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Strategic investment timing under asymmetric access charge regulation in telecommunications

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Abstract:

In a liberalized telecommunications market, an incumbent has several advantages over any entrant. An asymmetric access charge regulation for two such asymmetric firms stimulates competitive investment. We show that an entrant with a cost disadvantage has an incentive to invest as a leader under an asymmetric access charge regulation. These results fit well with the findings of previous empirical work. Moreover, we also investigate the effects of an asymmetric access charge regulation on competitive investment strategies.

Keywords: Investment timing; Competition; Regulation.

JEL classification: L96; L51; L43; G13;.

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1 Introduction

In a liberalized telecommunications market, the entrant has several disadvantages to any incumbent. This creates an asymmetric market environment, so there may be a role for asymmetric regulation. In policy debates, the desirability of asymmetric regulation has been recognized.[1] Asymmetric regulation is claimed to promote entry. At the same time, regulators are expected to increase consumer surplus (see, for instance, Peitz, 2005).

Several studies have examined the effect of access charges on the incentive to invest in telecommunications market. Gans (2001), Cave and Vogelsang (2003), Valletti (2003), Carter and Wright (2003), Peitz (2005), Kotakorpi (2006), and Hori and Mizuno (2006) examined an investment incentive in static models.[2] Hori and Mizuno (2006) considered an investment incentive between two symmetric firms in dynamic models, while Carter and Wright (2003) and Peitz (2005) explored the role of asymmetric regulation in asymmetric markets in static models.

To the best of our knowledge, however, there has *not* been an examination of strategic investment timing and project option values under asymmetric access charge regulation in an asymmetric market environment in a dynamic model. This paper examines competitive investment timing strategies under asymmetric access charge regulation in a recently liberalized telecommunications market, in which the incumbent typically enjoys several advantages over the entrants. This paper also investigates the effects of an asymmetric access charge regulation on competitive investment strategies. In particular, we extend the dynamic model between two symmetric firms developed by Hori and Mizuno (2006) to a dynamic model between two asymmetric firms.[3] This is because to address questions of regulation in a recently liberalized market, we must consider an asymmetric market. In other words, we extend the static model developed by Peitz (2005) to a dynamic model. Thus, we consider the effects of asymmetric access charge regulation on competitive investment timing strategies in an asymmetric market environment.

Our paper provides several important results. First, an asymmetric access charge regulation may lead the entrant to enter the new market as a leader. Second, an asymmetric

[1]Such asymmetric access charge regulation accords with legislation in many developed countries. For example, the regulation of access in the European Union and Japan differ between operators with and without significant market power.

[2]Laffont and Tirole (1997, 2000) Armstrong (1998), and Carter and Wright (1999) considered how access prices affect profits, consumers surplus, and welfare.

[3]Hori and Mizuno's study is based on an extended duopoly model of the standard monopoly model. See McDonald and Siegel (1986) and Dixit and Pindyck (1994, Chap. 5) for the standard monopoly model, and Dixit and Pindyck (1994, Chap. 7) and Weeds (2002) for the extended duopoly model in greater detail.

access charge regulation has the possibility of hastening the investment. This result can be regarded as an increase in consumer surplus. However, the producer surplus defined by the sum of the two firms' profits may be reduced. Finally, the larger an asymmetric access charge is, the smaller the investment trigger becomes. That is, an asymmetric access charge stimulates investment. However, when the asymmetric access charge is too large, the investment trigger is increased (i.e., investment is decreased). These results fit well with the findings of previous empirical work.

The remainder of the paper is organized as follows. Section 2 describes the setup of the model and the derivation of the project value function. Section 3 examines strategic investment between two asymmetric firms under asymmetric access charge regulation, and analyzes the effects of such regulation. Section 4 concludes. The appendix contains the derivation of the solution in detail.

2 Model

In this section, we begin with a description of the setup. We then provide the value functions for firms and compare them in any market situation. Finally, we provide the solution for the non-strategic equilibrium as a benchmark.

2.1 Setup

Consider two asymmetric firms, an incumbent (Firm A) and an entrant (Firm B). Two asymmetric firms have an investment opportunity in a new-technology full-coverage network, assuming the incumbent has an old-technology full-coverage network. The incumbent and the entrant are risk neutral.

The cash flow to provide the new-technology full-coverage network service depends on the number of operating firms in the market (i.e., monopoly or duopoly). The investment yields an instantaneous cash flow, $D_j X(t)$, where $D_j > 0$ is a constant amount depending on the number of operating firms, j ($j \in \{1, 2\}$). We assume that $D_1 > D_2$. This condition implies that an investment is less profitable when more firms have invested.[4] Also, let $X(t)$ be the price at time t given by the following geometric Brownian motion:

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}z(t), \quad t \geq 0, \tag{1}$$

where $z(t)$ denotes standard Brownian motion, and $\mu > 0$ and $\sigma > 0$ are positive constants.

[4]The assumption is reasonable in such a competitive market. It is exactly the same as in Dixit and Pindyck (1994), Pawlina and Kort (2006), and Bouis et al. (2009).

For convergence, we assume that $\mu < r$ where $r > 0$ denotes a risk-free interest rate.[5]

We assume that it is necessary for the first firm to enter the market to have two types of facilities to provide the new-technology full-coverage network service in the market. One is a long-distance network facility. The other is a local network facility. We denote by I_i^{S} and I_i^{N} the one-time fixed cost expenditures of Firm i for the long-distance and local network facilities, respectively ($i \in \{\mathrm{A}, \mathrm{B}\}$). Here, we assume the following relationship among these cost expenditures.

Assumption 1 (Asymmetric cost structure between two firms):

(i) $I^{\mathrm{S}} = I_{\mathrm{A}}^{\mathrm{S}} = I_{\mathrm{B}}^{\mathrm{S}} > 0.$

(ii) $I_{\mathrm{B}}^{\mathrm{N}} > I_{\mathrm{A}}^{\mathrm{N}} > 0.$

Condition (i) in Assumption 1 implies that the incumbent (Firm A) has the same cost of technology for a long-distance network facility as the entrant (Firm B). Condition (ii) means that the incumbent has a cost advantage for a local network facility compared with the entrant. This assumption is justified as follows. Recent market liberalization has allowed new competitors in the telecommunications market. The incumbent (e.g., Nippon Telegraph and Telephone Corporation; hereafter NTT) enjoys several advantages compared with the entrant. Typically, it is said that the incumbent has a cost advantage over the entrant.

We assume that the second firm to enter the market does not necessarily invest in a local network facility. The firm that only invests in the long-distance network facility provides the new-technology full-coverage network service by using an existing local network facility and paying an access charge. For example, Firm i without a local network facility can use the existing local network facility that Firm i' provides by paying an access charge $\nu_{i'} > 0$ for any i ($i, i' \in \{\mathrm{A}, \mathrm{B}\}, i \neq i'$). Then Firm i' incurs the access cost $c > 0$. Here, ν_i and c are exogenous and constant for any i. We assume the following relationship among them.

Assumption 2 (Asymmetric access charge regulation between two firms):

$\nu_{\mathrm{B}} > \nu = \nu_{\mathrm{A}} > c > 0.$

This assumption implies that there is asymmetric access charge regulation in an asymmetric market environment in that the incumbent has a cost advantage. This assumption

[5]Often, μ is assumed to be $\mu \in (\sigma^2/2, r)$, where $\sigma^2/2 < \mu$ is necessary for a meaningful expected time to exercise the investment option. See Shibata (2009) for greater detail.

is considered reasonable in an asymmetric market environment. In policy discussions, the need for asymmetric regulation has been recognized in an asymmetric market environment (See, e.g., Peitz, 2005). Such asymmetric access charge regulation is in accordance with legislation in many developed countries. For example, the regulation of access in the European Union and Japan differ between operators with and without significant market power.

Throughout the paper, it is assumed that the current state variable $X(0) = x$ is sufficiently low so that the investment is not undertaken immediately. We call the firm that invests first the *leader* and the other firm the *follower.*

There are two types of competitions in a duopoly market: *service-based competition* in which the follower has only a long-distance network facility while accessing the leader's local network, and *facility-based competition* in which the follower has both long-distance and local network facilities.[6] Thus, the follower adopts one of the two alternative strategies: the *sequential investment* strategy by which the firm enters the market by accessing leader's local network and builds its own local network later, or the *simultaneous investment* strategy by which the firm enters the market to have both long-distance and local network facilities.

The value functions are derived backwards. We begin by deriving a follower's value function. We then provide a leader's value function, given the follower's strategy. In our model, we must derive the follower's and leader's value functions in two scenarios. Two alternative scenarios are composed of the follower's *sequential* and *simultaneous* investment strategies. The follower's *sequential investment* strategy is that the follower enters the market *only* to adopt its own long-distance network by accessing the leader's local network until the market demand is sufficiently large. The follower's *simultaneous investment* strategy is that the follower enters the market to have both its own long-distance and local networks. Now we denote the rival firm of Firm i as Firm i'. For example, we have $i' =$ B when $i =$ A and vice versa. The follower's optimal strategy depends on the magnitude of the follower's values under two strategies. Thus, it is the *sequential investment* strategy if the follower's value under this strategy is larger than under the *simultaneous investment* strategy and vice versa. In our model, let "Q" and "M" stand for the follower's *sequential investment* and *simultaneous investment* strategies, respectively.

[6] See Bourreau and Doğan (2004) for service-based and facility-based competition in greater detail.

2.2 Value functions under the sequential investment strategy

In this subsection, we consider the leader's and follower's value functions on the condition that the follower adopts the *sequential investment* strategy.

2.2.1 Follower's value functions

Let $x^{\mathrm{S}}_{\mathrm{F}i}$ and $x^{\mathrm{N}}_{\mathrm{F}i}$ be the follower's *long-distance network* and *local network* investment triggers, respectively, under the follower's *sequential investment* strategy. Here, the subscript "Fi" denotes for the Firm i's strategy as a follower, and the superscripts "S" and "N" denote the long-distance network and local network investments, respectively. Mathematically, we define the stopping times by $\tau^{j}_{\mathrm{F}i} = \inf\{t \geq 0; X(t) \geq x^{j}_{\mathrm{F}i}\}$ for any i and j $(i \in \{\mathrm{A}, \mathrm{B}\}; j \in \{\mathrm{S}, \mathrm{N}\})$.[7]

The follower's value function, $F^{\mathrm{Q}}_i(x)$, is defined by

$$F^{\mathrm{Q}}_i(x) = \sup_{\tau^{\mathrm{S}}_{\mathrm{F}i}, \tau^{\mathrm{N}}_{\mathrm{F}i}} \mathbb{E}\Big[\int_{\tau^{\mathrm{S}}_{\mathrm{F}i}}^{\tau^{\mathrm{N}}_{\mathrm{F}i}} \mathrm{e}^{-r(u-t)} (D_2 - \nu_{i'}) X(u) \mathrm{d}u + \int_{\tau^{\mathrm{N}}_{\mathrm{F}i}}^{+\infty} \mathrm{e}^{-r(u-t)} D_2 X(u) \mathrm{d}u - \mathrm{e}^{-r(\tau^{\mathrm{S}}_{\mathrm{F}i} - t)} I^{\mathrm{S}} - \mathrm{e}^{-r(\tau^{\mathrm{N}}_{\mathrm{F}i} - t)} I^{\mathrm{N}}_i | X(0) = x \Big], \quad (2)$$

for all i $(i, i' \in \{\mathrm{A}, \mathrm{B}\}, i \neq i')$ where $\mathbb{E}[\cdot | X(0) = x]$ denotes the conditional expectation operator given that $X(0) = x$. Let the superscript "Q" stand for the follower's *sequential investment* strategy. Note that the first term is the present value of the payoff in the *service-based competition duopoly* by adopting its long-distance network, the second term is the present value of the payoff in the *facility-based competition duopoly* by adopting its own local network, the third and fourth terms are the present values of the one-time costs of the long-distance and local networks, respectively.

Under the standard argument (see, e.g., Dixit and Pindyck, 1994), the follower's value function, $F^{\mathrm{Q}}_i(x)$, is given by

$$F^{\mathrm{Q}}_i(x) = \sup_{x^{\mathrm{S}}_{\mathrm{F}i}, x^{\mathrm{N}}_{\mathrm{F}i}} \Big(\frac{x}{x^{\mathrm{S}}_{\mathrm{F}i}}\Big)^{\beta} \Big\{ \frac{D_2 - \nu_{i'}}{r - \mu} x^{\mathrm{S}}_{\mathrm{F}i} - I^{\mathrm{S}} \Big\} + \Big(\frac{x}{x^{\mathrm{N}}_{\mathrm{F}i}}\Big)^{\beta} \Big\{ \frac{\nu_{i'}}{r - \mu} x^{\mathrm{N}}_{\mathrm{F}i} - I^{\mathrm{N}}_i \Big\}, \quad (3)$$

where $x < \min\{x^{\mathrm{S}}_{\mathrm{F}i}, x^{\mathrm{N}}_{\mathrm{F}i}\}$ and β is positive constant and strictly larger than 1, i.e.,

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{(\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + \frac{2r}{\sigma^2}} > 1. \quad (4)$$

[7] To more precise, the stopping times are defined by $\tau^{\mathrm{S}}_{\mathrm{F}i} = \inf\{t \geq 0; X(t) \geq x^{\mathrm{S}}_{\mathrm{F}i}, x^{\mathrm{N}}_{\mathrm{F}i} \geq x^{\mathrm{S}}_{\mathrm{F}i}\}$ and $\tau^{\mathrm{N}}_{\mathrm{F}i} = \inf\{t \geq 0; X(t) \geq x^{\mathrm{N}}_{\mathrm{F}i} > x^{\mathrm{S}}_{\mathrm{F}i}\}$ for any i $(i \in \{\mathrm{A}, \mathrm{B}\})$. We show below that the *sequential investment* strategies are not defined when $x^{\mathrm{S}}_{\mathrm{F}i} \geq x^{\mathrm{N}}_{\mathrm{F}i}$.

Then, the triggers that the follower adopts for the long-distance and local networks, respectively, are obtained by

$$x_{\mathrm{F}i}^{\mathrm{S}*} = \frac{\beta}{\beta-1}\frac{r-\mu}{D_2-v_{i'}}I^{\mathrm{S}}, \tag{5}$$

$$x_{\mathrm{F}i}^{\mathrm{N}*} = \frac{\beta}{\beta-1}\frac{r-\mu}{v_{i'}}I_i^{\mathrm{N}}, \tag{6}$$

for all i $(i, i' \in \{\mathrm{A}, \mathrm{B}\}, i \neq i')$. Note that we have $x_{\mathrm{FA}}^{\mathrm{S}*} > x_{\mathrm{FB}}^{\mathrm{S}*}$ because of asymmetric access charge regulation $\nu_{\mathrm{A}} < \nu_{\mathrm{B}}$, and $x_{\mathrm{FA}}^{\mathrm{N}*} < x_{\mathrm{FB}}^{\mathrm{N}*}$ because of asymmetric cost technology and access charge regulation, $I_{\mathrm{A}}^{\mathrm{N}} < I_{\mathrm{B}}^{\mathrm{N}}$ and $\nu_{\mathrm{A}} < \nu_{\mathrm{B}}$.

It is straightforward that (3) is rewritten as

$$F_i^{\mathrm{Q}}(x) = \begin{cases} \left(\frac{x}{x_{\mathrm{F}i}^{\mathrm{S}*}}\right)^{\beta}\left\{\frac{D_2-\nu_{i'}}{r-\mu}x_{\mathrm{F}i}^{\mathrm{S}*} - I^{\mathrm{S}}\right\} + \left(\frac{x}{x_{\mathrm{F}i}^{\mathrm{N}*}}\right)^{\beta}\left\{\frac{\nu_{i'}}{r-\mu}x_{\mathrm{F}i}^{\mathrm{N}*} - I_i^{\mathrm{N}}\right\}, & x < x_{\mathrm{F}i}^{\mathrm{S}*}, \\ \frac{D_2-\nu_{i'}}{r-\mu}x - I^S + \left(\frac{x}{x_{\mathrm{F}i}^{\mathrm{N}*}}\right)^{\beta}\left\{\frac{\nu_{i'}}{r-\mu}x_{\mathrm{F}i}^{\mathrm{N}*} - I_i^{\mathrm{N}}\right\}, & x_{\mathrm{F}i}^{\mathrm{S}*} \leq x < x_{\mathrm{F}i}^{\mathrm{N}*}, \\ \frac{D_2}{r-\mu}x - I^{\mathrm{S}} - I_i^{\mathrm{N}}, & x_{\mathrm{F}i}^{\mathrm{N}*} \leq x. \end{cases}$$

2.2.2 Leader's value functions

Let $x_{\mathrm{L}i}^{\mathrm{Q}}$ be the triggers for entry as a leader under the follower's *sequential investment* strategy. Here, the subscript "Li" stands for the Firm i's strategy as a leader. Mathematically, we define the stopping times by $\tau_{\mathrm{L}i}^{\mathrm{Q}} = \inf\{t \geq 0; X(t) \geq x_{\mathrm{L}i}^{\mathrm{Q}}\}$ for all i $(i \in \{\mathrm{A}, \mathrm{B}\})$.

The leader's value function, $L_i^{\mathrm{Q}}(x)$, is defined by

$$\begin{aligned} L_i^{\mathrm{Q}}(x) &= \sup_{\tau_{\mathrm{L}i}^{\mathrm{Q}}} \mathbb{E}\Big[\int_{\tau_{\mathrm{L}i}^{\mathrm{Q}}}^{\tau_{\mathrm{F}i'}^{\mathrm{S}*}} \mathrm{e}^{-r(u-t)}D_1X(u)\mathrm{d}u + \int_{\tau_{\mathrm{F}i'}^{\mathrm{S}*}}^{\tau_{\mathrm{F}i'}^{\mathrm{N}*}} \mathrm{e}^{-r(u-t)}(D_2+\nu_i-c)X(u)\mathrm{d}u \\ &\quad + \int_{\tau_{\mathrm{F}i'}^{\mathrm{N}*}}^{+\infty} \mathrm{e}^{-r(u-t)}D_2X(u)\mathrm{d}u - \mathrm{e}^{-r(\tau_{\mathrm{L}i}^{*}-t)}(I^S + I_i^{\mathrm{N}})|X(0)=x\Big], \end{aligned} \tag{7}$$

for all i $(i, i' \in \{\mathrm{A}, \mathrm{B}\}, i \neq i')$. As in Dixit and Pindyck (1994), the leader's value function, $L_i^{\mathrm{Q}}(x)$, is written as

$$\begin{aligned} L_i^{\mathrm{Q}}(x) &= \sup_{x_{\mathrm{L}i}^{\mathrm{Q}}} \left(\frac{x}{x_{\mathrm{L}i}^{\mathrm{Q}}}\right)^{\beta}\left\{\frac{D_1}{r-\mu}x_{\mathrm{L}i}^{\mathrm{Q}} - I^{\mathrm{S}} - I_i^{\mathrm{N}}\right\} + \left(\frac{x}{x_{\mathrm{F}i'}^{\mathrm{S}*}}\right)^{\beta}\left\{\frac{D_2-D_1+\nu_i-c}{r-\mu}x_{\mathrm{F}i'}^{\mathrm{S}*}\right\} \\ &\quad + \left(\frac{x}{x_{\mathrm{F}i'}^{\mathrm{N}*}}\right)^{\beta}\frac{c-\nu_i}{r-\mu}x_{\mathrm{F}i'}^{\mathrm{N}*}, \end{aligned} \tag{8}$$

where $x < x_{\mathrm{L}i}^{\mathrm{Q}}$. Here, the first term is the present value of the payoff in the *monopoly*, the second term is the present value of the payoff in the *service-based competition duopoly*, and the third term is the present value of the payoff in the *facility-based competition duopoly.*

Then, the trigger for the leader to undertake both the long-distance and local network investments is obtained by

$$x_{\mathrm{L}i}^{\mathrm{Q}*} = \frac{\beta}{\beta - 1}\frac{r-\mu}{D_1}(I^{\mathrm{S}} + I_i^{\mathrm{N}}), \tag{9}$$

for all i $(i, i' \in \{\mathrm{A}, \mathrm{B}\},\ i \neq i')$. Obviously we have $x_{\mathrm{LA}}^{\mathrm{Q}*} < x_{\mathrm{LB}}^{\mathrm{Q}*}$ due to $I_{\mathrm{A}}^{\mathrm{N}} < I_{\mathrm{B}}^{\mathrm{N}}$. Note that (8) is rewritten as

$$L_i^{\mathrm{Q}}(x) = \begin{cases} \left(\frac{x}{x_{\mathrm{L}i}^{\mathrm{Q}*}}\right)^{\beta}\left\{\frac{D_1}{r-\mu}x_{\mathrm{L}i}^{\mathrm{Q}*} - I^{\mathrm{S}} - I_i^{\mathrm{N}}\right\} & \\ +\left(\frac{x}{x_{\mathrm{F}i'}^{\mathrm{S}*}}\right)^{\beta}\left\{\frac{D_2 - D_1 + \nu_i - c}{r-\mu}x_{\mathrm{F}i'}^{\mathrm{S}*}\right\} + \left(\frac{x}{x_{\mathrm{F}i'}^{\mathrm{N}*}}\right)^{\beta}\frac{c-\nu_i}{r-\mu}x_{\mathrm{F}i'}^{\mathrm{N}*}, & x < x_{\mathrm{L}i}^{\mathrm{Q}*}, \\ \frac{D_1}{r-\mu}x - I^{\mathrm{S}} - I_i^{\mathrm{N}} + \left(\frac{x}{x_{\mathrm{F}i'}^{\mathrm{S}*}}\right)^{\beta}\frac{D_2 - D_1\nu_i - c}{r-\mu}x_{\mathrm{F}i'}^{\mathrm{S}*} & \\ +\left(\frac{x}{x_{\mathrm{F}i'}^{\mathrm{N}*}}\right)^{\beta}\frac{c-\nu_i}{r-\mu}x_{\mathrm{F}i'}^{\mathrm{N}*}, & x_{\mathrm{L}i}^{\mathrm{Q}*} \leq x < x_{\mathrm{F}i'}^{\mathrm{S}*}, \\ \frac{D_2 + \nu_i - c}{r-\mu}x - I^{\mathrm{S}} - I_i^{\mathrm{N}} + \left(\frac{x}{x_{\mathrm{F}i'}^{\mathrm{N}*}}\right)^{\beta}\frac{c-\nu_i}{r-\mu}x_{\mathrm{F}i'}^{\mathrm{N}*}, & x_{\mathrm{F}i'}^{\mathrm{S}*} \leq x < x_{\mathrm{F}i'}^{\mathrm{N}*} \\ \frac{D_2}{r-\mu}x - I^{\mathrm{S}} - I_i^{\mathrm{N}}, & x_{\mathrm{F}i'}^{\mathrm{N}*} \leq x. \end{cases}$$

for all i $(i, i' \in \{\mathrm{A}, \mathrm{B}\},\ i \neq i')$.

2.3 Value functions under the simultaneous investment strategy

In this subsection, we consider the follower's and leader's value functions on the condition that the follower adopts the *simultaneous investment* strategy.

2.3.1 Follower's value functions

The follower's value, $F_i^{\mathrm{M}}(x)$, is given by

$$\begin{aligned} F_i^{\mathrm{M}}(x) &= \sup_{\tau_{\mathrm{F}i}^{\mathrm{M}}} \mathbb{E}\Big[\int_{\tau_{\mathrm{F}i}^{\mathrm{M}}}^{+\infty} \mathrm{e}^{-r(u-t)}D_2X(u)\mathrm{d}u - \mathrm{e}^{-r(\tau_{\mathrm{F}i}^{\mathrm{M}}-t)}(I^{\mathrm{S}} + I_i^{\mathrm{N}})|X(0) = x\Big] \\ &= \left(\frac{x}{x_{\mathrm{F}i}^{\mathrm{M}*}}\right)^{\beta}\left\{\frac{D_2}{r-\mu}x_{\mathrm{F}i}^{\mathrm{M}*} - I^{\mathrm{S}} - I_i^{\mathrm{N}}\right\}, \end{aligned} \tag{10}$$

where $x < x_{\mathrm{F}i}^{\mathrm{M}*}$ and $x_{\mathrm{F}i}^{\mathrm{M}*}$ is obtained by

$$x_{\mathrm{F}i}^{\mathrm{M}*} = \frac{\beta}{\beta - 1}\frac{r-\mu}{D_2}(I^{\mathrm{S}} + I_i^{\mathrm{N}}). \tag{11}$$

2.3.2 Leader's value functions

We have the leader's value $L_i^{\mathrm{M}}(x)$ as

$$\begin{aligned} L_i^{\mathrm{M}}(x) &= \sup_{\tau_{\mathrm{L}i}^{\mathrm{M}}} \mathbb{E}\Big[\int_{\tau_{\mathrm{L}i}^{\mathrm{M}}}^{\tau_{\mathrm{F}i'}^{\mathrm{M}}} \mathrm{e}^{-r(u-t)} D_1 X(u) \mathrm{d}u + \int_{\tau_{\mathrm{F}i'}^{\mathrm{M}}}^{+\infty} \mathrm{e}^{-r(u-t)} D_2 X(u) \mathrm{d}u \\ &\qquad -\mathrm{e}^{-r(\tau_{\mathrm{L}i}^{\mathrm{M}}-t)} (I^{\mathrm{S}} + I_i^{\mathrm{N}}) | X(0) = x \Big] \\ &= \left(\frac{x}{x_{\mathrm{L}i}^{\mathrm{M}*}} \right)^{\beta} \left\{ \frac{D_2}{r-\mu} x_{\mathrm{L}i}^{\mathrm{M}*} - I^{\mathrm{S}} - I_i^{\mathrm{N}} \right\} + \left(\frac{x}{x_{\mathrm{F}i'}^{\mathrm{M}*}} \right)^{\beta} \left(\frac{D_2 - D_1}{r-\mu} x_{\mathrm{F}i'}^{\mathrm{M}*} \right), \end{aligned} \tag{12}$$

where $x < x_{\mathrm{L}i}^{\mathrm{M}*}$ and $x_{\mathrm{L}i}^{\mathrm{M}*}$ is obtained by

$$x_{\mathrm{L}i}^{\mathrm{M}*} = \frac{\beta}{\beta - 1} \frac{r-\mu}{D_1} (I^{\mathrm{S}} + I_i^{\mathrm{N}}), \quad i \in \{\mathrm{A}, \mathrm{B}\}. \tag{13}$$

Note that (13) is exactly the same as (9), i.e., $x_{\mathrm{L}i}^{\mathrm{M}*} = x_{\mathrm{L}i}^{\mathrm{Q}*}$. Hereafter, the triggers $x_{\mathrm{L}i}^{\mathrm{Q}*}$ and $x_{\mathrm{L}i}^{\mathrm{M}*}$ are written as $x_{\mathrm{L}i}^{*}$ for simplicity. Also, $x_{\mathrm{F}i'}^{\mathrm{M}*}$ is given by (11) for i' ($i' \in \{\mathrm{A}, \mathrm{B}\}$).

2.4 Relationship between the values and triggers as a follower

In this subsection, we investigate the relationship between the followers' values and triggers. We obtain the following lemma. The proof is given in the appendix.

Lemma 1 *Suppose $x_{\mathrm{F}i}^{\mathrm{S}*} < x_{\mathrm{F}i}^{\mathrm{N}*}$ for all i ($i \in \{\mathrm{A}, \mathrm{B}\}$). Then we have the following orderings $x_{\mathrm{F}i}^{\mathrm{S}*} < x_{\mathrm{F}i}^{\mathrm{M}*} < x_{\mathrm{F}i}^{\mathrm{N}*}$ and $F_i^{\mathrm{Q}}(x) \geq F_i^{\mathrm{M}}(x)$ where $x < x_{\mathrm{F}i}^{\mathrm{S}*}$ for all i.*

From Lemma 1, $x_{\mathrm{F}i}^{\mathrm{S}*} < x_{\mathrm{F}i}^{\mathrm{N}*}$ implies that Firm i's optimal strategy as a follower is the *sequential investment* strategy. In other words, if $x_{\mathrm{F}i}^{\mathrm{S}*} \geq x_{\mathrm{F}i}^{\mathrm{N}*}$ is satisfied, then Firm i's optimal strategy is the *simultaneous investment* strategy.

2.5 Non-strategic equilibrium (Benchmark case)

In this subsection, we consider as a benchmark the non-strategic cases. This problem is equivalent to that in which there is always a dominant firm (i.e., an incumbent with a great cost advantage).

Recall that $x_{\mathrm{LA}}^{*} < x_{\mathrm{LB}}^{*}$ because $I_{\mathrm{A}}^{\mathrm{N}}$ is sufficiently smaller than $I_{\mathrm{B}}^{\mathrm{N}}$. Because the entrant (Firm B) is always better off by acting as a follower, the incumbent (Firm A) enters the market without taking the entrant's strategy into consideration. This implies that there is no strategic relationship between two firms. Thus, we summarize non-strategic equilibrium as follows:

Lemma 2 *The incumbent (Firm A) enters the market as a leader to adopt the long-distance and local networks once the price process $X(t)$ starting at x arrives at x^*_{LA}, while the entrant (Firm B) enters the service-based competition market to adopt a long-distance network at $x^{\mathrm{S}*}_{\mathrm{FB}}$ if $x^{\mathrm{S}*}_{\mathrm{FB}} < x^{\mathrm{N}*}_{\mathrm{FB}}$, and the entrant (Firm B) enters a facility-based competition duopoly to adopt the long-distance and local networks at $x^{\mathrm{M}*}_{\mathrm{FB}}$ if $x^{\mathrm{S}*}_{\mathrm{FB}} \geq x^{\mathrm{N}*}_{\mathrm{FB}}$.*

3 Strategic equilibrium

We solve the firm's decision problem by working backwards using dynamic programming.

We first consider Firm i's problem as a follower after the rival firm (Firm i') enters the market. Firm i's optimization problem as a follower is deciding which of the sequential and simultaneous investment strategies to adopt and choosing its investment timing strategy. The problem, $\max\{F_i^{\mathrm{Q}}(x), F_i^{\mathrm{M}}(x)\}$, can be simplified as

$$\max\{x^{\mathrm{S}*}_{\mathrm{F}i}, x^{\mathrm{N}*}_{\mathrm{F}i}\}, \tag{14}$$

because of Lemma 1. Also, it is clear that $L_i^{\mathrm{Q}}(x) > L_i^{\mathrm{M}}(x)$ when $F_i^{\mathrm{Q}}(x) > F_i^{\mathrm{M}}(x)$ for all i ($i \in \{\mathrm{A}, \mathrm{B}\}$), and vice versa.[8]

Next, we investigate Firm i's problem as a leader. Then, the important thing is whether Firm i has an incentive to become a leader. If so, Firm i must consider the fact that Firm i' will aim to preempt it as soon as a certain trigger is reached. These triggers, $x^{\mathrm{Q}*}_{\mathrm{P}i}$ and $x^{\mathrm{M}*}_{\mathrm{P}i}$, are the lowest realization of the process $X(t)$, in which Firm i is indifferent between being the leader and the follower. Here, let the superscript "Pi" denote the Firm i's *preemptive investment* strategy. Mathematically, $x^{\mathrm{Q}*}_{\mathrm{P}i}$ and $x^{\mathrm{M}*}_{\mathrm{P}i}$ are defined by

$$x^{\mathrm{Q}*}_{\mathrm{P}i} := \inf\{x > 0 : L_i^{\mathrm{Q}}(x) \geq F_i^{\mathrm{Q}}(x) = \max\{F_i^{\mathrm{Q}}(x), F_i^{\mathrm{M}}(x)\}\}, \tag{15}$$

$$x^{\mathrm{M}*}_{\mathrm{P}i} := \inf\{x > 0 : L_i^{\mathrm{M}}(x) \geq F_i^{\mathrm{M}}(x) = \max\{F_i^{\mathrm{Q}}(x), F_i^{\mathrm{M}}(x)\}\}, \tag{16}$$

for all i ($i \in \{\mathrm{A}, \mathrm{B}\}$). Importantly, $x^{k*}_{\mathrm{P}i}$ does *not always* exist for all i and k ($i \in \{\mathrm{A}, \mathrm{B}\}; k \in \{\mathrm{Q}, \mathrm{M}\}$). Suppose, for example, $\max\{L_i^{\mathrm{Q}}(x), L_i^{\mathrm{M}}(x)\} \leq \max\{F_i^{\mathrm{Q}}(x), F_i^{\mathrm{M}}(x)\}$ for all values of x. Then $x^{k*}_{\mathrm{P}i}$ does not exist. As a result, we show that there exists $x^{k*}_{\mathrm{P}i} \in (0, x^{k*}_{\mathrm{F}i})$ such that $\max\{L_i^{\mathrm{Q}}(x), L_i^{\mathrm{M}}(x)\} \geq \max\{F_i^{\mathrm{Q}}(x), F_i^{\mathrm{M}}(x)\}$ for some values of x ($x \in (0, x^{k*}_{\mathrm{F}i})$). The proof is that value functions L_i^k and F_i^k are continuous with x, subject to $L_i^k(0) < 0 = F_i^k(0)$ for all i and k ($i \in \{\mathrm{A}, \mathrm{B}\}$, $k \in \{\mathrm{Q}, \mathrm{M}\}$). These triggers, $x^{\mathrm{Q}*}_{\mathrm{P}i}$ and $x^{\mathrm{M}*}_{\mathrm{P}i}$, are called the preemption triggers, at which Firm i is indifferent between the payoffs of acting as a leader or a follower. Here, it is straightforward to obtain the following result, because of the definition of preemption.

[8]For any i ($i \in \{\mathrm{A}, \mathrm{B}\}$), we have $L_i^{\mathrm{Q}}(x) = \max\{L_i^{\mathrm{Q}}(x), L_i^{\mathrm{M}}(x)\}$ when $F_i^{\mathrm{Q}}(x) = \max\{F_i^{\mathrm{Q}}(x), F_i^{\mathrm{M}}(x)\}$.

Lemma 3 *Suppose that there exists* $x^{k*}_{\mathrm{P}i}$ *for all* i *and* k *(*$i \in \{\mathrm{A}, \mathrm{B}\}; k \in \{\mathrm{Q}, \mathrm{M}\}$*). Then, obviously, we have*

$$x^{k*}_{\mathrm{P}i} < x^{*}_{\mathrm{L}i}. \tag{17}$$

for all i *and* k *(*$i \in \{\mathrm{A}, \mathrm{B}\}; k \in \{\mathrm{Q}, \mathrm{M}\}$*).*

In the next subsection, we consider strategic equilibrium with asymmetric cost technologies and asymmetric regulation between two firms. Strategic optimal strategies for leaders in equilibrium are attributed to the optimal strategies of followers. There are three cases. The first is that the optimal strategies as a follower for the incumbent (Firm A) and the entrant (Firm B) are sequential investment strategies. The second is that the optimal strategies as a follower for the incumbent and the entrant are the sequential and simultaneous investment strategies, respectively. The third is that the optimal strategies as followers for the incumbent and the entrant are the simultaneous investment strategies.

3.1 Equilibrium when $F^{\mathrm{Q}}_i(x) \geq F^{\mathrm{M}}_i(x)$ for all i

In this subsection, we consider equilibrium on the grounds that the optimal strategies for a follower are *sequential investment* strategies for all i ($i \in \{\mathrm{A}, \mathrm{B}\}$). This problem is equivalent to the problem in which the inequalities, $x^{\mathrm{S}}_{\mathrm{F}i} < x^{\mathrm{N}}_{\mathrm{F}i}$ (i.e., $F^{\mathrm{Q}}_i(x) > F^{\mathrm{M}}_i(x)$), are satisfied for all i ($i \in \{\mathrm{A}, \mathrm{B}\}$). These inequalities are given by

$$\frac{D_2 - \nu_{i'}}{v_{i'}} > \frac{I^{\mathrm{S}}}{I^{\mathrm{N}}_i}, \tag{18}$$

for all i ($i, i' \in \{\mathrm{A}, \mathrm{B}\}; i \neq i'$). The fact that these two inequalities for the incumbent (Firm A) and the entrant (Firm B) in (18) are satisfied is equivalent to

$$\frac{D_2 - \nu_{\mathrm{B}}}{v_{\mathrm{B}}} > \frac{I^{\mathrm{S}}}{I^{\mathrm{N}}_{\mathrm{A}}}. \tag{19}$$

The proof is easily shown because $D_2 > \nu_{\mathrm{B}} > \nu_{\mathrm{A}} \geq 0$, $I^{\mathrm{N}}_{\mathrm{B}} > I^{\mathrm{N}}_{\mathrm{A}} \geq 0$ and $I^{\mathrm{S}} > 0$.[9]

The optimal investment strategies depend on the existence and the magnitude of the preemptive triggers. We summarize the results as follows.

Proposition 1 *Suppose that inequality (19) is satisfied. Then we have*

[9]Clearly, we have the following orderings

$$\frac{D_2 - \nu_{\mathrm{A}}}{v_{\mathrm{A}}} > \frac{D_2 - \nu_{\mathrm{B}}}{v_{\mathrm{B}}} > \frac{I^{\mathrm{S}}}{I^{\mathrm{N}}_{\mathrm{A}}} > \frac{I^{\mathrm{S}}}{I^{\mathrm{N}}_{\mathrm{B}}}.$$

That is, if (18) for the incumbent (Firm A) is satisfied, (18) for the entrant (Firm B) is automatically satisfied.

(i) If there exists a unique trigger $x_{\rm PB}^{\rm Q*}$ such that $x_{\rm PB}^{\rm Q*} < x_{\rm PA}^{\rm Q*}$, the entrant (Firm B) invests two networks once the price process $X(t)$ starting at x arrives at $x_{\rm PA}^{\rm Q*}$, the incumbent (Firm A) invests in long-distance and local networks at $x_{\rm FA}^{\rm S*}$ and $x_{\rm FA}^{\rm N*}$, respectively.

(ii) If there exist unique triggers $x_{\rm PA}^{\rm Q*}$ and $x_{\rm PB}^{\rm Q*}$ such that $x_{\rm PA}^{\rm Q*} < x_{\rm PB}^{\rm Q*} < x_{\rm LA}^{*}$, the incumbent (Firm A) invests in two networks at $x_{\rm PB}^{\rm Q*}$, and the entrant (Firm B) invests in long-distance and local networks at $x_{\rm FB}^{\rm S*}$ and $x_{\rm FB}^{\rm N*}$, respectively.

(iii) Otherwise, the incumbent (Firm A) invests in two networks at $x_{\rm LA}^{*}$, and the entrant (Firm B) invests in long-distance and local networks at $x_{\rm FB}^{\rm S*}$ and $x_{\rm FB}^{\rm N*}$, respectively.

Most interestingly in Proposition 1, the entrant with the cost disadvantage (i.e., Firm B) has the possibility of adopting two networks as a leader under asymmetric access charge regulation.

For the results presented below, we choose appropriate parameters to reflect the practice in the Japanese telecommunications industry. The parameters are $D_1 = 8$, $D_2 = 4$, $\sigma = 0.2$, $r = 0.09$, $\mu = 0.04$, $I^{\rm S} = 10$, $I_{\rm A}^{\rm N} = 20$, $I_{\rm B}^{\rm N} = 21$, $\nu_{\rm A} = 2$, $\nu_{\rm B} = 2.5$, and $c = 1.5$. Under these parameters, note that (19) is satisfied, i.e., $x_{{\rm F}i}^{\rm S} < x_{{\rm F}i}^{\rm N}$ and $F_i^{\rm Q}(x) > F_i^{\rm M}(x)$ in for all i ($i \in \{{\rm A}, {\rm B}\}$).

Table 1 shows the numerical results. Importantly, the preemption triggers $x_{\rm PA}^{\rm Q*}$ and $x_{\rm PB}^{\rm Q*}$ exist under the above parameters.

[Insert Table 1 about here]

Figure 1 demonstrates the value functions with respect to the state variable. The left and right panels of Figure 1 depict the leader's and follower's value functions for the incumbent (Firm A) and the entrant (Firm B), respectively. The investment triggers for the incumbent (Firm A) and the entrant (Firm B) in Figure 1 are summarized in Table 1.

[Insert Figure 1 about here]

Most importantly, we have $x_{\rm PA}^{\rm Q*} = 0.3754 > 0.3341 = x_{\rm PB}^{\rm Q*}$ under the above parameters. Then at equilibrium, the entrant with a cost disadvantage (Firm B) enters the new market as a leader at $x_{\rm PA}^{\rm Q*} = 0.3754$ while the incumbent with a cost advantage (Firm A) enters the market as a follower in the service-based competition duopoly at $x_{\rm FA}^{\rm S*} = 0.8239$. Thus, under asymmetric regulation of two asymmetric firms, the entrant with a cost disadvantage adopts the investment as a leader in the market.

3.2 Equilibrium when not $F_i^{\mathrm{Q}}(x) \geq F_i^{\mathrm{M}}(x)$ for all i

In this subsection, we assume that (18) is not satisfied. Then there are two other cases. One is that (18) for the incumbent (Firm A) is not satisfied and (18) for the entrant (Firm B) is satisfied. The other is that (18) for the incumbent (Firm A) and the entrant (Firm B) is not satisfied. In this subsection, we consider these two cases.

3.2.1 Equilibrium when $F_{\mathrm{A}}^{\mathrm{Q}}(x) < F_{\mathrm{A}}^{\mathrm{M}}(x)$ and $F_{\mathrm{B}}^{\mathrm{Q}}(x) \geq F_{\mathrm{B}}^{\mathrm{M}}(x)$

In this first case, the optimal strategies as a follower for the incumbent (Firm A) and the entrant (Firm B) are the *simultaneous investment* and *sequential investment* strategies, respectively. We then have the following result.

Corollary 1 *Suppose that the inequality (18) for the entrant (Firm B) is satisfied. Then we have*

(i) If there exist unique triggers $x_{\mathrm{PA}}^{\mathrm{Q}*}$ and $x_{\mathrm{PB}}^{\mathrm{M}*}$ such that $x_{\mathrm{PA}}^{\mathrm{Q}*} < x_{\mathrm{PB}}^{\mathrm{M}*} < x_{\mathrm{LA}}^{*}$, the incumbent (Firm A) invests in two networks once the price process $X(t)$ starting at x arrives at $x_{\mathrm{PB}}^{\mathrm{M}*}$, and the entrant (Firm B) invests in long-distance and local networks at $x_{\mathrm{FB}}^{\mathrm{S}*}$ and $x_{\mathrm{FB}}^{\mathrm{N}*}$, respectively.

(ii) Otherwise, the incumbent (Firm A) invests in two networks at x_{LA}^{*}, and the entrant (Firm B) invests in long-distance and local networks at $x_{\mathrm{FB}}^{\mathrm{S}*}$ and $x_{\mathrm{FB}}^{\mathrm{N}*}$, respectively.

In this case, the entrant with a cost disadvantage (i.e., Firm B) never enters the market as a leader.

3.2.2 Equilibrium when $F_{\mathrm{A}}^{\mathrm{Q}}(x) < F_{\mathrm{A}}^{\mathrm{M}}(x)$ and $F_{\mathrm{B}}^{\mathrm{Q}}(x) < F_{\mathrm{B}}^{\mathrm{M}}(x)$

In this second case, the optimal follower strategies for the incumbent and the entrant are *simultaneous investment* strategies. This equilibrium is the same as in Huisman (2001), Nielsen (2002), Pawlina and Kort (2006), Kong and Kwok (2007), and Kijima and Shibata (2009).[10] This problem is equivalent to the problem in which the inequality $F_i^{\mathrm{Q}}(x) < F_i^{\mathrm{M}}(x)$ is satisfied, i.e.,

$$x_{\mathrm{F}i}^{\mathrm{S}*} > x_{\mathrm{F}i}^{\mathrm{N}*}, \tag{20}$$

for all i ($i \in \{\mathrm{A}, \mathrm{B}\}$).

[10]See these papers in greater detail. We have omitted the numerical examples in this situation.

There is always $x_{\mathrm{PA}}^{\mathrm{M}*}$ while there is not always $x_{\mathrm{PB}}^{\mathrm{M}*}$. We always have

$$x_{\mathrm{PA}}^{\mathrm{M}*} < x_{\mathrm{PB}}^{\mathrm{M}*}. \tag{21}$$

on the grounds that there is $x_{\mathrm{PB}}^{\mathrm{M}*}$. These results imply that the incumbent (Firm A) always enters the market as a leader.

Corollary 2 *Suppose that the inequality (20) is satisfied for all i ($i \in \{\mathrm{A}, \mathrm{B}\}$). Then we have*

(i) If there exists $x_{\mathrm{PB}}^{\mathrm{M}*}$ such that $x_{\mathrm{PB}}^{\mathrm{M}*} < x_{\mathrm{LA}}^{*}$, the incumbent (Firm A) invests in two networks once the price process $X(t)$ starting at x arrives at $x_{\mathrm{PB}}^{\mathrm{M}*}$, and the entrant (Firm B) invests in them at $x_{\mathrm{FB}}^{\mathrm{M}*}$.

(ii) If there exists $x_{\mathrm{PB}}^{\mathrm{M}*}$ such that $x_{\mathrm{PB}}^{\mathrm{M}*} \geq x_{\mathrm{LA}}^{*}$, the incumbent (Firm A) invests in two networks at x_{LA}^{*}, and the entrant (Firm B) invests in them at $x_{\mathrm{FB}}^{\mathrm{M}*}$.

(iii) If there is not $x_{\mathrm{PB}}^{\mathrm{M}*}$, the incumbent (Firm A) invests in two networks at x_{LA}^{*}, and the entrant (Firm B) invests in them at $x_{\mathrm{FB}}^{\mathrm{M}*}$.

Other than on the grounds that the optimal follower strategies for both the incumbent (Firm A) and the entrant (Firm B) are the sequential investment strategies, the entrant with a cost disadvantage never enters the market as a leader. That is to say, the incumbent with a cost advantage always enters the market as a leader.

3.3 Effect of asymmetric regulation

To gain more insight into the solution, we consider the effect of asymmetric access charge regulation with respect to the investment strategies of two firms.

[Insert Figure 2 about here]

Figure 2 demonstrates the effects on the investment triggers of asymmetric access charge regulation. The left and right panels of Figure 2 depict the investment triggers of a leader and a follower, respectively. Here, the access charge ν_{B} is changed from 2.00 to 3.10 for fixed ν_{A} ($\nu_{\mathrm{A}} = 2.0$).[11] The other parameters are exactly the same as in the previous subsection, i.e., $D_1 = 8$, $D_2 = 4$, $\sigma = 0.2$, $r = 0.09$, $\mu = 0.04$, $I^{\mathrm{S}} = 10$, $I_{\mathrm{A}}^{\mathrm{N}} = 20$, $I_{\mathrm{B}}^{\mathrm{N}} = 21$, and $c = 1.5$. Under these parameters, there exists an incumbent's (Firm A's) preemptive investment trigger $x_{\mathrm{PA}}^{\mathrm{Q}*}$ such that $x_{\mathrm{PA}}^{\mathrm{A}*} < x_{\mathrm{LB}}^{*}$ for all ν_{B} while there

[11] To clarify the panels of Figure 2, ν_{B} is defined from 1.90 to 3.10.

exits an entrant's (Firm B's) preemptive investment trigger $x_{\mathrm{PB}}^{\mathrm{Q}*}$ such that $x_{\mathrm{PB}}^{\mathrm{A}*} < x_{\mathrm{LA}}^{*}$ for $\nu_{\mathrm{B}} \in (2.04, 2.66)$. The fact that there is no $x_{\mathrm{PB}}^{\mathrm{Q}*}$ for $\nu_{\mathrm{B}} > 2.66$ in the left panel is because of $x_{\mathrm{FA}}^{\mathrm{S}*} > x_{\mathrm{FA}}^{\mathrm{N}*}$ in the right panel. That is, when $x_{\mathrm{FA}}^{\mathrm{S}*} > x_{\mathrm{FA}}^{\mathrm{N}*}$, there is no $x_{\mathrm{PB}}^{\mathrm{Q}*}$ because the incumbent's (Firm A's) optimal strategy as a follower is to adopt the *simultaneous investment* strategy.

In the left panel, there are two important properties. The first property is the existence of the investment triggers in the left panel. For all ν_{B}, there exists $x_{\mathrm{PA}}^{\mathrm{Q}*}$, which has a U-shaped curve with ν_{B}. For $\nu_{\mathrm{B}} \in (2.04, 2.66)$, there exists $x_{\mathrm{PB}}^{\mathrm{Q}*}$ which is decreasing with ν_{B}. On the other hand, there are always nonpreemption investment triggers x_{LA}^{*} and x_{LB}^{*}, which are constant with ν_{B}. The second property is the magnitude of the investment triggers. We have $x_{\mathrm{PA}}^{\mathrm{Q}*} < x_{\mathrm{PB}}^{\mathrm{Q}*}$ for $\nu_{\mathrm{B}} \in (2.04, 2.15)$ while $x_{\mathrm{PA}}^{\mathrm{Q}*} > x_{\mathrm{PB}}^{\mathrm{Q}*}$ for $\nu_{\mathrm{B}} \in (2.15, 2.66)$. Note that we consider the magnitude of these only for $\nu_{\mathrm{B}} \in (2.04, 2.66)$ because there is no $x_{\mathrm{PB}}^{\mathrm{Q}*}$ for $\nu_{\mathrm{B}} \notin (2.04, 2.66)$. On the other hand, we can see $x_{\mathrm{LA}}^{*} < x_{\mathrm{LB}}^{*}$ for all ν_{B}. Consequently, we consider the investment strategies as a leader. For $\nu_{\mathrm{B}} \in (1.90, 2.04]$ and $\nu_{\mathrm{B}} \geq 2.66$, the incumbent (Firm A) invests as a leader at x_{LA}^{*} This is because there does not exist $x_{\mathrm{PB}}^{\mathrm{Q}*}$ for these regions. For $\nu_{\mathrm{B}} \in (2.04, 2.15)$, the incumbent (Firm A) invests as a leader at $x_{\mathrm{PB}}^{\mathrm{Q}*}$ due to $x_{\mathrm{PA}}^{\mathrm{Q}*} < x_{\mathrm{PB}}^{\mathrm{Q}*}$. For $\nu_{\mathrm{B}} = 2.15$, each firm invests as a leader at $x_{\mathrm{PA}}^{\mathrm{Q}*} = x_{\mathrm{PB}}^{\mathrm{Q}*}$ with probabilities one half. For $\nu_{\mathrm{B}} \in (2.15, 2.66)$, the entrant (Firm B) invests as a leader at $x_{\mathrm{PA}}^{\mathrm{Q}*}$ because of $x_{\mathrm{PA}}^{\mathrm{Q}*} > x_{\mathrm{PB}}^{\mathrm{Q}*}$. Interestingly, when ν_{B} is sufficiently large, although the entrant (Firm B) seems to have an asymmetric regulation advantage, the entrant (Firm B) never enjoys it. This is because the incumbent (Firm A) never accesses the entrant's local network.

In the right panel, we can see that $x_{\mathrm{FB}}^{\mathrm{S}*}$, $x_{\mathrm{FB}}^{\mathrm{N}*}$, $x_{\mathrm{FB}}^{\mathrm{M}*}$, and $x_{\mathrm{FA}}^{\mathrm{M}*}$ are constant with ν_{B}. On the other hand, $x_{\mathrm{FA}}^{\mathrm{S}*}$ and $x_{\mathrm{FA}}^{\mathrm{N}*}$ are increasing and decreasing in ν_{B}, respectively. As shown in Lemma 1, we have $x_{\mathrm{FA}}^{\mathrm{S}*} < x_{\mathrm{FA}}^{\mathrm{M}*} < x_{\mathrm{FA}}^{\mathrm{N}*}$ for $\nu_{\mathrm{B}} < 2.66$, $x_{\mathrm{FA}}^{\mathrm{S}*} = x_{\mathrm{FA}}^{\mathrm{M}*} = x_{\mathrm{FA}}^{\mathrm{N}*}$ for $\nu_{\mathrm{B}} = 2.66$, and $x_{\mathrm{FA}}^{\mathrm{S}*} > x_{\mathrm{FA}}^{\mathrm{M}*} > x_{\mathrm{FA}}^{\mathrm{N}*}$ for $\nu_{\mathrm{B}} > 2.66$. For $\nu_{\mathrm{B}} \leq 2.15$ and $\nu_{\mathrm{B}} \geq 2.66$, at equilibrium, the entrant (Firm B) invests in long-distance and local networks at $x_{\mathrm{FB}}^{\mathrm{S}*}$ and $x_{\mathrm{FB}}^{\mathrm{N}*}$, respectively. For $\nu_{\mathrm{B}} \in (2.15, 2.66)$, the incumbent (Firm A) invests in long-distance and local networks at $x_{\mathrm{FA}}^{\mathrm{S}*}$ and $x_{\mathrm{FA}}^{\mathrm{N}*}$, respectively.[12]

In summary, the incumbent (Firm A) invests as a leader for $\nu_{\mathrm{B}} \leq 2.15$ and $\nu_{\mathrm{B}} \geq 2.66$, while the entrant (Firm B) invests as a leader for $\nu_{\mathrm{B}} \in (2.15, 2.66)$. Most importantly, for $\nu_{\mathrm{B}} \in (2.04, 2.66)$, the larger the asymmetric access charge, the smaller the leader's

[12]For $\nu_{\mathrm{B}} = 2.15$, to more precise, two firms (i.e., Firms A and B) have an incentive to enter the market as a leader at $x_{\mathrm{FA}}^{\mathrm{Q}*} = x_{\mathrm{FB}}^{\mathrm{Q}*}$. Because the two firms are then completely symmetric under asymmetric regulation, each firm enters the market with probability one half. This discussion is the same as in Weeds (2002), Huisman and Kort (2004), and Nishihara and Shibata (2009).

investment trigger. The result is the same as in Hori and Mizuno (2006). For $\nu_{\rm B} \leq 2.04$ and $\nu_{\rm B} \geq 2.66$, however, the investment trigger is constant at $x^*_{\rm LA}$. When the asymmetric access charge is too small or large, the investment trigger turns out to be constant. These results are consistent with the empirical results.

Figure 3 depicts the incumbent's (Firm A's) and the entrant's (Firm B's) values with respect to $\nu_{\rm B}$ at equilibrium. Here, if there exists $x^{\rm Q*}_{{\rm P}i}$ such that $x^{\rm Q*}_{{\rm P}i} < x^*_{{\rm L}i'}$, $L^{\rm Q}_i(x)$ is defined as $L^{\rm P}_i(x)$ where

$$\begin{aligned} L^{\rm P}_i(x) &= \left(\frac{x}{x^*_{{\rm P}i}}\right)^\beta \left\{\frac{D_1}{r-\mu}x^*_{{\rm P}i} - I^{\rm S} - I^{\rm N}_i\right\} + \left(\frac{x}{x^{\rm S*}_{{\rm F}i'}}\right)^\beta \left\{\frac{D_2 - D_1 + \nu_i - c}{r-\mu}x^{\rm S*}_{{\rm F}i'}\right\} \\ &\quad + \left(\frac{x}{x^{\rm N*}_{{\rm F}i'}}\right)^\beta \frac{c-\nu_i}{r-\mu}x^{\rm N*}_{{\rm F}i'}, \end{aligned} \tag{22}$$

where $x < x^*_{{\rm P}i}$ for all i $(i, i' \in \{{\rm A}, {\rm B}\}; i \neq i')$. If there does not exist $x^{\rm Q*}_{{\rm P}i}$ such that $x^{\rm Q*}_{{\rm P}i} < x^*_{{\rm L}i'}$, $L^{\rm Q}_i(x)$ is given as (8). Under the above parameters, the incumbent's (Firm A's) value is $L^{\rm Q}_{\rm A}(x)$ if $\nu_{\rm B} \leq 2.06$ and $\nu_{\rm B} \geq 2.66$, $L^{\rm P}_{\rm A}(x)$ if $2.06 \leq \nu_{\rm B} \leq 2.15$, and $F^{\rm Q}_{\rm A}(x)$ if $2.15 \leq \nu_{\rm B} \leq 2.66$. The entrant's (Firm B's) value is $F^{\rm Q}_{\rm B}(x)$ if $\nu_{\rm B} \leq 2.15$ and $\nu_{\rm B} \geq 2.66$ and $L^{\rm P}_{\rm B}(x)$ if $2.15 \leq \nu_{\rm B} \leq 2.66$.

When $\nu_{\rm B} = 2.00$, the access charge regulation turns out be *symmetric*. Then, the incumbent's (Firm A's) value is larger than the entrant's (Firm B's) because of the asymmetric cost structure $I^{\rm N}_{\rm A} < I^{\rm N}_{\rm B}$. When $\nu_{\rm B} > 2.00$, the access charge regulation is regarded as *asymmetric*. Naturally, for $\nu_{\rm B} \in (2.00, 2.66)$, the incumbent's (Firm A's) value is monotonically decreasing in $\nu_{\rm B}$ while the entrant's (Firm B's) value is monotonically increasing in $\nu_{\rm B}$. For $\nu_{\rm B} \in (2.00, 2.66)$, an increase in $\nu_{\rm B}$ shifts wealth from the incumbent (Firm A) to the entrant (Firm B). This possibility of transferring wealth is known as "asset substitution" from the incumbent (Firm A) to the entrant (Firm B) via asymmetric access charge regulation. Also, there are two turning points at $\nu_{\rm B} = 2.15$ and $\nu_{\rm B} = 2.66$ where the strategies of Firms A and B as a leader and a follower, respectively, interchange. At $\nu_{\rm B} = 2.15$, the incumbent's (Firm A's) and the entrant's (Firm B's) values are the same because both firms are symmetric under asymmetric regulation. That is, the competitive environment between the two firms is completely *symmetric*. At $\nu_{\rm B} = 2.66$, on the other hand, the incumbent's (Firm A's) value jumps while the entrant's (Firm B's) value drops. For $\nu_{\rm B} > 2.66$, the incumbent's (Firm A's) and the entrant's (Firm B's) values are constant at $\nu_{\rm B}$.

As a result, the incumbent (Firm A) has a competitive advantage at $\nu_{\rm B} < 2.15$ and $\nu_{\rm B} \geq 2.66$, while the entrant (Firm B) has a competitive advantage at $\nu_{\rm B} \in (2.15, 2.66)$. The incumbent's (Firm A's) and the entrant's (Firm B's) competitive environment is completely *symmetric* at $\nu_{\rm B} = 2.15$.

Most interestingly, when the asymmetric access charge is too large (i.e., $\nu_{\mathrm{B}} \geq 2.66$), the incumbent (Firm A) becomes a dominant firm. We conclude that the competitive market environment weakens when the asymmetric access charge is too large. The implied suggestion for the regulation authority is that too severe an asymmetric access charge should not be imposed to protect a newly entrant under an asymmetric competitive market.

3.4 Efficiency in welfare

In this subsection, we consider efficiency in welfare with respect to asymmetric access charge regulation. In the economics literature, the measure of efficiency in welfare called the "total surplus" is defined by the sum of the "consumer surplus" and "producer surplus."

First, the consumer is *explicitly* not considered in our model.[13] However, the utility of the consumer depends on the speed that the investment is adopted. That is, the earlier the investment is adopted, the larger the utility is. This is because the consumer can enjoy the service once the investment is adopted. From this viewpoint, just before $\nu_{\mathrm{B}} = 2.66$, the preemptive investment trigger is lowest (see the left panel of Figure 2). We conclude that consumers then enjoy the highest surplus at just before $\nu_{\mathrm{B}} = 2.66$.

Second, we consider the producer surplus defined by the sum of the incumbent's (Firm A's) and the entrant's (Firm B's) values.[14] Under the basic parameters, the producers surplus, $W(x)$, is

$$W(x) = \begin{cases} L_{\mathrm{A}}^{\mathrm{Q}}(x) + F_{\mathrm{B}}^{\mathrm{Q}}(x), & \text{if } \nu_B \leq 2.04, \\ L_{\mathrm{A}}^{\mathrm{P}}(x) + F_{\mathrm{B}}^{\mathrm{Q}}(x), & \text{if } 2.04 \leq \nu_B \leq 2.15, \\ L_{\mathrm{B}}^{\mathrm{P}}(x) + F_{\mathrm{A}}^{\mathrm{Q}}(x), & \text{if } 2.15 \leq \nu_B \leq 2.66, \\ L_{\mathrm{A}}^{\mathrm{Q}}(x) + F_{\mathrm{B}}^{\mathrm{Q}}(x), & \text{otherwise.} \end{cases} \tag{23}$$

Figure 3 depicts the producer surplus with respect to ν_{B}. Note that the producers surplus is decreasing in ν_{B} for $\nu_B < 2.15$ and increasing in ν_{B} for $\nu_{\mathrm{B}} \geq 2.15$. Thus, the producers surplus is highest for $\nu_{\mathrm{B}} \geq 2.66$, and lowest for $\nu_{\mathrm{B}} = 2.15$ when the competitive environment between two firms is completely symmetric under asymmetric access charge regulation. Here, the fact that the producer surplus is lowest when the two firms are completely symmetric fits well with the findings of previous theoretical and empirical work.

[13] To consider consumer surplus, a value function of consumer is required.

[14] This definition is equivalent to the total social surplus because the consumer surplus is considered in our model. See Shibata (2009) and Shibata and Nishihara (2009) for a similar definition.

Thus, we investigate efficiency in welfare via the total surplus defined by the sum of the consumer and producer surplus. The discussion is thus standard in the microeconomics literature. For $\nu_{\mathrm{B}} < 2.66$, the total surplus is highest at just below $\nu_{\mathrm{B}} = 2.66$. For $\nu_{\mathrm{B}} \geq 2.66$, the total surplus is constant. We cannot discuss here whether the asymmetric access charge is imposed at more than $\nu_{\mathrm{B}} = 2.66$. Of course, that depends on the magnitude of the gap between the decrease in the consumer surplus and the increase in the producer surplus.

4 Concluding remarks

This paper extends the static model developed by Peitz (2005) to the dynamic model. We consider the effects of asymmetric access charge regulation on a preemptive investment strategy in an asymmetric market environment. We show that an entrant with a cost disadvantage has an incentive to invest as a leader under asymmetric regulation. Our results fit well with the findings of previous work.

Some extensions of the model would prove interesting. For example, because regulation may be eliminated once the entrant overcomes several disadvantages compared with the incumbent, it would be interesting to include the effects of deregulation with uncertainty. Then two asymmetric firms must adopt the investment, considering that a firm would face deregulation with uncertainty after investment.

Appendix

Proof of Lemma 1 First, we show that $x_{\mathrm{F}i}^{\mathrm{S}*} < x_{\mathrm{F}i}^{\mathrm{N}*}$ implies $x_{\mathrm{F}i}^{\mathrm{S}*} < x_{\mathrm{F}i}^{\mathrm{M}*} < x_{\mathrm{F}i}^{\mathrm{N}*}$ for all i ($i \in \{\mathrm{A}, \mathrm{B}\}$). Suppose $x_{\mathrm{F}i}^{\mathrm{S}*} < x_{\mathrm{F}i}^{\mathrm{N}*}$. Then we have

$$\frac{I^{\mathrm{S}}}{D_2 - \nu_{i'}} < \frac{I_i^{\mathrm{N}}}{I^{\mathrm{S}}}. \tag{A.1}$$

Rearranging (A.1) gives

$$\frac{I^{\mathrm{S}} + I_i^{\mathrm{N}}}{D_2} < \frac{I_i^{\mathrm{N}}}{\nu_{i'}}, \tag{A.2}$$

which leads to $x_{\mathrm{F}i}^{\mathrm{M}*} < x_{\mathrm{F}i}^{\mathrm{N}*}$. Similarly, rearranging (A.1) yields

$$\frac{I^{\mathrm{S}}}{D_2 - \nu_{i'}} < \frac{I^{\mathrm{S}} + I_i^{\mathrm{N}}}{D_2}, \tag{A.3}$$

i.e., $x_{\mathrm{F}i}^{\mathrm{S}*} < x_{\mathrm{F}i}^{\mathrm{M}*}$. Thus we have $x_{\mathrm{F}i}^{\mathrm{S}*} < x_{\mathrm{F}i}^{\mathrm{M}*} < x_{\mathrm{F}i}^{\mathrm{N}*}$.

Second, we show that $x_{\mathrm{F}i}^{\mathrm{S}*} < x_{\mathrm{F}i}^{\mathrm{N}*}$ implies $F_i^{\mathrm{Q}}(x) \geq F_i^{\mathrm{M}}(x)$. From the above result, we have already obtained that $x_{\mathrm{F}i}^{\mathrm{S}*} < x_{\mathrm{F}i}^{\mathrm{N}*}$ leads to $x_{\mathrm{F}i}^{\mathrm{S}*} < x_{\mathrm{F}i}^{\mathrm{M}*} < x_{\mathrm{F}i}^{\mathrm{N}*}$. These orderings are rewritten as

$$\Big(\frac{(D_2 - \nu_{i'})/D_2}{I^{\mathrm{S}}/(I^{\mathrm{S}} + I_i^{\mathrm{N}})}\Big)^{\beta} > 1 > \Big(\frac{\nu_{i'}/D_2}{I_i^{\mathrm{N}}/(I^{\mathrm{S}} + I_i^{\mathrm{N}})}\Big)^{\beta}. \tag{A.4}$$

On the other hand, the inequality, $F_i^{\mathrm{Q}}(x) \geq F_i^{\mathrm{M}}(x)$, is equivalent to

$$\begin{aligned}&(x_{\mathrm{F}i}^{\mathrm{S}*})^{-\beta}\Big\{\frac{D_2 - \nu_{i'}}{r-\mu}x_{\mathrm{F}i}^{\mathrm{S}*} - I^{\mathrm{S}}\Big\} + (x_{\mathrm{F}i}^{\mathrm{N}*})^{-\beta}\Big\{\frac{\nu_{i'}}{r-\mu}x_{\mathrm{F}i}^{\mathrm{N}*} - I_i^{\mathrm{N}}\Big\} \\ &\geq (x_{\mathrm{F}i}^{\mathrm{M}*})^{-\beta}\Big\{\frac{D_2}{r-\mu}x_{\mathrm{F}i}^{\mathrm{M}*} - (I_i^{\mathrm{N}} + I^{\mathrm{S}})\Big\}.\end{aligned} \tag{A.5}$$

Substituting three triggers, $x_{\mathrm{F}i}^{\mathrm{S}*}$ in (5), $x_{\mathrm{F}i}^{\mathrm{N}*}$ in (6), and $x_{\mathrm{F}i}^{\mathrm{M}*}$ in (11), into (A.5) yields

$$\Big(\frac{(D_2 - \nu_{i'})/D_2}{I^{\mathrm{S}}/(I^{\mathrm{S}} + I_i^{\mathrm{N}})}\Big)^{\beta}\frac{I^{\mathrm{S}}}{I^{\mathrm{S}} + I_i^{\mathrm{N}}} + \Big(\frac{\nu_{i'}/D_2}{I_i^{\mathrm{N}}/(I^{\mathrm{S}} + I_i^{\mathrm{N}})}\Big)^{\beta}\frac{I_i^{\mathrm{N}}}{I^{\mathrm{S}} + I_i^{\mathrm{N}}} \geq 1. \tag{A.6}$$

Using Jensen's inequality, we obtain that (A.4) implies (A.6) with $\beta > 1$. These two results complete the proof. □

References

[1] Armstrong, M., 1998. Network interconnection in telecommunications. Economic Journal, 108, 545–564.

[2] Bouis, R., Huisman, K.J.M., Kort, P.M., 2009. Investment in oligopoly under uncertainty: The accordion effect. International Journal of Industrial Organization, 27, 320–331.

[3] Bourreau, M., and Doğan, P., 2004. Service-based vs. facility-based competition in local access networks. Information Economics and Policy, 16, 287-306.

[4] Carter, M., and Wright, J., 1999. Interconnection in network industries. Review of Industrial Organization, 14, 1–25.

[5] Carter, M., and Wright, J., 2003. Asymmetric network interconnection. Review of Industrial Organization, 22, 27–46.

[6] Cave, M., and Volelsang, I., 2003. How access pricing and entry interact. Telecommunications Policy, 27, 717–727.

[7] Dixit, A. K. and Pindyck, R. S., 1994. Investment under uncertainty. Princeton University Press, Princeton, NJ.

[8] Gans, J. S., 2001. Regulating private infrastructure investment: optimal pricing for access to essential facilities. Journal of Regulatory Economics, 20, 167–189.

[9] Hori, K. and Mizuno, K., 2006. Access pricing and investment with stochastically growing demand. International Journal of Industrial Organization, 24, 795–808.

[10] Huisman, K., 2001. Technology investment: A game theoretic real options approach. Kluwer Academic Publishers.

[11] Huisman, K., and Kort, P. M., 2004. Strategic technology adoption taking into account future technological improvements: A real options approach. European Journal of Operational Research, 159, 705–728.

[12] Kijima, M., and Shibta, T., 2009. Real Options in a duopoly market with general volatility structure. World Scientific Publishers.

[13] Kong, J. J. and Kwok, Y. K., 2007. Real options in strategic investment games between two asymmetric firms. European Journal of Operational Research, 181, 967–985.

[14] Kotakorpi, K., 2006. Access price regulation, investment and entry in telecommunications. International Journal of Industrial Organization. 24, 1013–1020.

[15] Laffont, J., and Tirole, J., 2000. Competitions in Telecommunications. The MIT Press.

[16] Laffont, J., Rey, P., and Tirole, J., 1997. Competition between telecommunications operators. European Economic Review, 41, 701–711.

[17] McDonald, R., and Siegel, D. R., 1986. The value of waiting to invest. Quarterly Journal of Economics, 101, 707–727.

[18] Nielsen, M. J., 2002. Competition and irreversible investments. International Journal of Industrial Organization, 20, 731–743.

[19] Nishihara, M., and Shibta, T., 2009. Investment game with debt financing. Recent Advances in Financial Engineering, 161–188, World Scientific Publishing, Singapore.

[20] Pawlina, G. , and Kort, P. M., 2006. Real options in an asymmetric duopoly: Who benefits from your competitive disadvantage? Journal of Economics and Management Strategy, 15, 1–35.

[21] Peitz, M., 2005. Asymmetric access price regulation in telecommunications markets. European Economic Review, 49, 341–358.

[22] Shibata, T., 2009. Investment timing, asymmetric information, and audit structure: A real options framework. Journal of Economic Dynamics and Control, 33, 903–921.

[23] Shibata, T., and Nishihara, M., 2009. Dynamic investment and capital structure under manager-shaholder conflict. Journal of Economic Dynamics and Control, forthcoming.

[24] Valletti, T. M., 2003. The theory of access pricing and its linkage with investment incentives. Telecommunications Policy, 27, 659–675.

[25] Weeds, H., 2002. Strategic Delay in a Real Options Model of R&D Competition. Review of Economic Studies, 69, 729–747.

	$x_{\mathrm{P}i}^{\mathrm{Q}*}$	$x_{\mathrm{L}i}^{\mathrm{Q}*}$	$x_{\mathrm{F}i}^{\mathrm{S}*}$	$x_{\mathrm{F}i}^{\mathrm{N}*}$	$x_{\mathrm{F}i}^{\mathrm{M}*}$
Incumbent (Firm A)	0.3754	0.4635	0.8239	0.9887	0.9269
Entrant (Firm B)	0.3341	0.4789	0.6179	1.2977	0.9578

Table 1: Investment triggers

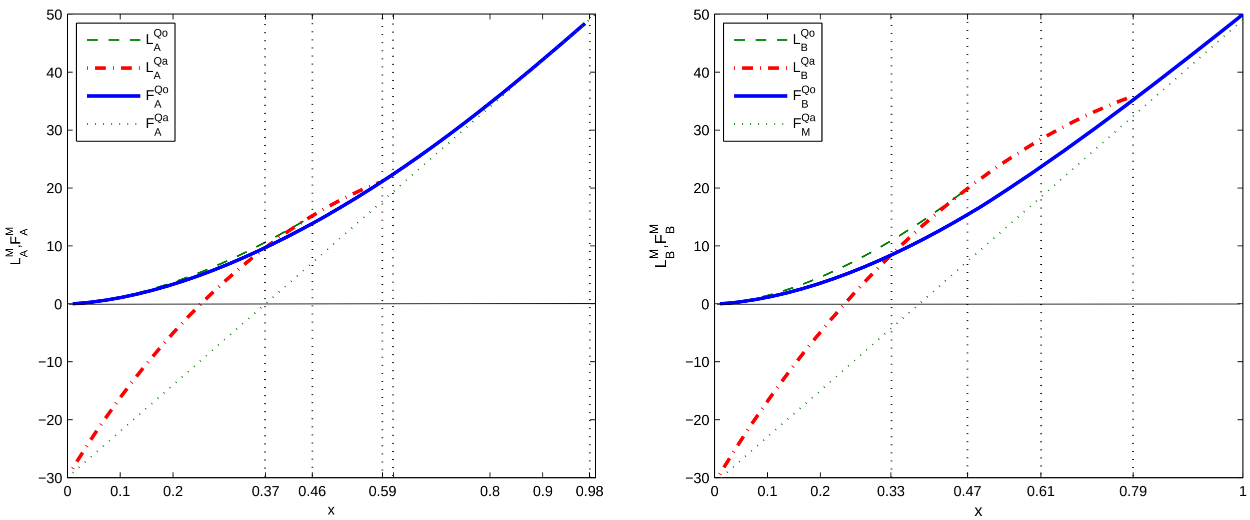

Figure 1: Value functions for Firms A and B

The left panel depicts the value function for the incumbent (Firm A). The right panel demonstrate the value function for the entrant (Firm B). Here, we have $x_{\mathrm{PA}}^{\mathrm{Q}*} = 0.3754 > 0.3341 = x_{\mathrm{PB}}^{\mathrm{Q}*}$ under these parameters. Then at equilibrium, the entrant (Firm B) enters the market as a leader at $x_{\mathrm{PA}}^{\mathrm{Q}*}$ while the incumbent (Firm A) enters the market as a follower in the service-based competition duopoly at $x_{\mathrm{FA}}^{\mathrm{S}*}$.

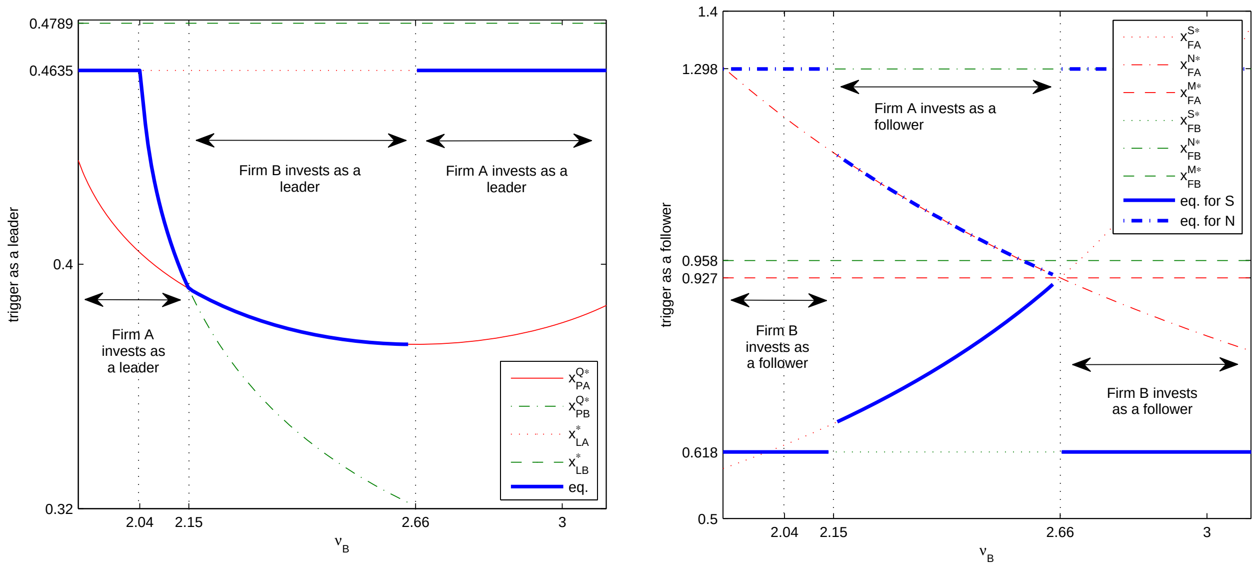

Figure 2: Investment triggers for leader and follower at equilibrium

The left panel depicts the investment trigger to enter the market as a leader. The right panel demonstrates the investment trigger to enter the market as a follower. For $\nu_{\mathrm{B}} < 2.15$ and $\nu_{\mathrm{B}} \geq 2.66$, the incumbent (Firm A) adopts the investment as a leader. For $\nu_{\mathrm{B}} \in (2.15, 2.66)$, the entrant (Firm B) adopts the investment as a leader.

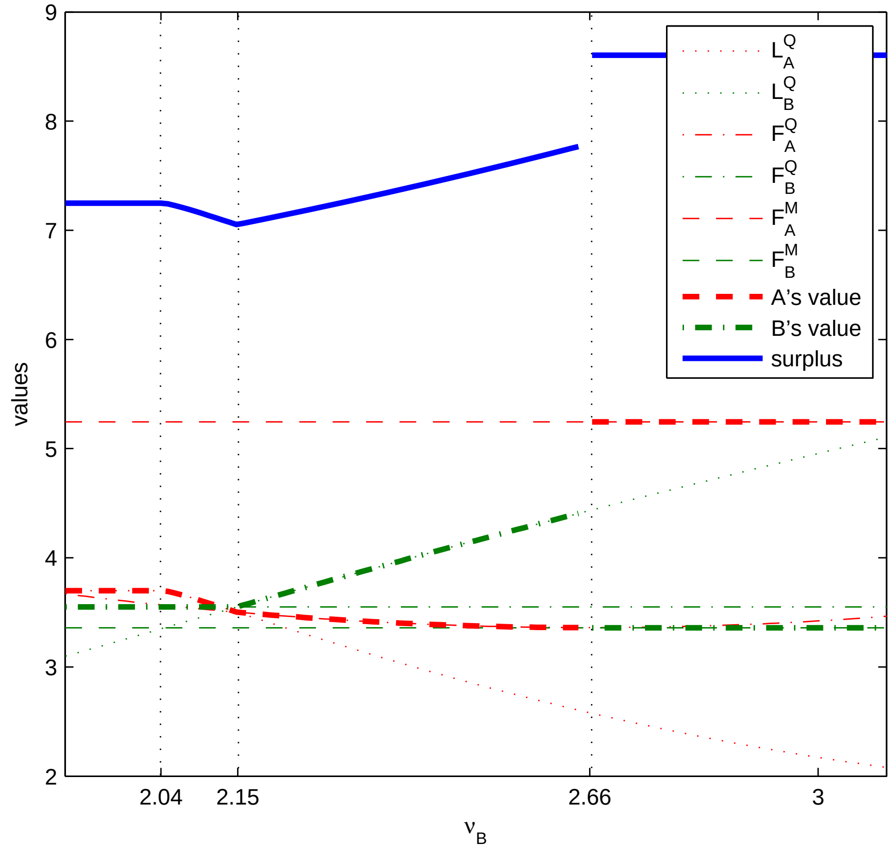

Figure 3: Leader's and follower's values with access charge at equilibrium

For $\nu_{\mathrm{B}} < 2.66$, the incumbent's value is increasing with ν_{B} while the entrant's value is decreasing with ν_{B}. At $\nu_{\mathrm{B}} = 2.15$, the incumbent's and entrant's values are exactly same when the competitive environment between two firms is completely symmetric. The (producers) surplus defined by the sum of two values is then the lowest.