a deterministic profit in domestic country with no default risk. A plausible explanation for the risk asymmetry in domestic and foreign countries is given by the fact that operating in the domestic country may be less risky than operating abroad. The parent firm is more likely to perceive the characteristics of its own country, and thus can easily predict and offset changes on their domestic business environment.

Let τ_d and τ_f denote the corporate tax rates in countries d and f, respectively, where we assume $\tau_d > \tau_f$. Without debt shifting, the MNC's instantaneous profit from foreign subsidiary (in domestic currency) at time t is given by

$$(1 - \tau_f)(X(t) - c)Q(t). \tag{3}$$

Note that the tax rate τ_f applies not only to the EBIT $X(t)$ as the effective tax rate but also to the coupon c as the effective deductible-tax rate.

On the other hand, if debt shifting is possible, the MNC can enjoy more tax benefits through debt shifting since coupon payments are tax deductible. More specifically, the

[7]Mathematically speaking, we begin with the probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The canonical filtration generated by the underlying stochastic structure is denoted by $\{\mathcal{F}_t\}$, where $\mathcal{F}_t$ defines the information available at time t.

[8]We set $\mu_x = r_f - \delta$, where $\delta = r_f - \mu_x > 0$ is the convenience yield.

[9]This assumption reflects the well-known empirical evidence (see, e.g., Phylaktis and Ravazzolo, 2005; Hau and Rey, 2006) that the equity market dynamics is correlated to the associated exchange rate dynamics.

MNC has an incentive not to issue debt locally in foreign country with low tax rate, but to issue debt in domestic country with high tax rate and shift the debt to foreign subsidiary. The MNC can then make profits by utilizing the tax rate differential.

However, as Mills and Newberry (2004) report, although debt shifting may bring tax benefits, it is also associated with transaction costs. Hence, following Panteghini (2009), we assume that the MNC shifts a percentage k of the foreign subsidiary's coupon c with a quadratic cost function $\nu(k) = nk^2/2,\ k \in [0,1]$, where $n \geq 0$ measures how costly the debt shifting is. Note that $\nu(k)$ is increasing and convex in k with $\nu(0) = 0$.

It follows that, under the debt shifting, the MNC's instantaneous profit from foreign subsidiary (in domestic currency) at time t is given by

$$\begin{aligned} & (1-\tau_f)\left(X(t) - c + kc\right)Q(t) - (1-\tau_d)(k+\nu(k))cQ(t) \\ = \; & (1-\tau_f)X(t)Q(t) - (1-\tilde{\tau})cQ(t), \end{aligned} \tag{4}$$

where

$$\tilde{\tau} \equiv \tau_f + \phi(k), \quad \phi(k) \equiv (\tau_d - \tau_f)k - (1-\tau_d)\nu(k). \tag{5}$$

Note from (4) that, while the effective tax rate for the EBIT in foreign market is τ_f, the effective deductible-tax rate for coupon is $\tilde{\tau}$; cf. (3).

It is readily shown from (5) that the optimal percentage k^* of coupon shifting is obtained as[10]

$$k^* = \operatorname*{argmax}_k \phi(k) = \begin{cases} 1, & 0 \leq n \leq \bar{n}, \\ \dfrac{\bar{n}}{n}, & n \geq \bar{n}, \end{cases} \tag{6}$$

where

$$\bar{n} = \frac{\tau_d - \tau_f}{1-\tau_d}. \tag{7}$$

The optimal percentage k^* increases with respect to the tax rate differential $\tau_d - \tau_f$. Substituting Eq. (6) into Eq. (5), we obtain

$$\tilde{\tau} = \begin{cases} \tau_d - \dfrac{n}{2}(1-\tau_d) & \in \left[\dfrac{1}{2}(\tau_d + \tau_f), \tau_d\right], \quad 0 \leq n \leq \bar{n}, \\ \tau_f + \dfrac{\bar{n}}{2n}(\tau_d - \tau_f) & \in \left[\tau_f, \dfrac{1}{2}(\tau_d + \tau_f)\right], \quad n \geq \bar{n}. \end{cases} \tag{8}$$

Note that if $\tau_d > \tau_f$ then $\tilde{\tau} > \tau_f$. Hence, the tax rate differential makes the effective deductible-tax rate $\tilde{\tau}$ larger than the foreign tax rate τ_f. If $\tau_d = \tau_f$ then $\tilde{\tau} = \tau_f$, and the instantaneous profit (4) with debt shifting is reduced to the profit (3) without debt shifting. In the following, we consider the case $\tilde{\tau} = \tau_f$ (i.e., $\tau_d = \tau_f$) as the benchmark case (no debt shifting) of our model.

Although issuing debt can accompany with tax benefits, it is also subject to default risk. As in Leland (1994), we consider an equity-based definition of default whereby

[10] Panteghini (2009) neglects the case that $k^* = 1$ if $0 \leq n \leq \bar{n}$.

equityholders inject funds into the foreign subsidiary as long as the equity value of the subsidiary is positive. In other words, equityholders of the subsidiary default the debt obligation when the equity value becomes equal to zero for the first time. Once they decide to default, bankruptcy immediately occurs. That is, there is no possibility of debt renegotiation (see, e.g., Mella-Barral and Perraudin, 1997). Also, we assume that the debt of foreign subsidiary is not guaranteed legally by the parent firm, because they are separate entities.[11] Upon default, a fraction of $\alpha \in (0,1)$ of the subsidiary's firm value is lost as default cost, while the remaining $(1-\alpha)$ part belongs to debtholders. After default of the foreign subsidiary, the parent firm becomes a pure domestic firm.

Let T^i (superscript "i" stands for investment) denote the time that the foreign subsidiary is established, and let T^b (superscript "b" stands for bankruptcy) denote the time of default. Note that the MNC's instantaneous profit (4) from the foreign subsidiary is first-order homogeneous with respect to $(X(t)Q(t), Q(t))$. Hence, according to McDonald and Siegel (1986), we can define the stopping times T^i and T^b as

$$\begin{aligned} T^i &= \inf\left\{t \geq 0,\ X(t)Q(t)/Q(t) \geq x^i\right\} = \inf\{t \geq 0,\ X(t) \geq x^i\}, \\ T^b &= \inf\left\{t \geq T^i,\ X(t)Q(t)/Q(t) \leq x^b\right\} = \inf\{t \geq T^i,\ X(t) \leq x^b\}, \end{aligned} \tag{9}$$

for some thresholds x^i and x^b determined optimally by equityholders of the foreign subsidiary.

3 Investment and capital structure of foreign subsidiary

In this section, we investigate the investment and capital structure decisions of foreign subsidiary. The investment decision is characterized by an endogenously determined threshold. When the foreign EBIT process $X(t)$ reaches the investment threshold x^i for the first time, the MNC establishes the foreign subsidiary. On the other hand, the capital structure decision involves the choice of debt level and default threshold of the foreign subsidiary. The coupon level $c(x^i)$ of debt, which is characterized by the trade-off between tax benefits and default costs of debt financing, is determined simultaneously with the investment decision. In contrast, the default threshold $x^b(c)$, which depends on the coupon level, is determined after foreign subsidiary is established. Note that the three endogenous variables (x^i, $c(x^i)$ and $x^b(c)$) form a nested structure in our model, whence enabling us to examine the interaction between investment and capital structure decisions.

We derive the equityholders' optimal decisions using backward induction. Section 3.1 examines the default threshold of foreign subsidiary based on the equity value. Section 3.2 analyzes the coupon level of debt and the investment threshold of foreign subsidiary.

[11] Kolasinski (2009) finds that a firm tends to have nonguaranteed subsidiary debt in its capital structure when the divisions vary more in risk.

3.1 Default decision

The default threshold of foreign subsidiary is determined by equityholders so as to maximize the equity value after investment. Let $\mathbb{E}^{(x,q)}$ denote the expectation operator when $(X(t), Q(t)) = (x, q)$ under the probability measure $\mathbb{P}$. For $T^i \le t \le T^b$, the equity value of foreign subsidiary (in domestic currency) is evaluated from (4) as

$$\begin{aligned} E(x,q) =& \mathbb{E}^{(x,q)}\left[\int_t^{T^b} \mathrm{e}^{-r_d(s-t)}\big[(1-\tau_f)X(s)Q(s) - (1-\tilde{\tau})cQ(s)\big]\mathrm{d}s\right] \\ =& \mathbb{E}^{(x,q)}\left[\int_t^{T^b} \mathrm{e}^{-r_d(s-t)}Q(s)\big[(1-\tau_f)X(s) - (1-\tilde{\tau})c\big]\mathrm{d}s\right]. \end{aligned} \tag{10}$$

Now, define a new probability measure $\tilde{\mathbb{P}}$ by

$$\left.\frac{\mathrm{d}\tilde{\mathbb{P}}}{\mathrm{d}\mathbb{P}}\right|_{\mathcal{F}_t} = Y(t) = \mathrm{e}^{\sigma_q w_2(t) - \frac{1}{2}\sigma_q^2 t}. \tag{11}$$

Since $Q(s) = Q(0)\mathrm{e}^{(r_d - r_f - \frac{1}{2}\sigma_q^2)s + \sigma_q w_2(s)}$ from Eq. (2), we can rewrite the equity value (10) as

$$\begin{aligned} E(x,q) &= \frac{q}{Y(t)}\mathbb{E}^{(x,q)}\left[\int_t^{T^b} \mathrm{e}^{-r_f(s-t)}Y(s)\big[(1-\tau_f)X(s) - (1-\tilde{\tau})c\big]\mathrm{d}s\right] \\ &= q\tilde{\mathbb{E}}^{x}\left[\int_t^{T^b} \mathrm{e}^{-r_f(s-t)}\big[(1-\tau_f)X(s) - (1-\tilde{\tau})c\big]\mathrm{d}s\right], \end{aligned} \tag{12}$$

using the change-of-measure technique, where $\tilde{\mathbb{E}}^x$ denotes the expectation operator under the new probability measure $\tilde{\mathbb{P}}$ given that $X(t) = x$.

Remark 3.1. Mathematically speaking, the exchange rate $Q(t)$ is nothing but the vehicle of the change of measure, and $X(t)$ is the only state variable under the new measure $\tilde{\mathbb{P}}$ when evaluating the equity value. By looking at (12), because $X(t)$ is discounted by the risk-free interest rate r_f in foreign country, the pricing measure $\tilde{\mathbb{P}}$ is considered to be the *foreign* risk-neutral measure.

According to the Girsanov theorem, $\tilde{w}_1(t)$ and $\tilde{w}_2(t)$ defined by

$$\tilde{w}_1(t) = w_1(t) - \rho\sigma_q t, \qquad \tilde{w}_2(t) = w_2(t) - \sigma_q t$$

are standard Brownian motions under the new probability measure $\tilde{\mathbb{P}}$ with correlation $\mathrm{d}\tilde{w}_1(t)\mathrm{d}\tilde{w}_2(t) = \rho\mathrm{d}t$. Also, the process $X(t)$ under $\tilde{\mathbb{P}}$ is given by

$$\frac{\mathrm{d}X(t)}{X(t)} = (\mu_x + \rho\sigma_q\sigma_x)\mathrm{d}t + \sigma_x\mathrm{d}\tilde{w}_2(t), \tag{13}$$

where $\mu_x + \rho\sigma_q\sigma_x$ is the instantaneous expected growth rate of $X(t)$ under $\tilde{\mathbb{P}}$.[12] The exchange rate uncertainty σ_q and the correlation ρ together enter the drift term of the process $X(t)$ as an adjustment under the new probability measure $\tilde{\mathbb{P}}$.

Remark 3.2. We start our discussion by specifying the model for $(X(t), Q(t))$ under $\mathbb{P}$ and then apply the change-of-measure technique to derive the dynamics of $X(t)$ under $\tilde{\mathbb{P}}$ in Eq. (13). Consequently, we can explicitly examine the exchange rate volatility σ_q as well as the correlation ρ between $X(t)$ and $Q(t)$ in the investment and capital structure decisions in what follows. If we started by assuming the dynamics of $X(t)$ under $\tilde{\mathbb{P}}$ directly as $\mathrm{d}X(t)/X(t) = \tilde{\mu}_x\mathrm{d}t + \sigma_x\mathrm{d}\tilde{w}_2(t)$, there is no means to identify the adjustment term $\rho\sigma_q\sigma_x$ in the drift $\tilde{\mu}_x$.

From Ito's formula, the equity value $E(x) \equiv E(x, q)$ must satisfy the following second-order ordinary differential equation (ODE):

$$r_f E(x) = (\mu_x + \rho\sigma_q\sigma_x)E'(x) + \frac{1}{2}\sigma_x^2 x^2 E''(x) + (1-\tau_f)x - (1-\tilde{\tau})c. \tag{14}$$

The general solution to the ODE (14) is given by

$$E(x) = \Pi x + \frac{1-\tilde{\tau}}{r_f}c + A_1 x^{\beta} + A_2 x^{\gamma}, \qquad x \geq x^b, \tag{15}$$

where

$$\Pi \equiv \frac{1-\tau_f}{r_f - \mu_x - \rho\sigma_q\sigma_x}, \tag{16}$$

and where β and γ are the positive and negative roots, respectively, of the quadratic equation

$$\mathbb{Q}(y) \equiv \frac{1}{2}\sigma_x^2 y(y-1) + (\mu_x + \rho\sigma_q\sigma_x)y - r_f = 0. \tag{17}$$

That is, we have

$$\begin{aligned} \beta &= \frac{1}{\sigma_x^2}\left[-\left(\mu_x + \rho\sigma_q\sigma_x - \frac{1}{2}\sigma_x^2\right) + \sqrt{\left(\mu_x + \rho\sigma_q\sigma_x - \frac{1}{2}\sigma_x^2\right)^2 + 2r_f\sigma_x^2}\right] > 1, \\ \gamma &= -\frac{1}{\sigma_x^2}\left[\left(\mu_x + \rho\sigma_q\sigma_x - \frac{1}{2}\sigma_x^2\right) + \sqrt{\left(\mu_x + \rho\sigma_q\sigma_x - \frac{1}{2}\sigma_x^2\right)^2 + 2r_f\sigma_x^2}\right] < 0. \end{aligned} \tag{18}$$

The coefficients A_1 and A_2 as well as the threshold x^b in (15) are some constants to be determined.

[12] Again, we set $\mu_x + \rho\sigma_q\sigma_x = r_f - \tilde{\delta}$, where $\tilde{\delta} = r_f - \mu_x - \rho\sigma_q\sigma_x > 0$ is the *risk-adjusted* convenience yield under $\tilde{\mathbb{P}}$.

The three unknowns A_1, A_2 and x^b invloved in (15) can be determined by the following three boundary conditions:

$$\lim_{x\to\infty}\frac{E(x)}{x}<\infty,\qquad E(x^b)=0,\qquad E'(x^b)=0. \tag{19}$$

The first condition is known as the no-bubbles condition, which implies that the coefficient A_1 corresponding to the positive root β in Eq. (18) must equal zero. The second condition is the value-matching condition, because the equity value becomes zero upon default (i.e., the equity value evaluated at default threshold x^b is zero). The third condition is called the smooth-pasting condition, which ensures that the default threshold x^b is chosen optimally so as to maximize the equity value.

Substituting $E(x)$ in Eq. (15) with $A_1=0$ into the boundary conditions (19), we get simultaneous equations with respect to A_2 and x^b, which can be solved with ease. The default threshold x^b is obtained as

$$x^b(c)=Ac,\qquad A\equiv\frac{\gamma}{\gamma-1}\frac{1-\tilde{\tau}}{r_f\Pi}, \tag{20}$$

Note that the default threshold is a linear function of coupon c. The equity value of foreign subsidiary after investment (in domestic currency) is given by

$$E(x,q)=\Pi q\left[x-x^b\left(\frac{x}{x^b}\right)^{\gamma}\right]-\frac{1-\tilde{\tau}}{r_f}cq\left[1-\left(\frac{x}{x^b}\right)^{\gamma}\right], \tag{21}$$

which consists of the two components: (i) the present value of the EBIT without default, and (ii) the present value of the coupon payments paid to debtholders without default.

Similarly, the debt value after investment can be derived as

$$D(x,q)=\frac{cq}{r_f}\left[1-\left(\frac{x}{x^b}\right)^{\gamma}\right]+(1-\alpha)\Pi x^b q\left(\frac{x}{x^b}\right)^{\gamma}, \tag{22}$$

which also consists of the two components: (i) the present value of coupon payments without default, and (ii) the remaining firm value upon default.

Finally, the firm value $V(x,q)$ is the sum of the equity and debt values and given by

$$V(x,q)=\Pi q\left[x-\alpha x^b\left(\frac{x}{x^b}\right)^{\gamma}\right]+\frac{\tilde{\tau}}{r_f}cq\left[1-\left(\frac{x}{x^b}\right)^{\gamma}\right]. \tag{23}$$

3.2 Coupon level and investment decisions

Before turning to the analysis of coupon level and investment decisions, it is important to distinguish the *ex ante* equity value and the *ex post* equity value. While the *ex post* equity value is given by the present value of the cash flows accruing to equityholders after debt has been issued (see Eq. (10)), the *ex ante* equity value is given by the sum of the *ex post* equity value and the debt value (see Eq. (23)) at the time it is issued. As a result, although equityholders choose the default threshold to maximize the *ex post* equity value, they choose the coupon level to maximize the firm value (the *ex ante* equity value) upon investment, internalizing both the tax benefits and default costs of debt financing. That is, given the investment threshold x^i, the coupon level c is determined from Eq. (23) as

$$c^*(x^i) = \underset{c}{\operatorname{argmax}}\, V(x^i, Q(T^i)), \tag{24}$$

where $x^i = X(T^i)$. Recall that the default threshold x^b involved in the firm value $V(x, q)$ in (23) is a linear function of coupon c.

Substituting Eq. (20) into Eq. (23) with $(x, q) = (x^i, Q(T^i))$ and rearranging the terms, the firm value upon investment can be expressed as

$$V(x^i, Q(T^i)) = \left[\Pi + \frac{\tilde{\tau}}{r_f}\frac{f(m)}{A}\right] x^i Q(T^i), \tag{25}$$

where

$$f(m) = m\left(1 - \frac{B}{1-\gamma} m^{-\gamma}\right), \qquad m \equiv \frac{x^b}{x^i} = \frac{Ac}{x^i}, \tag{26}$$

and

$$B \equiv 1 - \gamma\left(1 - \alpha + \frac{\alpha}{\tilde{\tau}}\right) > 1. \tag{27}$$

It follows that

$$c^*(x^i) = \frac{x^i m^*}{A}, \tag{28}$$

where

$$m^* = \underset{m>0}{\operatorname{argmax}}\, f(m). \tag{29}$$

The first-order condition (FOC) for (29) is given by

$$f'(m^*) = 1 - B(m^*)^{-\gamma} = 0,$$

which is solved as

$$m^* = B^{\frac{1}{\gamma}} \in (0, 1), \tag{30}$$

where B is defined in (27). The fact $m^* \in (0, 1)$ follows, since m^* is the ratio of default threshold to investment threshold. The second-order condition (SOC) for (29) is given by

$$f''(m^*) = \gamma B(m^*)^{-\gamma-1} < 0,$$

which ensures that m^* is in fact the unique solution of the maximization problem (29). Using Eq. (26), the firm value upon investment in Eq. (25) can be rewritten as

$$V(x^i, Q(T^i)) = \psi x^i Q(T^i), \tag{31}$$

where

$$\psi \equiv \left(1 + \frac{\tilde{\tau}}{1-\tilde{\tau}} m^*\right) \Pi. \tag{32}$$

Having derived the firm value, we next analyze the optimal investment threshold. Since the investment cost financed by equity is $IQ(T^i) - D(x^i, Q(T^i))$, equityholders choose the optimal investment threshold of foreign subsidiary as

$$x^{i*} = \underset{x^i}{\operatorname{argmax}}\, \mathbb{E}^{(x,q)} \left[\mathrm{e}^{-r_d T^i} \left[E(x^i, Q(T^i)) - (IQ(T^i) - D(x^i, Q(T^i))) \right] \right]. \tag{33}$$

The objective function in the maximization problem (33) is exactly the same as the *ex ante* firm value of foreign subsidiary, V^o (superscript "o" stands for option value), which is defined by

$$\begin{aligned} V^o(x, q; x^i) &\equiv \mathbb{E}^{(x,q)} \left[\mathrm{e}^{-r_d T^i} \left[V(x^i, Q(T^i)) - IQ(T^i) \right] \right] \\ &= \left(\psi x^i - I \right) \mathbb{E}^{(x,q)} \left[e^{-r_d T^i} Q(T^i) \right]. \end{aligned} \tag{34}$$

By changing the probability measure from $\mathbb{P}$ to $\tilde{\mathbb{P}}$, we have

$$\mathbb{E}^{(x,q)} \left[e^{-r_d T^i} Q(T^i) \right] = q \tilde{\mathbb{E}}^x \left[e^{-r_f T^i} \right].$$

It is well known (see, e.g., Dixit and Pindyck, 1994) that the Laplace transform of the first hitting time T^i, when the process follows (13), is given by

$$\tilde{\mathbb{E}}^x \left[e^{-r_f T^i} \right] = \left(\frac{x}{x^i} \right)^{\beta}, \tag{35}$$

where β is defined in Eq. (18). It follows that

$$V^o(x, q; x^i) = \left(\psi x^i - I \right) q \left(\frac{x}{x^i} \right)^{\beta}. \tag{36}$$

Therefore, we obtain the following results.

Proposition 3.1. *For the foreign subsidiary, the optimal investment threshold is given by*

$$x^{i*} = \frac{\beta}{\beta - 1} \frac{I}{\psi}, \tag{37}$$

where ψ is defined by (32)*. The default threshold is obtained as*

$$x^{b*} = \frac{\beta}{\beta - 1} \frac{m^*}{\psi} I,$$

where m^* is defined by (30)*. The coupon level is given by*

$$c^* = \frac{\beta}{\beta - 1} \frac{\gamma - 1}{\gamma} \frac{r_f m^*}{1 - \tilde{\tau} + \tilde{\tau} m^*} I.$$

The ex ante firm value is obtained as

$$V^{o*}(x, q; x^{i*}) = \frac{qI}{\beta - 1}\left(\frac{x}{x^{i*}}\right)^{\beta}, \qquad x < x^{i*}. \tag{38}$$

Finally, the optimal leverage upon investment is given by

$$L^*(x^{i*}, Q(T^{i*})) \equiv \frac{D(x^{i*}, Q(T^{i*}))}{V(x^{i*}, Q(T^{i*}))} = \frac{\gamma - 1}{\gamma}\frac{(1-\xi)m^*}{1 - \tilde{\tau} + \tilde{\tau}m^*}, \tag{39}$$

where

$$\xi \equiv \left[1 - (1-\alpha)(1-\tilde{\tau})\frac{\gamma}{\gamma - 1}\right] B^{-\gamma} \in (0, 1). \tag{40}$$

Note that the optimal investment and default thresholds, the coupon level, and the *ex ante* firm value of foreign subsidiary are all proportional to the investment cost I, thanks to the GBM assumption for the EBIT and exchange rate processes. The optimal leverage upon investment is constant and independent of investment cost I.

By setting $\tilde{\tau} = \tau_f$ in Proposition 3.1, we have the results for our benchmark case (no debt shifting). Furthermore, assuming $\sigma_q = 0$ (no exchange rate uncertainty) in addition, the results in Proposition 3.1 are reduced to those derived in Sundaresan and Wang (2007).

4 Model implications

In this section, we analyze the model characteristics to provide several model predictions. Section 4.1 compares the results with debt shifting to those in the benchmark case (without debt shifting). Section 4.2 investigates the inefficiency in investment and capital structure policies due to ignoring debt shifting. While Section 4.3 examines the impact of tax rate differential on the leverage of foreign subsidiary upon investment (corresponding to Huizinga *et al.*, 2008), Section 4.4 considers two policies for attracting FDI (corresponding to Yu *et al.*, 2007). Then, we investigate the impact of exchange rate and foreign EBIT uncertainties as well as their correlation on FDI in Section 4.5. Finally, in Section 4.6, we consider the inefficiency in investment and capital structure policies due to ignoring exchange rate uncertainty.

4.1 Impact of effective deductible-tax rate and debt shifting cost

In this subsection, we consider the impact of effective deductible-tax rate $\tilde{\tau}$ as well as debt shifting cost n on the model solutions obtained in Proposition 3.1 to infer the effect of debt shifting. Recall that $\tilde{\tau} > \tau_f$ when $\tau_d > \tau_f$, where τ_f is the effective tax rate for EBIT.

Proposition 4.1. *With respect to the effective deductible-tax rate $\tilde{\tau}$, we have:*

(a) $\mathrm{d}x^{i*}/\mathrm{d}\tilde{\tau} < 0$, *i.e., the optimal investment threshold of foreign subsidiary is decreasing;*

(b) $\mathrm{d}x^{b*}/\mathrm{d}\tilde{\tau} > 0$ *when* $G_1 < 0$*, where*

$$G_1 \equiv \frac{1}{(1-\tilde{\tau})^2} - \frac{\alpha}{B}\frac{1}{\tilde{\tau}^2 m^*}. \tag{41}$$

That is, when $G_1 < 0$*, the optimal default threshold is increasing;*

(c) $\mathrm{d}c^*/\mathrm{d}\tilde{\tau} > 0$*, i.e. the coupon level is increasing;*

(d) $\mathrm{d}V^{o*}/\mathrm{d}\tilde{\tau} > 0$*, i.e. the ex ante firm value of foreign subsidiary is increasing;*

(e) $\mathrm{d}L^*/\mathrm{d}\tilde{\tau} > 0$ *when* $G_2 > 0$*, where*

$$G_2 \equiv \left(1 - m^* + \frac{\alpha}{B}\frac{1-\tilde{\tau}}{\tilde{\tau}^2}\right)(1-\xi) - \frac{\gamma}{B}\left(\frac{1-\alpha}{\gamma-1} - \frac{\xi\alpha}{\tilde{\tau}^2}\right)(1-\tilde{\tau}+\tilde{\tau}m^*). \tag{42}$$

That is, when $G_2 > 0$*, the optimal leverage of foreign subsidiary is increasing.*

Proof. See Appendix A.1. □

The results (b) and (e) in Proposition 4.1 look restrictive at first glance, because of the conditions $G_1 < 0$ and $G_2 > 0$. However, for a wide range of reasonable parameter values, we find that $G_1 < 0$ and $G_2 > 0$ are always satisfied through extensive numerical experiments. Remember $\tilde{\tau} = \tau_f$ in our benchmark case (no debt shifting). Hence, we conclude that debt shifting induces (a) earlier investment, (b) earlier default, (c) higher coupon level, (d) larger *ex ante* firm value, and (e) higher leverage of foreign subsidiary.

Panteghini (2009) also concludes that debt shifting stimulates debt financing. However, default delays with debt financing in Panteghini (2009), because the coupon level is fixed when the comparative analysis is conducted. In fact, default occurs earlier because of the higher coupon level with debt shifting, *ceteris paribus*. Moreover, by taking into consideration the investment decision, we complement Panteghini (2009) by demonstrating that debt shifting increases the *ex ante* firm value of foreign subsidiary, and consequently stimulates FDI.

Next, we examine the impact of debt shifting cost n. Recall that $n \geq 0$ measures how costly the debt shifting is. Since $\mathrm{d}\tilde{\tau}/\mathrm{d}n < 0$, we immediately obtain the following result from Proposition 4.1.

Corollary 4.1. *With respect to the debt shifting cost* n*, we conclude that:*

(a) *the optimal investment threshold of foreign subsidiary is increasing;*

(b) *the optimal default threshold is decreasing;*

(c) *the coupon level is decreasing;*

(d) *the ex ante firm value of foreign subsidiary is decreasing;*

(e) *the optimal leverage of foreign subsidiary is decreasing.*

The results in Corollary 4.3 are intuitively sound, because lower debt shifting cost stimulates debt financing and increases the *ex ante* firm value, and thus accelerates FDI.

4.2 Inefficiency due to ignoring debt shifting

In this subsection, we consider the inefficiency that results from ignoring debt shifting. The inefficiency in the *ex ante* firm value (Eq. (38)) and leverage (Eq. (39)) are defined, respectively, as

$$\Delta\overline{V}^{o*}(x,q;x^{i*}) \equiv 1 - \frac{V^{o*}(x,q;\overline{x}^{i*})}{V^{o*}(x,q;x^{i*})}, \qquad \Delta\overline{L}^{*}(x^{i*},Q(T^{i*})) \equiv 1 - \frac{L^{*}(\overline{x}^{i*},Q(\overline{T}^{i*}))}{L^{*}(x^{i*},Q(T^{i*}))}, \quad (43)$$

where $\overline{x}^{i*}$ and $Q(\overline{T}^{i*})$ correspond to the case without debt shifting (i.e., $\tilde{\tau} = \tau_f$ or, equivalently, $\tau_d = \tau_f$). Since $x^{i*} \leq \overline{x}^{i*}$ and $L^{*}(x^{i*},Q(T^{i*})) \geq L^{*}(\overline{x}^{i*},Q(\overline{T}^{i*}))$ from the results (a) and (e) in Proposition 4.1, the underinvestment and underleverage problems occur when debt shifting is ignored. We use these $\Delta\overline{V}^{o*}$ and $\Delta\overline{L}^{*}$ to measure the inefficiencies in investment policy and capital structure policy, respectively.

Figure 1 depicts the inefficiencies in investment and capital structure policies with respect to the tax rate differential $\Delta\tau \equiv \tau_d - \tau_f$. The base parameter values are listed in Table 2, and the initial values are set as $X(0) = 1$ and $Q(0) = 12$. We draw the figures for $0 \leq \Delta\tau \leq 0.25$, because the maximum differential between the listed countries in Table 1 is about 0.25. Under these parameter values, we have $0 \leq \bar{n} \leq 0.42$. While the case $n = 0$ (complete debt shifting) means that debt shifting costs nothing, the case $n = 0.5$ (partial debt shifting) corresponds to a costly debt shifting. From Eq. (6), when $n = 0.5$, the optimal percentage of debt shifting is $0 \leq k^{*} \leq 84\%$ for $0 \leq \Delta\tau \leq 0.25$.

Table 2: Base parameter values

Parameter	Value
Growth rate of foreign EBIT	$\mu_x = 0.01$
Volatility of foreign EBIT	$\sigma_x = 0.35$
Volatility of exchange rate	$\sigma_q = 0.1$
Correlation between exchange rate and foreign EBIT	$\rho\ = 0.5$
Risk-free interest rate in foreign country	$r_f = 0.06$
Corporate tax rate in foreign country	$\tau_f = 0.15$
Proportional default cost	$\alpha\ = 0.4$
Investment cost	$I\ = 10$

Figure 1: Inefficiency with respect to tax rate differential.

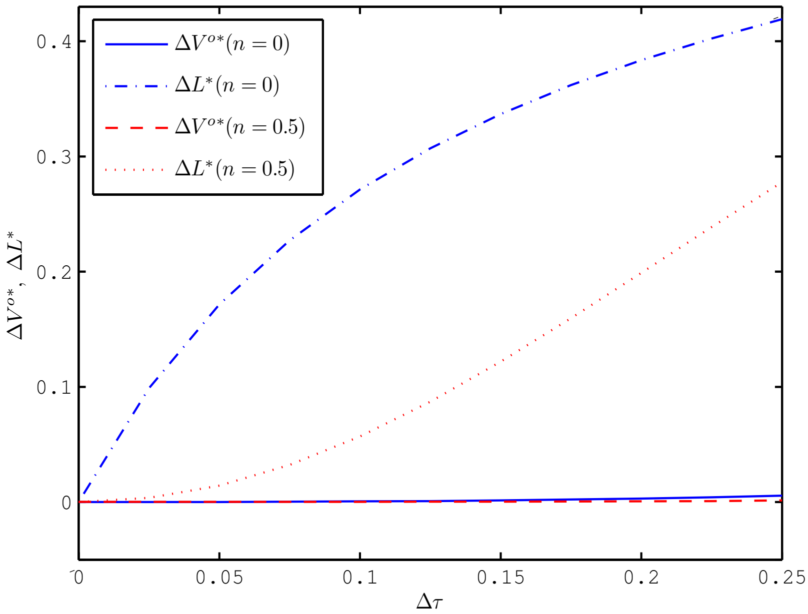


Figure shows the inefficiencies in investment and capital structure policies with respect to the tax rate differential $\Delta\tau$. The base parameter values listed in Table 2 are used and the initial values are set as $X(0) = 1$ and $Q(0) = 12$.

We observe from Fig. 1 that, while the inefficiency in the *ex ante* firm value (investment policy) is very small, the inefficiency in leverage (capital structure policy) is large. The upper panel of Fig. 1 shows that, when $n = 0$ and $\Delta\tau = 0.25$, the inefficiency in leverage (underleverage) is over 40%. Even in the case of costly debt shifting ($n = 0.5$), the inefficiency in leverage (underleverage) is over 25% when $\Delta\tau = 0.25$. Therefore, we conclude that the impact of debt shifting on capital structure policy cannot be ignored. This result echoes the empirical evidence reported in Huizinga *et al.* (2008), who claim that ignoring debt shifting would understate the impact of tax rate on debt policy by about 25%.

4.3 Impact of tax rate differential

In this subsection, we examine the impact of tax rate differential $\Delta\tau$ on the optimal leverage of foreign subsidiary. The empirical results in Huizinga *et al.* (2008) show that the optimal leverage always increases with respect to the tax rate differential, when the debt shifting cost is *not* considered. However, as the next result reveals, the optimal leverage may be decreasing in the tax rate differential, when the debt shifting is costly.

Proposition 4.2. *With respect to the tax rate differential* $\Delta\tau$*, we have*

$$\frac{\mathrm{d}L^*}{\mathrm{d}\Delta\tau}\begin{cases}\geq 0, & \textit{if } 0 \leq n \leq \bar{n}_1,\\ \leq 0, & \textit{if } n \geq \bar{n}_1,\end{cases} \tag{44}$$

where

$$\bar{n}_1 \equiv \bar{n}\left(2+\frac{\bar{n}}{2}\right) > \bar{n}. \tag{45}$$

That is, the optimal leverage of foreign subsidiary increases (decreases, respectively) with respect to the tax rate differential when the debt shifting cost is lower (higher) than $\bar{n}_1$*.*

Proof. See Appendix A.2. □

In the appendix, we show that both $\mathrm{d}L^*/\mathrm{d}\tau_d > 0$ and $\mathrm{d}L^*/\mathrm{d}\tau_f > 0$ when $G_2 > 0$, which is always satisfied under reasonable parameter values.[13] Although the impact of both domestic and foreign tax rates is positive on the optimal leverage, the reasons for the positivity are different. Without debt shifting, the optimal leverage of foreign subsidiary is determined by the trade-off between the tax benefits and default costs of debt financing. An increase in the foreign tax rate leads to an increase in the tax benefits (through the effective tax rate τ_f), and consequently an increase in the optimal leverage of foreign subsidiary. In contrast, an increase in the domestic tax rate leads to an increase in the tax benefits (through the effective deductible-tax rate $\tilde{\tau}$), and an increase in the optimal leverage of foreign subsidiary because of debt shifting. When $n \leq \bar{n}_1$, the impact of the domestic tax rate dominates that of the foreign tax rate. Therefore, the optimal leverage of foreign subsidiary increases with the tax rate differential. However, when $n \geq \bar{n}_1$, the impact of the foreign tax rate dominates that of the domestic tax rate, which leads to a decrease of the optimal leverage with respect to the tax rate differential. When the debt shifting cost is not considered, Huizinga *et al.* (2008) show that the optimal leverage increases with respect to the tax rate differential. Our result is consistent with this empirical result. Moreover, our results in Proposition 4.2 complements their results, when the debt shifting is costly.

4.4 Policies for attracting FDI: investment cost subsidy versus tax rate reduction

In this subsection, we follow Yu *et al.* (2007) to examine two policies for the host government to attract FDI: investment cost subsidy and tax rate reduction. We compare the FDI

[13]The empirical results in Huizinga *et al.* (2008) show that the optimal leverage L^* increases with respect to the domestic tax rate τ_d when the debt shifting cost is not considered. This result holds in our model for reasonable parameter values even when the debt shifting is costly.

thresholds and the expected costs of the two policies to find which one is more efficient for attracting FDI. Let T_u^{i*}, x_u^{i*} and $\tilde{\mathbb{E}}[u]$ denote the FDI timing, investment threshold and expected cost under policy u, where u is either investment cost subsidy (S) or tax rate reduction (R).

We first consider the case that the host government implements a proportional subsidy sI, $s \in (0,1)$, for the investment cost to attract FDI. That is, the investment cost is reduced from I to $(1-s)I$. From the result (a) in Proposition 3.1, we immediately have

$$x_{\mathrm{S}}^{i*} = \frac{\beta}{\beta-1}\frac{(1-s)I}{\psi} < x^{i*}. \tag{46}$$

That is, investment cost subsidy induces earlier FDI. The expected cost of subsidy for the host goverment is given by

$$\tilde{\mathbb{E}}[\mathrm{S}] = \tilde{\mathbb{E}}[e^{-r_f T_{\mathrm{S}}^{i*}}]sI = \left(\frac{x}{x_{\mathrm{S}}^{i*}}\right)^{\beta} sI, \tag{47}$$

where the second equality follows from Eq. (35).

Next, we consider the case that the host government reduces the tax rate τ_f to $\tau_f^{'}$ $(< \tau_f)$ for attracting FDI. Since τ_f affects $\tilde{\tau}$, m^*, Π, and consequently ψ in Eq. (32), we have the investment threshold under the tax rate reduction as

$$x_{\mathrm{R}}^{i*} = \frac{\beta}{\beta-1}\frac{I}{\psi^{'}}, \tag{48}$$

where

$$\psi^{'} \equiv \left(1 + \frac{\tilde{\tau}^{'}}{1-\tilde{\tau}^{'}} m^{*'}\right)\Pi^{'}. \tag{49}$$

Here, $\tilde{\tau}^{'}$, $m^{*'}$, $\Pi^{'}$ denote the values corresponding to the case that the foreign tax rate is $\tau_f^{'}$.

Proposition 4.3. *With respect to the foreign tax rate τ_f, we have $\frac{\mathrm{d}x^{i*}}{\mathrm{d}\tau_f} > 0$ when $G_3 < 0$, where*

$$G_3 \equiv \left(\frac{\alpha}{\tilde{\tau}B} + \frac{1}{1-\tilde{\tau}}\right)\left(1 - \frac{\bar{n}}{n}\right) - (1-\tilde{\tau})m^* - \tilde{\tau}. \tag{50}$$

That is, when $G_3 < 0$, the optimal investment threshold increases with respect to the foreign tax rate.

Proof. See Appendix A.3. □

For a wide range of reasonable parameter values, we find that the condition $G_3 < 0$ in Proposition 4.3 is always satisfied through extensive numerical experiments. Hence, we conclude that $x_{\mathrm{R}}^{i*} < x^{i*}$, if $\tau_f^{'} < \tau_f$. In other words, foreign tax rate reduction induces earlier FDI. From Eq. (36), the expected cost of tax rate reduction for the host goverment is given by

$$\tilde{\mathbb{E}}[\mathrm{R}] = \tilde{\mathbb{E}}[e^{-r_f T_{\mathrm{R}}^{i*}}](\psi' - \psi)x_{\mathrm{R}}^{i*} = \left(\frac{x}{x_{\mathrm{R}}^{i*}}\right)^{\beta} (\psi' - \psi)x_{\mathrm{R}}^{i*}. \tag{51}$$

Now, we compare the two policies. If $x_{\mathrm{S}}^{i*} = x_{\mathrm{R}}^{i*}$, we have $s = 1 - \psi/\psi'$. Substituting it into Eq. (47), we obtain

$$\tilde{\mathbb{E}}[\mathrm{S}] = \left(\frac{x}{x_{\mathrm{S}}^{i*}}\right)^{\beta} \left(1 - \frac{\psi}{\psi'}\right) I. \tag{52}$$

On the other hand, substituting $x_{\mathrm{S}}^{i*} = x_{\mathrm{R}}^{i*}$ with (48) into Eq. (51), we obtain

$$\tilde{\mathbb{E}}[\mathrm{R}] = \left(\frac{x}{x_{\mathrm{R}}^{i*}}\right)^{\beta} (\psi' - \psi)\frac{\beta}{\beta - 1}\frac{I}{\psi'} = \frac{\beta}{\beta - 1}\tilde{\mathbb{E}}[\mathrm{S}] > \tilde{\mathbb{E}}[\mathrm{S}], \tag{53}$$

since $\beta > 1$. Hence, the expected cost of subsidy is smaller than that of tax rate reduction, when the investment thresholds are kept the same. Similarly, if we start from $\tilde{\mathbb{E}}[\mathrm{S}] = \tilde{\mathbb{E}}[\mathrm{R}]$, we obtain $x_{\mathrm{S}}^{i*} < x_{\mathrm{R}}^{i*}$.

Therefore, we conclude that investment cost subsidy is more efficient for attracting earlier FDI, as Yu *et al.* (2007) assert. Note that we generalize the result in Yu *et al.* (2007) by showing that their result holds even in our framework where debt shifting and exchange rate uncertianty are considered.

If $\sigma_x \to 0$, we have $\beta \to r_f/\mu_x$ by virtue of l'Hôpital's rule. If $\sigma_x \to 0$ and $\mu_x \to 0$, we have $\beta \to \infty$ and consequently $\tilde{\mathbb{E}}[\mathrm{R}] \to \tilde{\mathbb{E}}[\mathrm{S}]$ from (53). In other words, if the foreign EBIT is very stable, there is almost no difference between the two policies.

4.5 Impact of exchange rate and foreign EBIT uncertainties

In our model, there are two uncertainties on exchange rate and foreign EBIT with correlation. In the standard real options model (see Campa, 1993; Dixit and Pindyck, 1994), option values to wait for investment become higher as uncertainty gets larger. Hence, we expect that the optimal investment threshold becomes higher when the uncertainties get larger, because the MNC tends to wait more for the investment. However, this is not always the case for the exchange rate uncertainty in our model, as the following results reveal. To this end, we note that

$$\frac{\mathrm{d}x^{i*}}{\mathrm{d}z} = \frac{\partial x^{i*}}{\partial \beta}\frac{\partial \beta}{\partial z} + \frac{\partial x^{i*}}{\partial \psi}\left(\frac{\partial \psi}{\partial m^*}\frac{\partial m^*}{\partial \gamma}\frac{\partial \gamma}{\partial z} + \frac{\partial \psi}{\partial z}\right), \qquad z = \sigma_q, \sigma_x, \rho. \tag{54}$$

Hence, the impacts of uncertainties and correlation on investment are divided into two terms. While the first term in the right-hand side is the usual direct effect in the real options literature, the second term is an indirect effect through debt financing.

Proposition 4.4. (a) *With respect to the exchange rate uncertainty σ_q, we have*

$$\frac{\mathrm{d}x^{i*}}{\mathrm{d}\sigma_q}\begin{cases}\geq 0, & \text{if } -1\leq\rho\leq 0,\\ \leq 0, & \text{if } \quad 0\leq\rho\leq 1,\end{cases} \tag{55}$$

when $G_4 < 0$, where

$$G_4 \equiv \frac{1}{\beta(\beta-1)}\left[1-\frac{\mu_x'}{\sqrt{\mu_x'^2+2r_f\sigma_x^2}}\right]+\frac{\Pi-\psi}{\psi}\frac{F}{\gamma^2}\left[1+\frac{\mu_x'}{\sqrt{\mu_x'^2+2r_f\sigma_x^2}}\right]-\frac{\sigma_x^2\Pi}{1-\tau_f}, \tag{56}$$

$\mu_x' \equiv \mu_x+\rho\sigma_q\sigma_x-\frac{1}{2}\sigma_x^2$, *and* $F\equiv \ln B+B^{-1}-1$. *That is, when $G_4<0$, the optimal investment threshold increases (decreases, respectively) with respect to the exchange rate uncertainty when the correlation between exchange rate and foreign EBIT is negative (positive).*

(b) *With respect to the foreign EBIT uncertainty σ_x, we have $\mathrm{d}x^{i*}/\mathrm{d}\sigma_x > 0$ when $G_5>0$, where*

$$G_5 \equiv \frac{1}{\beta-1}\frac{\rho\sigma_q+(\beta-1)\sigma_x}{\beta\sigma_x^2+\mu_x'}+\frac{\Pi-\psi}{\psi}\frac{F}{\gamma}\frac{\rho\sigma_q+(\gamma-1)\sigma_x}{\gamma\sigma_x^2+\mu_x'}-\frac{\rho\sigma_q\Pi}{1-\tau_f}. \tag{57}$$

That is, when $G_5>0$, the optimal investment threshold increases with respect to the foreign EBIT uncertainty.

(c) *With respect to the correlation ρ, we have $\mathrm{d}x^{i*}/\mathrm{d}\rho<0$, when $G_4<0$. That is, when $G_4<0$, the optimal investment threshold decreases with respect to the correlation between the foreign EBIT and exchange rate uncertainties.*

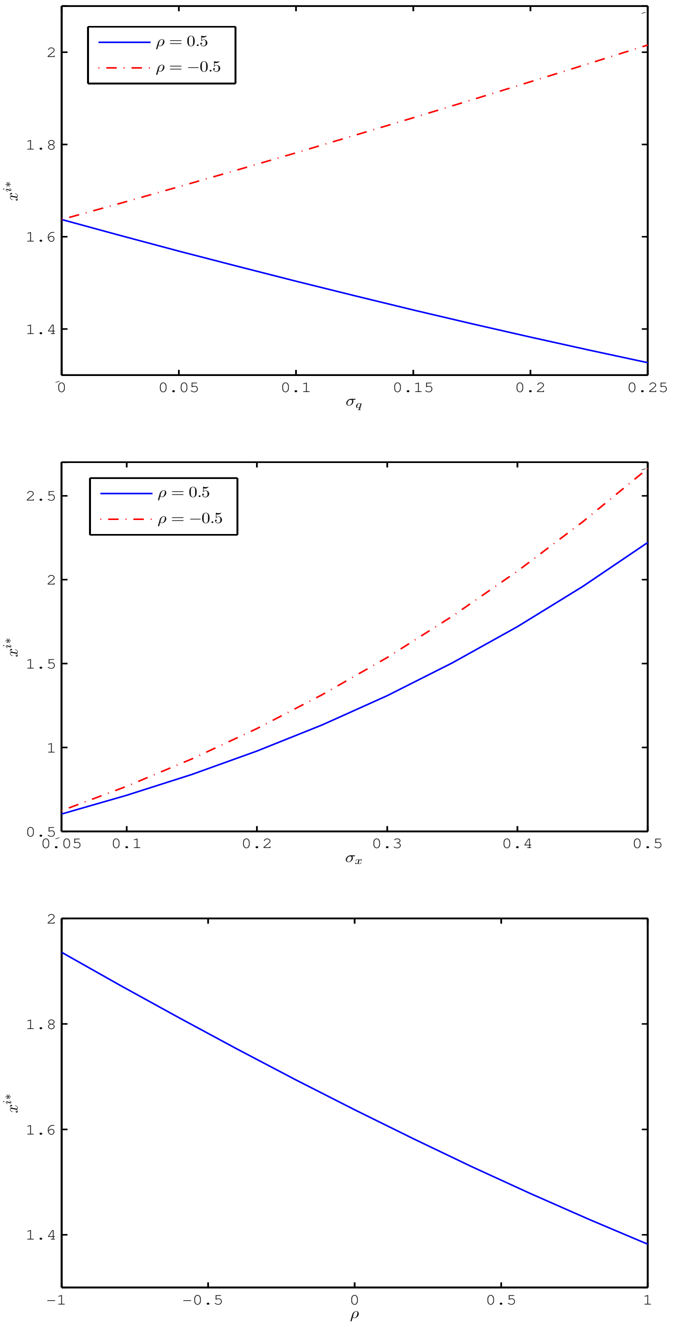
Proof. See Appendix A.4. □

Again, for a wide range of reasonable parameter values, we find that the conditions ($G_4<0$ and $G_5>0$) in Proposition 4.4 are always satisfied through extensive numerical experiments.[14] The results in Proposition 4.4 are illustrated by Fig. 2.

The upper panel in Fig. 2 shows that the impact of exchange rate uncertainty on FDI depends on the correlation between exchange rate and foreign EBIT uncertainties. As mentioned in Section 3.1, the exchange rate uncertainty σ_q enters the drift term of the process $X(t)$ only. More specifically, the drift term is adjusted by $\rho\sigma_q\sigma_x$ as in Eq. (13) under the new probability measure $\tilde{\mathbb{P}}$. This is the reason why the impact of exchange rate uncertainty on FDI depends on the sign of correlation ρ. Of course, when $\rho=0$, the exchange rate does not matter on FDI. Also, when $\sigma_x\to 0$, the optimal investment threshold becomes irrelevant to the exchange rate uncertainty, because the adjustment becomes zero.

[14] Surprisingly, even for abnormal parameter values (e.g., $\sigma_x=3$, $\mu_x=\pm1$, etc.), the conditions are satisfied. Hence, we conclude that our numerical results are robust.

Figure 2: FDI with respect to uncertainties and correlation.



These figures show the optimal investment threshold of foreign subsidiary with respect to exchange rate uncertainty for $0 \leq \sigma_q \leq 0.25$ (upper panel), with respect to foreign EBIT uncertainty for $0.05 \leq \sigma_x \leq 0.5$ (middle panel), and with respect to correlation for $-1 \leq \rho \leq 1$. The parameter region of σ_q and σ_x are set different, because the volatility of exchange rate is estimated around 10% and less than that of EBIT in practice. The other parameters are taken from Table 2. The domestic tax rate and debt shifting cost are set as $\tau_d = 0.4$ and $n = 0.5$, respectively.

Note that the exchange rate uncertainty works in the opposite direction to the usual uncertainty effect in the standard real options model. In other words, when the correlation is positive, exchange rate uncertainty can contribute to the internationalization of production activity. This is consistent with the empirical findings reported in Goldberg and Kolstad (1995).

One important implication of the result (a) in Proposition 4.4 is that, when performing empirical analyses of the impact of exchange rate volatility, it is crucial to know whether the correlation between exchange rate and foreign EBIT uncertainties is positive or negative. As mentioned in the introductory section, the relationship between exchange rate uncertainty and FDI remains ambiguous in the literature. Our result reveals that, for MNCs with EBIT uncertainty being positively (negatively, respectively) correlated to exchange rate uncertainty, there is a positive (negative) relationship between exchange rate uncertainty and FDI. Note that it is generally very difficult to determine the sign of correlation statistically unless the correlation is significantly away from zero.

The middle panel in Fig. 2 shows that the higher the foreign EBIT uncertainty, the higher the investment threshold of foreign subsidiary, irrespective of the correlation. That is, the foreign EBIT uncertainty works in the same direction to the usual uncertainty effect in the standard real options model. This is also consistent with the empirical evidence reported in Goldberg and Kolstad (1995), who claim that the higher the US GDP volatility, the smaller the share of Japanese FDI in the US market. By comparing the upper and middle panels in Fig. 2, we find that the quantitative impact of foreign EBIT uncertainty on the investment threshold is larger than that of exchange rate uncertainty. That is because the fundamental uncertainty for FDI stems from the foreign EBIT.

The bottom panel in Fig. 2 shows that the higher the correlation, the lower the investment threshold of foreign subsidiary. In other words, the correlation itself accelerates FDI, which echoes the results in Goldberg and Kolstad (1995).

4.6 Inefficiency due to ignoring exchange rate uncertainty

In this subsection, we consider the inefficiency resulting from ignoring exchange rate uncertainty. The inefficiency in the *ex ante* firm value (Eq. (38)) and leverage (Eq. (39)) are defined, respectively, as

$$\Delta\underline{V}^{o*}(x,q;x^{i*}) \equiv 1 - \frac{V^{o*}(x,q;\underline{x}^{i*})}{V^{o*}(x,q;x^{i*})}, \qquad \Delta\underline{L}^{*}(x^{i*},Q(T^{i*})) \equiv 1 - \frac{L^{*}(\underline{x}^{i*},Q(\underline{T}^{i*}))}{L^{*}(x^{i*},Q(T^{i*}))}, \tag{58}$$

where $\underline{x}^{i*}$ and $Q(\underline{T}^{i*})$ correspond to the case that $\sigma_q = 0$. We use these $\Delta\underline{V}^{o*}$ and $\Delta\underline{L}^{*}$ to measure the inefficiencies in investment and capital structure policies, respectively.

Figure 3 depicts the inefficiencies in investment and capital structure policies with respect to exchange rate uncertainty σ_q. We observe that, when the correlation is positive

Figure 3: Inefficiency with respect to exchange rate uncertainty.

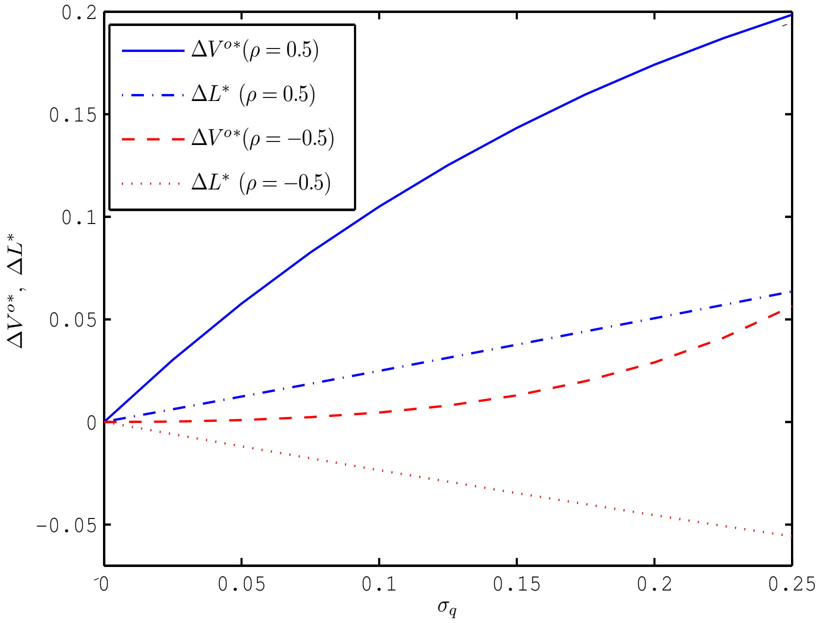


Figure shows the inefficiencies in investment and capital structure policies with respect to exchange rate uncertainty σ_q. The base parameter values (except σ_q) listed in Table 2 are used. The domestic tax rate and debt shifting cost are set as $\tau_d = 0.4$ and $n = 0.5$, respectively. The initial values are set as $X(0) = 1$ and $Q(0) = 12$.

(negative, respectively), the foreign subsidiary is underleveraged (overleveraged), although the magnitude of inefficiency in capital structure policy is relatively small; cf. Fig. 1. Moreover, the inefficiency in the *ex ante* firm value (investment policy) is relatively large for the case of positive correlation compared to the case of negative correlation. Recall that $x^{i*} \leq (\geq)\ \underline{x}^{i*}$ when $\rho \geq (\leq)\ 0$ from the result (a) in Proposition 4.4. Therefore, the magnitude of inefficiency due to underinvestment is larger than that due to overinvestment. When $\sigma_q = 0.25$, ignoring exchange rate uncertainty induces underinvestment about 20%. Even for our base case where $\sigma_q = 0.1$, the inefficiency of underinvestment is over 10%. Thus, we conclude that the impact of exchange rate uncertainty on investment policy cannot be ignored, especially when the correlation is positive. This result echoes the empirical evidence shown in Goldberg and Kolstad (1995), who report that exchange rate volatility has a positive and statistically significant impact on FDI.

5 Conclusions

In this paper, we examine the impact of international debt shifting on investment and capital structure decisions of foreign subsidiary by incorporating exchange rate uncertainty.

We find that debt shifting induces earlier investment, earlier default, higher leverage, and larger *ex ante* firm value of foreign subsidiary. The inefficiency in investment and capital structure policies due to ignoring debt shifting cannot be neglected.

Our results are consistent with several empirical findings. When debt shifting is not too costly, the optimal leverage of foreign subsidiary at its establishment increases as the tax rate differential increases, which is consistent with Huizinga *et al.* (2008). When the correlation between foreign EBIT and exchange rate is positive, FDI advances as exchange rate uncertainty rises and becomes more correlated to the foreign EBIT uncertainty, which echoes the results reported in Goldberg and Kolstad (1995). Moreover, ignoring the exchange rate uncertainty and the possibility of debt shifting, our model is reduced to Sundaresan and Wang (2007).

Finally, as a future research, we point out an important but difficult topic. In our model, in order to focus the impact of debt shifting on both investment and capital structure decisions of foreign subsidiary, we ignore the uncertainty in the domestic market. However, with the advance of globalization, the domestic market becomes more related to a foreign market than ever, and the exchange rate between the two currencies should be determined so as to reflect the economic conditions of the two countries. Therefore, it is of great interest to extend our model in such a way that the domestic EBIT is also stochastic with correlations to both foreign EBIT and exchange rate uncertainties.

A Appendix

A.1 Proof of Proposition 4.1

(a) Since

$$\frac{\mathrm{d}x^{i*}}{\mathrm{d}\tilde{\tau}} = \frac{\mathrm{d}x^{i*}}{\mathrm{d}\psi}\frac{\mathrm{d}\psi}{\mathrm{d}\tilde{\tau}}$$

and since $\mathrm{d}x^{i*}/\mathrm{d}\psi = -x^{i*}/\psi < 0$, we only need to show

$$\frac{\mathrm{d}\psi}{\mathrm{d}\tilde{\tau}} = \frac{\partial\psi}{\partial m^*}\frac{\partial m^*}{\partial\tilde{\tau}} + \frac{\partial\psi}{\partial\tilde{\tau}} > 0.$$

This can be verified, because

$$\frac{\partial\psi}{\partial m^*} = \frac{\tilde{\tau}}{1-\tilde{\tau}}\Pi > 0, \qquad \frac{\partial m^*}{\partial\tilde{\tau}} = \frac{\alpha}{\tilde{\tau}^2}(m^*)^{1-\gamma} > 0, \qquad \frac{\partial\psi}{\partial\tilde{\tau}} = \frac{m^*}{(1-\tilde{\tau})^2}\Pi > 0.$$

(b) First, we have

$$\frac{\mathrm{d}x^{b*}}{\mathrm{d}\tilde{\tau}} = \frac{\beta}{\beta-1}I\frac{\mathrm{d}(m^*/\psi)}{\mathrm{d}\tilde{\tau}}.$$

Since

$$\frac{m^*}{\psi} = \left(\frac{1}{m^*} + \frac{\tilde{\tau}}{1-\tilde{\tau}}\right)^{-1} \Pi^{-1},$$

it follows that

$$\frac{\mathrm{d}(m^*/\psi)}{\mathrm{d}\tilde{\tau}} = -\left(\frac{m^*}{\psi}\right)^2 \Pi G_1,$$

where G_1 is defined in (41), proving Part (b).

(c) Since $c^* = x^{b*}/A$, we have

$$\begin{aligned}
\frac{\mathrm{d}c^*}{\mathrm{d}\tilde{\tau}} &= \frac{\mathrm{d}x^{b*}}{\mathrm{d}\tilde{\tau}}\frac{1}{A} - \frac{x^{b*}}{A^2}\frac{\mathrm{d}A}{\mathrm{d}\tilde{\tau}} \\
&= \frac{c^*}{1-\tilde{\tau}}\frac{\Pi}{\psi}\left[-\frac{m^*}{1-\tilde{\tau}} + \frac{\alpha}{B}\frac{1-\tilde{\tau}}{\tilde{\tau}^2} + 1 + \frac{\tilde{\tau}}{1-\tilde{\tau}}m^*\right] \\
&= \frac{c^*}{1-\tilde{\tau}}\frac{\Pi}{\psi}\left[-m^* + \frac{\alpha}{B}\frac{1-\tilde{\tau}}{\tilde{\tau}^2} + 1\right] > 0,
\end{aligned}$$

because $m^* \in (0,1)$.

(d) Since $\mathrm{d}V^{o*}/\mathrm{d}x^{i*} < 0$ and $\mathrm{d}x^{i*}/\mathrm{d}\tilde{\tau} < 0$, we have

$$\frac{\mathrm{d}V^{o*}}{\mathrm{d}\tilde{\tau}} = \frac{\mathrm{d}V^{o*}}{\mathrm{d}x^{i*}}\frac{\mathrm{d}x^{i*}}{\mathrm{d}\tilde{\tau}} > 0.$$

(e) Since

$$L^* = c^*\frac{\beta-1}{\beta}\frac{1-\xi}{r_f I},$$

we have

$$\begin{aligned}
\frac{\mathrm{d}L^*}{\mathrm{d}\tilde{\tau}} &= \frac{\beta-1}{\beta}\frac{1}{r_f I}\left[\frac{\mathrm{d}c^*}{\mathrm{d}\tilde{\tau}}(1-\xi) - c^*\frac{\mathrm{d}\xi}{\mathrm{d}\tilde{\tau}}\right] \\
&= \frac{\beta-1}{\beta}\frac{1}{r_f I}\left[\frac{\mathrm{d}c^*}{\mathrm{d}\tilde{\tau}}(1-\xi) - c^*\left(\frac{\partial\xi}{\partial\tilde{\tau}} + \frac{\partial\xi}{\partial m^*}\frac{\partial m^*}{\partial\tilde{\tau}}\right)\right] \\
&= \frac{\beta-1}{\beta}\frac{c^*}{r_f I}\frac{\Pi}{\psi}\frac{1}{1-\tilde{\tau}}G_2,
\end{aligned}$$

where G_2 is defined in (42), proving Part (e).

A.2 Proof of Proposition 4.2

First, note that

$$\frac{\mathrm{d}L^*}{\mathrm{d}\Delta\tau} = \frac{\mathrm{d}L^*}{\mathrm{d}\tilde{\tau}}\left(\frac{\mathrm{d}\tilde{\tau}}{\mathrm{d}\tau_d} - \frac{\mathrm{d}\tilde{\tau}}{\mathrm{d}\tau_f}\right).$$

Since

$$\frac{\mathrm{d}\tilde{\tau}}{\mathrm{d}\tau_d} = \begin{cases} 1 + \frac{n}{2} > 0, & 0 \leq n \leq \bar{n}, \\ \frac{\bar{n}}{n}\left(1 + \frac{\bar{n}}{2}\right) > 0, & n \geq \bar{n}, \end{cases}$$

and since

$$\frac{\mathrm{d}\tilde{\tau}}{\mathrm{d}\tau_f} = \begin{cases} 0, & 0 \leq n \leq \bar{n}, \\ 1 - \frac{\bar{n}}{n} \geq 0, & n \geq \bar{n}, \end{cases}$$

we have

$$\frac{\mathrm{d}\tilde{\tau}}{\mathrm{d}\tau_d} - \frac{\mathrm{d}\tilde{\tau}}{\mathrm{d}\tau_f} \begin{cases} > 0, & 0 \leq n \leq \bar{n}, \\ = \frac{\bar{n}}{n}\left(2 + \frac{\bar{n}}{2}\right) - 1 \begin{cases} \geq 0, & n \leq \bar{n}_1, \\ \leq 0, & n \geq \bar{n}_1, \end{cases} \end{cases}$$

where $\bar{n}_1$ is defined in (45).

A.3 Proof of Proposition 4.3

Note that

$$\frac{\mathrm{d}x^{i*}}{\mathrm{d}\tau_f} = \frac{\mathrm{d}x^{i*}}{\mathrm{d}\psi}\left(\frac{\mathrm{d}\psi}{\mathrm{d}\tilde{\tau}}\frac{\mathrm{d}\tilde{\tau}}{\mathrm{d}\tau_f} + \frac{\mathrm{d}\psi}{\mathrm{d}\Pi}\frac{\mathrm{d}\Pi}{\mathrm{d}\tau_f}\right) \tag{A.1}$$

If $0 \leq n \leq \bar{n}$, since $\mathrm{d}\tilde{\tau}/\mathrm{d}\tau_f = 0$, we have

$$\frac{\mathrm{d}x^{i*}}{\mathrm{d}\tau_f} = \frac{\mathrm{d}x^{i*}}{\mathrm{d}\psi}\frac{\mathrm{d}\psi}{\mathrm{d}\Pi}\frac{\mathrm{d}\Pi}{\mathrm{d}\tau_f} = -\frac{x^{i*}}{\psi}\frac{\psi}{\Pi}\frac{-\Pi}{1-\tau_f} = \frac{x^{i*}}{1-\tau_f} > 0.$$

If $n \geq \bar{n}$, by substituting all the derivatives that appear in Eq. (A.1), we obtain

$$\frac{\mathrm{d}x^{i*}}{\mathrm{d}\tau_f} = -\frac{x^{i*}}{\psi}\frac{m^*\Pi}{1-\tilde{\tau}}G_3,$$

where G_3 is defined in Eq. (50). Therefore, we have $\mathrm{d}x^{i*}/\mathrm{d}\tau_f > 0$ if $G_{\tau_f} < 0$.

A.4 Proof of Proposition 4.4

(a) Note that

$$\frac{\mathrm{d}x^{i*}}{\mathrm{d}\sigma_q} = \frac{\partial x^{i*}}{\partial\beta}\frac{\partial\beta}{\partial\sigma_q} + \frac{\partial x^{i*}}{\partial\psi}\left(\frac{\partial\psi}{\partial m^*}\frac{\partial m^*}{\partial\gamma}\frac{\partial\gamma}{\partial\sigma_q} + \frac{\partial\psi}{\partial\sigma_q}\right). \tag{A.2}$$

First, we calculate $\partial m^*/\partial\gamma$:

$$\frac{\partial m^*}{\partial\gamma} = m^*\frac{\partial\ln m^*}{\partial\gamma} = -\frac{m^*}{\gamma^2}F(\gamma),$$

where

$$F(\gamma) \equiv \ln B(\gamma) + \frac{1}{B(\gamma)} - 1,$$

and $B(\gamma)$ is defined in Eq. (27). Since $F(0) = 0$ and $F'(\gamma) < 0$, we have $F(\gamma) > 0$ over the region $\gamma < 0$. Consequently $\partial m^*/\partial\gamma = -m^* F(\gamma)/\gamma^2 < 0$.

Next, we calculate $\partial\beta/\partial\sigma_q$ and $\partial\gamma/\partial\sigma_q$:

$$\frac{\partial\beta}{\partial\sigma_q} = -\frac{\rho}{\sigma_x}\left[1 - \frac{\mu'_x}{\sqrt{\mu'^2_x + 2r_f\sigma_x^2}}\right] \leq (\geq)\ 0, \qquad \text{when } \rho \geq (\leq)\ 0,$$

$$\frac{\partial\gamma}{\partial\sigma_q} = -\frac{\rho}{\sigma_x}\left[1 + \frac{\mu'_x}{\sqrt{\mu'^2_x + 2r_f\sigma_x^2}}\right] \leq (\geq)\ 0, \qquad \text{when } \rho \geq (\leq)\ 0,$$

where $\mu'_x \equiv \mu_x + \rho\sigma_q\sigma_x - \frac{1}{2}\sigma_x^2$.

We also have

$$\frac{\partial x^{i*}}{\partial\beta} = \frac{-I}{\psi(\beta-1)^2} < 0$$

and

$$\frac{\partial\psi}{\partial\sigma_q} = \frac{\rho\sigma_x\psi\Pi}{1-\tau_f} \geq (\leq)\ 0, \qquad \text{when } \rho \geq (\leq)\ 0.$$

Besides, we have already obtained $\partial x^{i*}/\partial\psi$ and $\partial\psi/\partial m^* > 0$ in A.1 (a). Substituting all the partial derivatives above into Eq. (A.2), we finally obtain

$$\frac{\mathrm{d}x^{i*}}{\mathrm{d}\sigma_q} = \frac{\rho}{\sigma_x}x^{i*}G_4,$$

where G_4 is defined in Eq. (56), proving Part (a).

(b) Note that

$$\frac{\mathrm{d}x^{i*}}{\mathrm{d}\sigma_x} = \frac{\partial x^{i*}}{\partial\beta}\frac{\partial\beta}{\partial\sigma_x} + \frac{\partial x^{i*}}{\partial\psi}\left(\frac{\partial\psi}{\partial m^*}\frac{\partial m^*}{\partial\gamma}\frac{\partial\gamma}{\partial\sigma_x} + \frac{\partial\psi}{\partial\sigma_x}\right). \tag{A.3}$$

All the partial derivatives in Eq. (A.3) have already been calculated in Part (a), except $\partial\beta/\partial\sigma_x$ and $\partial\gamma/\partial\sigma_x$.

By differentiating Eq. (17), we have

$$\frac{\partial\mathbb{Q}}{\partial y}\frac{\partial y}{\partial\sigma_x} + \frac{\partial\mathbb{Q}}{\partial\sigma_x} = 0, \qquad y = \beta, \gamma.$$

Since

$$\frac{\partial\mathbb{Q}}{\partial y} = (y - 1/2)\sigma_x^2 + \mu_x + \rho\sigma_q\sigma_x, \qquad \frac{\partial\mathbb{Q}}{\partial\sigma_x} = y[\rho\sigma_q + (y-1)\sigma_x],$$

we obtain

$$\frac{\partial y}{\partial\sigma_x} = -y\frac{\rho\sigma_q + (y-1)\sigma_x}{y\sigma_x^2 + \mu'_x}.$$

Substituting all the partial derivatives above into Eq. (A.3), we then obtain

$$\frac{\mathrm{d}x^{i*}}{\mathrm{d}\sigma_x} = x^{i*}G_5,$$

where G_5 is defined in Eq. (57), proving Part (b).

(c) Note that

$$\frac{\mathrm{d}x^{i*}}{\mathrm{d}\rho} = \frac{\partial x^{i*}}{\partial \beta}\frac{\partial \beta}{\partial \rho} + \frac{\partial x^{i*}}{\partial \psi}\left(\frac{\partial \psi}{\partial m^*}\frac{\partial m^*}{\partial \gamma}\frac{\partial \gamma}{\partial \rho} + \frac{\partial \psi}{\partial \rho}\right).$$

The proof is similar to the proof of Part (a), except that

$$\frac{\partial \beta}{\partial \rho} = -\frac{\sigma_q}{\sigma_x}\left[1 - \frac{\mu'_x}{\sqrt{\mu_x'^2 + 2r_f\sigma_x^2}}\right] < 0, \quad \frac{\partial \gamma}{\partial \rho} = -\frac{\sigma_q}{\sigma_x}\left[1 + \frac{\mu'_x}{\sqrt{\mu_x'^2 + 2r_f\sigma_x^2}}\right] < 0.$$

Therefore, we have $\mathrm{d}x^{i*}/\mathrm{d}\rho < 0$ when $G_4 < 0$ is satisfied.

References

[1] Amuedo-Dorantes, C., & Pozo, S. (2001). Foreign exchange rates and foreign direct investment in the United States. *International Trade Journal*, 15, 323–343.

[2] Campa, J. M. (1993). Entry by foreign firms in the United States under exchange rate uncertainty. *Review of Economics and Statistics*, 75, 614–622.

[3] Cushman, D. O. (1985). Real exchange rate risk, expectations, and the level of direct investment. *Review of Economics and Statistics*, 67, 297–308.

[4] Cushman, D. O. (1988). Exchange-rate uncertainty and foreign direct investment in the United States. *Review of World Economics*, 124, 322–335.

[5] Dharmapala, D., & Hines, J. R. (2009). Which countries become tax havens? *Journal of Public Economics*, 93, 1058–1068.

[6] Dixit, A. K., & Pindyck, R. S. (1994). *Investment under uncertainty*. Princeton University Press, Princeton, NJ.

[7] Fan, H., & Sundaresan, S. (2000). Debt valuation, strategic debt service and optimal dividend policy. *Review of Financial Studies*, 13, 1057–1099.

[8] Goldberg, L. S., & Kolstad, C. D. (1995). Foreign direct investment, exchange rate variability and demand uncertainty. *International Economic Review*, 36, 855–873.

[9] Grubert, H., & Mutti, J. (2000). Do taxes influence where U.S. corporations invest? *National Tax Journal*, 53, 825–839.

[10] Hassett, K. A., & Metcalf, G. E. (1995). Investment under alternative return assumptions: Comparing random walks and mean reversion. *Journal of Economic Dynamics and Control*, 19, 1471–1488.

[11] Hau, H., & Rey, H. (2006). Exchange rate, equity prices, and capital flows. *Review of Financial Studies*, 19, 273–317.

[12] Hines, J. R. (1999). Lessons from behavioral responses to international taxation. *National Tax Journal*, 52, 305–322.

[13] Huizinga, H., Laeven, L., & Nicodeme, G. (2008). Capital structure and international debt shifting. *Journal of Financial Economics*, 88, 80–118.

[14] Kiyota, K., & Urata, S. (2004). Exchange rate, exchange rate volatility and foreign direct investment. *The World Economy*, 27, 1501–1536.

[15] Kolasinski, A. C. (2009). Subsidiary debt, capital structure and internal capital markets. *Journal of Financial Economics*, 94, 327–343.

[16] Leland, H. (1994). Corporate debt value, bond covenants, and optimal capital structure. *Journal of Finance*, 49, 1213–1252.

[17] Leland, H., & Toft, K. (1996). Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *Journal of Finance*, 51, 987–1019.

[18] McDonald, R., & Siegel, D. (1986). The value of waiting to invest. *Quarterly Journal of Economics*, 101, 707–728.

[19] Mella-Barral, P., & Perraudin, W. (1997). Strategic debt service. *Journal of Finance*, 52, 531–556.

[20] Mills, L. F., & Newberry, K. J. (2004). Do foreign multinational's tax incentives influence their U.S. income reporting and debt policy? *National Tax Journal*, 57, 89–107.

[21] Mintz, J., & Smart, M. (2004). Income shifting, investment, and tax competition: Theory and evidence from provincial taxation in Canada, *Journal of Public Economics*, 88, 1149–1168.

[22] Panteghini, P. M. (2009). The capital structure of multinational companies under tax competition. *International Tax and Public Finance*, 16, 59–81.

[23] Phylaktis, K., & Ravazzolo, F. (2005). Stock prices and exchange rate dynamics. *Journal of International Money and Finance*, 24, 1031–1053.

[24] Servén, L. (2003). Real-exchange-rate uncertainty and private investment in LDCs. *Review of Economics and Statistics*, 85, 212–218.

[25] Sundaresan, S., & Wang, N. (2007). Dynamic investment, capital structure, and debt overhang. Working paper, Columbia University.

[26] Urata, S., & Kawai, H. (2000). The determinants of the location of foreign direct investment by Japanese small and medium-sized enterprises, *Small Business Economics*, 15, 79–103.

[27] Yu, C. F., Chang, T. C., & Fan, C. P. (2007). FDI timing: entry cost subsidy versus tax rate reduction. *Economic Modelling*, 24, 262–271.