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Investment timing under nationalization and privatization

Takashi Shibata and Michi Nishihara

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Takashi Shibata[a†] and Michi Nishihara[b‡]

[a] *Graduate School of Social Sciences, Tokyo Metropolitan University.*

[b] *Center for the Study of Finance and Insurance, Osaka University.*

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Abstract: We develop a theoretical framework for investment timing strategy under both nationalization and privatization. We show that investment timing under privatization is later than under nationalization. However, in almost all cases, the management effort under privatization is larger than under nationalization. As a result, there are trade-offs of efficiencies in allocation (investment timing) and productivity (management effort) under both nationalization and privatization. These results fit well with the findings of previous empirical work. Furthermore, we derive the total social values for investment under each of two institutional modes, by using the real options model in corporate finance.

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†Correspondence author. Email: `tshibata@tmu.ac.jp`

‡Email: `nishihara@sigmath.es.osaka-u.ac.jp`

1 Introduction

Privatization has been one of the most controversial economic and political debates since the late 1980s. In these debates, privatization is typically praised as resulting in "higher efficiency in productivity for management", and nationalization as "higher efficiency in allocation for investment." There is substantial empirical evidence supporting these issues under nationalization and privatization (Vickers and Yarrow, 1991; Megginson et al., 1994).

Several studies have examined the costs and benefits under both nationalization and privatization. Shapiro and Willig (1990), Laffont and Tirole (1993), De Fraja (1993), Schmidt (1996a, 1996b), and Corneo and Rob (2003), develop a comparative model of nationalization and privatization in an agency problem situation. They all argue that nationalization and privatization can be seen as different governance structures.[1] In particular, Schmidt (1996a, 1996b) considers the difference between two institutional modes as different governance structures, which give rise to different production levels (allocation) and different management efforts (productivity).[2] They find that there are trade-offs of efficiencies, and examine the costs and benefits under both nationalization and privatization.[3]

To the best of our knowledge, however, there has not been comparison of investment timings under nationalization and privatization. What is of great interest is to examine investment timings under each of the two organizational modes. This is because we can consider the costs and benefits under both organizational modes from different viewpoints to the existing studies above. Therefore, in this paper, we examine investment timing strategy under both nationalization and privatization by using the real options model in corporate finance.[4]

In this paper, we consider the costs and benefits under both nationalization and privatization. In particular, we extend a model developed by Schmidt (1996a, 1996b) to

[1]These ideas are based on Williamson (1985). Also, Laffont and Tirole (1991) summarize many of the conventional arguments and assumptions in this literature. In Shleifer (1998), the politics of government ownership and privatization are examined.

[2]Sappington and Stiglitz (1987) argue that the main difference between the two modes concerns the transactions costs faced by the government when attempting to intervene in the delegated production activities.

[3]Villalonga (2000) reviews the theoretical and empirical research about the effect of privatization on efficiency.

[4]The real options model has become a standard framework for investment timing decisions in corporate finance. The seminal paper in this literature is McDonald and Siegel (1986). An excellent overview of the real options approach is found in Dixit and Pindyck (1994).

the real options framework. Schmidt (1996a, 1996b) argues that allocations of ownership rights lead to different allocations of inside information about the firm, which in turn affect efficiency in both allocation and productivity. As in Schmidt (1996a, 1996b), we assume that the government has two possible courses of action. One is that the government *nationalizes* the firm and control production directly. The other is that the government *privatizes* the firm and delegates the investment to the private firm. As a consequence, in the subgame after nationalization, the government's problem is the *no-agency problem* in that there is no delegation of the investment decision. In the subgame after privatization, the government has less information about production in that there is delegation of the investment decision. Therefore, the government's problem is the *agency problem* under asymmetric information. Under each of these organizational modes, we examine investment timings (allocation), management efforts (productivity), and the total social value of investment.

We find that the investment timing under privatization is always later than under nationalization. Anticipating investment decisions, however, the management effort under privatization is larger than under nationalization in almost all cases. These results imply that efficiency in allocation (investment decision) under privatization is always *lower* than under nationalization, while efficiency in productivity (management effort) under privatization is *higher* than under nationalization. Under privatization, there are trade-offs between *higher efficiency in productivity* (a more efficient management effort) and *lower efficiency in allocation* (a less efficient investment decision). Under nationalization, on the other hand, there are trade-offs between *higher efficiency in allocation* and *lower efficiency in productivity*. As a result, the government should choose to either nationalize or privatize the firm depending on whether greater importance is attached to efficiency in allocation or efficiency in productivity.

The government may attach importance to efficiency in both allocation and productivity. Then, it is difficult to choose whether to nationalize or privatize the firm. In such a situation, it is natural that the government should choose one of the alternatives by comparing the total social values for investment under nationalization and privatization. When we choose one of the two alternatives from the viewpoint of the total social values, we assume two cost structures for management efforts: a *symmetric* and an *asymmetric* cost structures. Under a *symmetric* cost structure, the cost for the privatized firm is the same as for the nationalized firm. Then, the government *always* prefers nationalization to privatization in that the total social value for investment under nationalization is strictly larger than under privatization.

Under an *asymmetric* cost structure, we assume that the privatized firm has a cost efficient advantage for management effort, compared with the nationalized firm. This assumption is justified in that the advantage is the elimination of information disadvantage.[5] Then, the total social value for investment under privatization may be larger than under nationalization. In such a case, unlike in the case of a symmetric cost structure, the government prefers privatization to nationalization.

The remainder of the paper is organized as follows. In Section 2, we describe the setup of the model. In Sections 3 and 4, we formulate the optimization problem and derive the optimal contracts under both nationalization and privatization. In Section 5, we compare the costs and benefits under nationalization with those under privatization. Section 6 concludes.

2 Model

Consider a government (G) that has an option to invest in a single project. We assume that the government delegates the investment decision to a firm (F). Throughout our analysis, all agents are assumed to be risk neutral and aim to maximize their expected payoff.

The investment yields a revenue $(X_t)_{t\geq 0}$, which follows a geometric Brownian motion:

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}z_t, \quad X_0 = x, \tag{1}$$

where $(z_t)_{t\geq 0}$ denotes the standard Brownian motion, and where the mean growth rate μ, as well as the volatility σ, are positive constants. For convergence, we assume that $r > \mu$ where r is a constant interest rate.

We assume that the cost expenditure to undertake the investment that we denote by I is completely sunk. The cost expenditure, I, could take one of two possible values: I_1 or I_2 with $I_2 > I_1$ where $I_i > 0$ for all $i \in \{1, 2\}$. We denote $\Delta I = I_2 - I_1$. We assume that I_1 represents a "lower cost" expenditure and I_2 represents a "higher cost" expenditure.

Let $V(x; I_i)$ denote the project value function for $I = I_i$ $(i \in \{1, 2\})$. The value, $V(x; I_i)$, is formulated as

$$V(x; I_i) = \sup_{\tau_i} \ \mathbb{E}^x \left[\mathrm{e}^{-r\tau_i}(X_{\tau_i} - I_i) \right], \quad i \in \{1, 2\}, \tag{2}$$

for $\tau_i > 0$ at time zero $t = 0$. Here, $\mathbb{E}^x[\cdot]$ denotes the expectation operator given that $X_0 = x$, and τ_i is the time that the investment is exercised at the trigger $x_i = x(I_i)$ for each i $(i \in \{1, 2\})$, i.e., $\tau_i := \inf\{t \geq 0; X_t = x_i\}$. In this paper, it is assumed that the

[5] Under the privatized firm's cost advantage, the investment triggers under privatization are larger than under nationalization. From this viewpoint our assumption is justified.

current state value $X_0 = x$ is sufficiently low so that the investment is not undertaken immediately. Mathematically, we assume that $\tau_i > 0$ and $x < x_i$ for all i ($i \in \{1, 2\}$). Using the standard arguments, the value function for $I = I_i$ is

$$V(x; I_i) = \left(\frac{x}{x_i}\right)^{\beta} (x_i - I_i), \tag{3}$$

where β is defined by

$$\beta = \frac{1}{\sigma^2}\left(- \left(\mu - \frac{1}{2}\sigma^2\right) + \sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2r\sigma^2}\right) > 1. \tag{4}$$

The management effort has an impact on the likelihood of drawing a lower cost expenditure $I = I_1$. The firm affects the likelihood of drawing I_1 by exerting a one-time management effort, at *time zero*. If the firm exerts the effort ξ, it incurs a cost $c(\xi) = b\xi$, but increases the likelihood of drawing $I = I_1$, which is denoted by $q(\xi)$. That is, nature (N) draws $I = I_1$ with probability $q(\xi)$ and $I = I_2$ with probability $1 - q(\xi)$, where $q(\xi)$ is increasing and concave in ξ with $q(0) = 0$ and $\lim_{\xi \uparrow +\infty} q(\xi) = 1$. Immediately after exerting the management effort at *time zero*, the firm observes the realization of the cost expenditure.

The government (G) has two possible ways to affect investment timing and management effort. One way is that the government *nationalizes* the firm and control production directly. The other way is that the government *privatizes* the firm and delegates the investment to the privatized firm. The decision whether to nationalize or privatize the firm has to be made just before *time zero*. We assume that if the government gives up control of the firm as a privatized firm, then it will have less information about the firm's cost expenditure as compared with the situation where it controls the firm as a nationalized firm. This assumption is exactly the same as in Shapiro and Willig (1990) and Schmidt (1996a, 1996b) for which the assumption is justifiable.[6] Consequently, in the subgame after nationalization, the government possesses all the property rights on the project at *time zero* and keeps the firm *after that*. In the subgame after privatization, the government delegates the project investment and gives the regulation scheme to the privatized firm. The process of privatization is not modeled explicitly.

The structure of the game is summarized in reduced form in the game tree in Figure 1. Just before *time zero*, the government chooses whether to nationalize or privatize the firm. At *time zero*, the government determines the investment timing and the management effort. Here, all the actions are determined at *time zero* and locked in *after that*. This implies that renegotiation is not allowed after making the contract at *time zero*. The

[6] For example, in Schmidt (1996b), the British government presumably finds it more difficult to obtain detailed information about British Telecom now that British Telecom is a private company.

commitment increases the *ex ante* value, while it may cause *ex post* inefficiency in the actions. Immediately after the firm exerts the management effort offered by the government, one of two possible cost expenditures is realized. Then, the firm *always* observes the realization of the cost expenditure, while the government does not observe it in the subgame after nationalization although the government observes it in the subgame after privatization.

[Insert Figure 1 about here]

3 Nationalization

In this section, we first formulate the optimization problem in the subgame after *nationalization*. We then derive the optimal contract and value.

3.1 No-agency problem (Full information setting)

In the subgame after nationalization, the government observes the realized value of the cost expenditure I immediately after *time zero* and controls the production in that there is no delegation of the investment decision. Therefore, the government's optimization problem is the *no-agency problem*. The contract in the subgame after nationalization $\mathcal{M}^{\mathrm{n}}$ is modeled as

$$\mathcal{M}^{\mathrm{n}} = (\xi, x(I_i, \xi); i \in \{1, 2\}).$$

Here, the component ξ is the *ex ante* action, and the components $x(I_1, \xi)$ and $x(I_2, \xi)$, are the *ex post* actions, contingent on a true I_i given ξ. Let superscript "n" refer to the optimum under nationalization. For notational simplicity, we write the *ex post* actions as $x_1(\xi)$ and $x_2(\xi)$.

3.2 Optimal contract and value

Because the contract is composed of the *ex ante* and *ex post* actions, the contract is solved using backward induction. We first derive investment triggers as the *ex post* actions in the contract under nationalization. The government's optimization problem is formulated as

$$\max_{x_1, x_2} \quad q(\xi)\left(\frac{x}{x_1}\right)^{\beta}(x_1 - I_1) + (1 - q(\xi))\left(\frac{x}{x_2}\right)^{\beta}(x_2 - I_2) - b\xi. \tag{5}$$

Here, the first and second terms are the expected project value for $I = I_i$. The third term is the compensation payment for the cost incurred by the firm for exerting the management effort. The sum of these three terms is the government's *ex ante* option value defined by $\pi_{\mathrm{g}}(x)$.

Then, it is straightforward to obtain the optimal investment triggers:

$$(x_1^{\mathrm{n}}(\xi), x_2^{\mathrm{n}}(\xi)) = (x_1^*, x_2^*),$$

where

$$x_i^* = \frac{\beta}{\beta - 1} I_i, \quad i \in \{1, 2\}. \tag{6}$$

These results are exactly the same as in the standard real option model (e.g., McDonald and Siegel, 1986). Importantly, x_1^* and x_2^* do not depend on ξ. Once X_t starting at $X_0 = x$ hits x_i^* for $I = I_i$, the investment is exercised.

Anticipating the *ex post* actions, the government has to decide on the management effort level under nationalization. Then, the optimization problem for management effort is defined as

$$\max_{\xi} \quad q(\xi)\Big(\frac{x}{x_1^*}\Big)^{\beta}(x_1^* - I_1) + (1 - q(\xi))\Big(\frac{x}{x_2^*}\Big)^{\beta}(x_2^* - I_2) - b\xi. \tag{7}$$

Differentiating (7) with respect to ξ, the solution ξ^{n} is explicitly obtained by

$$\xi^{\mathrm{n}} = q'^{-1}\Big(\frac{b}{(x/x_1^*)^{\beta}(x_1^* - I_1) - (x/x_2^*)^{\beta}(x_2^* - I_2)}\Big) > 0. \tag{8}$$

Note that $(x/x_1^*)^{\beta}(x_1^* - I_1) > (x/x_2^*)^{\beta}(x_2^* - I_2)$ because of $x_1^* < x_2^*$. Recall that b is a measure of efficiency in the cost function for management effort. Obviously, a decrease in b increases ξ^{n}.[7]

Finally, substituting the solutions into the value function yields the optimal value. Therefore, we have the following results.

Lemma 1 *In the subgame after nationalization, the optimal contract* $\mathcal{M}^{\mathrm{n}}$ *is*

$$\mathcal{M}^{\mathrm{n}} = \{\xi^{\mathrm{n}}, x_1^*, x_2^*\}. \tag{9}$$

The total social value $\pi^{\mathrm{n}}(x)$ *is equal to the government's value, i.e.,*

$$\pi^{\mathrm{n}}(x) = q(\xi^{\mathrm{n}})\Big(\frac{x}{x_1^*}\Big)^{\beta}(x_1^* - I_1) + (1 - q(\xi^{\mathrm{n}}))\Big(\frac{x}{x_2^*}\Big)^{\beta}(x_2^* - I_2) - b\xi^{\mathrm{n}}. \tag{10}$$

[7] This is because we have

$$\frac{\mathrm{d}\xi^{\mathrm{n}}}{\mathrm{d}b} = \big((x/x_1^*)^{\beta}(x_1^* - I_1) - (x/x_2^*)^{\beta}(x_2^* - I_2)\big)^{-1}(q''(\xi^{\mathrm{n}}))^{-1} < 0.$$

4 Privatization

In this section, we first formulate the optimization problem in the subgame after privatization. We then derive the optimal contract and values.

4.1 Agency problem (Asymmetric information setting)

In the subgame after *privatization*, the government delegates the investment to the privatized firm and cannot observe the realized value of the cost expenditure I. Therefore, the government's optimization problem for the *ex post* actions is defined under the *asymmetric information problem*. In such a situation, the government's optimization problem is to choose the regulation scheme that maximizes its payoff by inducing the firm to choose a socially more efficient investment decision. Such a regulation must be designed to provide some incentives for the firm to truthfully reveal private information.[8] Without any regulations to enforce truthful revelation of private information, the government suffers some further losses.[9]

We assume that the government gives some incentives to the firm to truthfully reveal private information. In particular, the government gives a subsidy $s(\cdot)$ and/or imposes a penalty $P > 0$ to the firm when the firm's false announcement is detected by auditing, contingent on the investment trigger $x(\cdot)$. Here, the audit technology allows the government to verify the state announced by the firm with probability $p(\cdot)$ at a cost $c(p(\cdot))$ with $c(0) = 0$, $c' > 0$, $c'' > 0$, and $\lim_{p(\cdot)\uparrow 1} c(p(\cdot)) = +\infty$.[10] Here, the penalty P and the cost function $c(p(\cdot))$ are given exogenously.

Therefore, in the subgame after privatization, the government designs the optimal regulation scheme as

$$\mathcal{M}^{\mathrm{p}} = (\xi, x(\tilde{I}_i, \xi), s(\tilde{I}_i, \xi), p(\tilde{I}_i, \xi); i \in \{1, 2\}).$$

Here, the component ξ is the *ex ante* action, the components, $x(\tilde{I}_i, \xi)$, $s(\tilde{I}_i, \xi)$, and $p(\tilde{I}_i, \xi)$, are the *ex post* actions contingent on a reported $\tilde{I}$ given ξ. Let superscript "p" refer to the

[8]The agency problem in this paper is based on in the real options model. See Grenadier and Wang (2005), Mæland (2002), Nishihara and Shibata (2007), Shibata (2007), Shibata and Nishihara (2007) for more details.

[9]Although any regulation turns out to be suboptimal, it will reduce the government's losses arising from asymmetric information.

[10]These assumptions are intuitively reasonable. The first assumption is that there is no cost incurred if the government does not use the audit technology. The second and third assumptions imply that $c(p)$ is strictly increasing and convex in p. The final assumption is that complete auditing incurs a huge cost that the owner cannot pay.

optimum under privatization. Because the *revelation principle* ensures that the manager reveals a true I as private information, we make no distinction between a reported $\tilde{I}$ and a true I.[11] We drop the suffix "tilde" on the reported $\tilde{I}$. Therefore, we simply write the *ex post* actions as $x_i(\xi)$, $s_i(\xi)$, and $p_i(\xi)$.

4.2 Optimal investment triggers

As in the previous section, we first derive the *ex post* actions in the contract under privatization.[12] The government's optimization problem is formulated as

$$\max_{x_i(\xi),s_i(\xi),p_i(\xi)} \quad q(\xi)\Big(\frac{x}{x_1(\xi)}\Big)^{\beta}(x_1(\xi)-I_1-s_1(\xi)-c(p_1(\xi))) \tag{11}$$
$$+(1-q(\xi))\Big(\frac{x}{x_2(\xi)}\Big)^{\beta}(x_2(\xi)-I_2-s_2(\xi)-c(p_2(\xi)))-b\xi,$$

subject to

$$\Big(\frac{x}{x_1(\xi)}\Big)^{\beta}s_1(\xi) \geq \Big(\frac{x}{x_2(\xi)}\Big)^{\beta}(s_2(\xi)+\Delta I-p_2(\xi)P), \tag{12}$$
$$\Big(\frac{x}{x_2(\xi)}\Big)^{\beta}s_2(\xi) \geq \Big(\frac{x}{x_1(\xi)}\Big)^{\beta}(s_1(\xi)-\Delta I-p_1(\xi)P), \tag{13}$$
$$s_i(\xi) \geq 0, \quad i\in\{1,2\}, \tag{14}$$
$$q(\xi)\Big(\frac{x}{x_1(\xi)}\Big)^{\beta}s_1(\xi)+(1-q(\xi))\Big(\frac{x}{x_2(\xi)}\Big)^{\beta}s_2(\xi) \geq 0, \tag{15}$$
$$1 \geq p_i(\xi) \geq 0, \quad i\in\{1,2\}. \tag{16}$$

Here, the objective function (11) is the government's *ex ante* option value $\pi_{\mathrm{g}}(x)$.

Constraints (12) and (13) are the incentive-compatibility constraints for the firm under states I_1 and I_2, respectively.[13] Constraints (14) and (15) are the limited-liability constraints and the participation constraint, respectively.[14] Constraints (16) are obvious, where $p_i(\xi)$ is the probability of an audit.

Although the optimization problem is subject to seven inequality constraints, we can simplify the problem in the following three steps. First, (15) is automatically satisfied. This is because (14) implies (15). Second, the manager in state I_2 does not have the incentive to tell a lie as does a firm in state I_1. This is because the firm in state I_2 suffers

[11]See Fudenberg and Tirole (1991) and Salanié (2005) for the *revelation principle*.

[12]This problem is the same as in Nishihara and Shibata (2007), and Shibata and Nishihara (2007).

[13]Consider, for example, constraint (12). The firm's payoff in state I_1 is $(x/x_1(\xi))^{\beta}s_1(\xi)$ if it tells the truth, but it is $(x/x_2(\xi))^{\beta}(s_2(\xi)+\Delta I-p_2(\xi)P)$ if the firm instead claims that it is state I_2. Therefore, the firm tells the truth if (12) is satisfied. Constraint (13) follows similarly.

[14]The nonnegative subsidies $s_1(\xi)$ and $s_2(\xi)$ ensure that the firm makes an agreement about delegation. For example, if $s_2(\xi)<0$, then the firm would refuse the contract on learning that $I=I_2$.

a loss from such a false announcement. Therefore, (13) is automatically satisfied, and $p_1^{\mathrm{P}}(\xi) = 0$ and $w_2^{\mathrm{P}}(\xi) = 0$ are obtained in optimum. Finally, $p_2 \leq 1$ in (16) is automatically satisfied. This statement is shown by $\lim_{p_2(\xi)\uparrow 1} c(p_2(\xi)) = +\infty$ and $c'(p_2(\xi)) > 0$ for any $p_2(\xi)$.

As a result, the simplified optimization problem is

$$\max_{x_1(\xi),x_2(\xi),s_1(\xi),p_2(\xi)} \quad q(\xi)\Big(\frac{x}{x_1(\xi)}\Big)^{\beta}(x_1(\xi) - I_1 - s_1(\xi)) + (1-q(\xi))\Big(\frac{x}{x_2(\xi)}\Big)^{\beta}(x_2(\xi) - I_2 - c(p_2(\xi))) - b\xi, \tag{17}$$

subject to

$$\Big(\frac{x}{x_1(\xi)}\Big)^{\beta} s_1(\xi) \;\geq\; \Big(\frac{x}{x_2(\xi)}\Big)^{\beta}(\Delta I - p_2(\xi)P), \quad s_1(\xi) \geq 0, \quad p_2(\xi) \geq 0. \tag{18}$$

As shown in the appendix, the *ex post* optimal actions $(x_i^{\mathrm{P}}(\cdot), s_i^{\mathrm{P}}(\cdot), p_i^{\mathrm{P}}(\cdot))$ are obtained as

$$\begin{aligned}
(x_1^{\mathrm{P}}(\xi), s_1^{\mathrm{P}}(\xi), p_1^{\mathrm{P}}(\xi)) &= \Big(x_1^*, \quad \Big(\frac{x_1^*}{x_2^{**}(\xi)}\Big)^{\beta}(\Delta I - p_2^{**}(\xi)P), \quad 0\Big), \\
(x_2^{\mathrm{P}}(\xi), s_2^{\mathrm{P}}(\xi), p_2^{\mathrm{P}}(\xi)) &= \Big(x_2^{**}(\xi), \quad 0, \quad p_2^{**}(\xi)\Big),
\end{aligned}$$

where

$$x_2^{**}(\xi) = \frac{\beta}{\beta-1}\Big(I_2 + c(p_2^{**}(\xi)) + \frac{q(\xi)}{1-q(\xi)} \cdot (\Delta I - p_2^{**}(\xi)P)\Big),$$

and

$$p_2^{**}(\xi) = \begin{cases} 0, & \text{if} \quad 0 \leq P < \frac{1-q(\xi)}{q(\xi)}c'(0), \\ c'^{-1}(\frac{q(\xi)}{1-q(\xi)}P), & \text{if} \quad \frac{1-q(\xi)}{q(\xi)}c'(0) \leq P < \max\{\Delta I, \ \frac{1-q(\xi)}{q(\xi)}c'(\frac{\Delta I}{P})\}, \\ \frac{\Delta I}{P}, & \text{otherwise.} \end{cases}$$

Note that we have $(x_1^{\mathrm{P}}(\xi), s_2^{\mathrm{P}}(\xi), p_1^{\mathrm{P}}(\xi)) = (x_1^*, 0, 0)$ for all the three regions. Here, $p_2^{\mathrm{P}}(\xi)$ is first determined depending on the magnitude of the penalty P. Then, $x_2^{\mathrm{P}}(\xi)$ is decided through $p_2^{\mathrm{P}}(\xi)$, and $s_1^{\mathrm{P}}(\xi)$ is decided through $x_2^{\mathrm{P}}(\xi)$ and $p_2^{\mathrm{P}}(\xi)$.

In this contract, the government induces the firm to truthfully reveal private information, with only the subsidy if $0 \leq P < \frac{1-q(\xi)}{q(\xi)}c'(0)$, with a combination of the subsidy and audit if $\frac{1-q(\xi)}{q(\xi)}c'(0) \leq P < \max\{\Delta I, \ \frac{1-q(\xi)}{q(\xi)}c'(\frac{\Delta I}{P})\}$, and with only the audit, respectively. Therefore, we call these regions the *subsidy-only region*, the *joint subsidy and audit region*, and the *audit-only region*, respectively. In other word, an increase in P changes the solution from the *subsidy-only region* to the *audit-only region* via the *joint region*. Intuitively, the larger P is, the more available the audit technology is.

It is important to obtain $x_1^{\mathrm{p}}(\xi) = x_1^{\mathrm{n}}(\xi) = x_1^*$ and $x_2^{\mathrm{p}}(\xi) > x_2^{\mathrm{n}}(\xi) = x_2^*$ for any ξ.[15] These results imply that the firm exercises investment under privatization later than the government does under nationalization. As a result, efficiency in allocation (investment timing) under privatization is lower than under nationalization.[16] We consider these results in greater detail in Section 5.

4.3 Optimal management effort

Anticipating the *ex post* actions, the government determines the *ex ante* action in the subgame after *privatization*. Then the optimization problem for management effort is formulated by

$$\max_{\xi} \quad q(\xi)\Big(\frac{x}{x_1^*}\Big)^{\beta}\Big(x_1^* - I_1\Big) + (1-q(\xi))\Big(\frac{x}{x_2^{**}(\xi)}\Big)^{\beta}\Big(x_2^{**}(\xi) - I_2^{**}(\xi)\Big) - b\xi, \tag{19}$$

where

$$I_2^{**}(\xi) = I_2 + \frac{q(\xi)}{1-q(\xi)}(\Delta I - p_2^{**}(\xi)P) + c(p_2^{**}(\xi)). \tag{20}$$

Differentiating (19) with respect to ξ yields

$$\frac{\mathrm{d}\pi_{\mathrm{g}}}{\mathrm{d}\xi} = \Big(\frac{\partial \pi_{\mathrm{g}}}{\partial x_2^{**}}\frac{\partial x_2^{**}}{\partial \xi} + \frac{\partial \pi_{\mathrm{g}}}{\partial p_2^{**}}\frac{\partial p_2^{**}}{\partial \xi} + \frac{\partial \pi_{\mathrm{g}}}{\partial \xi}\Big) = \frac{\partial \pi_{\mathrm{g}}}{\partial \xi},$$

where we have used $\frac{\partial \pi_{\mathrm{g}}}{\partial x_2^{**}} = \frac{\partial \pi_{\mathrm{g}}}{\partial p_2^{**}} = 0$ by the *envelope theorem*.[17] We assume that the second-order condition is satisfied.[18] Then, the optimal management effort ξ^{p} is implicitly obtained as

$$q'(\xi^{\mathrm{p}}) = \frac{b}{(x/x_1^*)^{\beta}(x_1^* - I_1) - (x/x_2^{**}(\xi^{\mathrm{p}}))^{\beta}\Big(x_2^{**}(\xi^{\mathrm{p}}) - I_2 + (\Delta I - p_2^{**}(\xi^{\mathrm{p}})P) - c(p_2^{**}(\xi^{\mathrm{p}}))\Big)}. \tag{21}$$

Here, on the one hand, we consider the optimal management effort ξ^{p} on the *audit-only region*. Since $(\Delta I - p_2^{**}(\xi^{\mathrm{p}})P) = 0$ in the *audit-only region*, we have

$$\Big(\frac{x}{x_2^*}\Big)^{\beta}(x_2^* - I_2) > \Big(\frac{x}{x_2^{**}(\xi^{\mathrm{p}})}\Big)^{\beta}(x_2^{**}(\xi^{\mathrm{p}}) - I_2 - c(p_2^{**}(\xi^{\mathrm{p}}))). \tag{22}$$

[15]See Shibata and Nishihara (2007) about the properties of the solution in greater detail.

[16]Recall that the investment decision can be regarded as the allocation problem.

[17]See Mas-Colell et al. (1995) for the *envelope theorem*.

[18]The second-order condition is $\frac{\mathrm{d}^2\pi_{\mathrm{g}}}{\mathrm{d}\xi^{\mathrm{p}2}} = (q''(\xi^{\mathrm{p}})\theta(\xi^{\mathrm{p}}) + q'(\xi^{\mathrm{p}})\theta'(\xi^{\mathrm{p}})) < 0$ where $\theta(\xi^{\mathrm{p}})$ is defined as

$$\theta(\xi^{\mathrm{p}}) = \big(\frac{x}{x_1^*}\big)^{\beta}(x_1^* - I_1) - \big(\frac{x}{x_2^{**}(\xi^{\mathrm{p}})}\big)^{\beta}\big(x_2^{**}(\xi^{\mathrm{p}}) - I_2 - (\Delta I - p_2^{**}(\xi^{\mathrm{p}})P) - c(p^{**}(\xi^{\mathrm{p}}))\big).$$

Here, whether or not the second-order condition is satisfied in all the cases is not clear. However, the second-order condition is satisfied in almost all cases.

Therefore, it is straightforward to obtain $\xi^{\mathrm{p}} > \xi^{\mathrm{n}}$.

On the other hand, we investigate the optimal management effort in the *other (joint and subsidy-only) regions*. Suppose that $c(p_2^{**}(\xi^{\mathrm{p}})) > (\Delta I - p_2^{**}(\xi^{\mathrm{p}})P) \geq 0$. This assumption leads to the following inequality[19]

$$\left(\frac{x}{x_2^*}\right)^{\beta}(x_2^* - I_2) > \left(\frac{x}{x_2^{**}(\xi^{\mathrm{p}})}\right)^{\beta}(x_2^{**}(\xi^{\mathrm{p}}) - I_2 + (\Delta I - p_2^{**}(\xi^{\mathrm{p}})P) - c(p_2^{**}(\xi^{\mathrm{p}}))). \tag{23}$$

Therefore, under this condition we obtain $\xi^{\mathrm{p}} > \xi^{\mathrm{n}}$ on the *joint and subsidy only regions*. To the extent that we numerically solved (19) for various parameters, we could not find any example of $\xi^{\mathrm{p}} \leq \xi^{\mathrm{n}}$. This is because we obtain the inequity (23) in almost all cases.[20]

As a result, we have $\xi^{\mathrm{p}} > \xi^{\mathrm{n}}$ in almost all cases. We conclude that efficiency in productivity (management effort) under privatization can be higher than under nationalization in almost all cases. Then, there are trade-offs of efficiencies in allocation and productivity between nationalization and privatization. We consider these trade-offs in greater detail in Section 5.

Finally, we consider the comparative statics on ξ^{p} with respect to b. In order to do so, totally differentiating (21) and rearranging yields

$$\frac{\mathrm{d}\xi^{\mathrm{p}}}{\mathrm{d}b} = \frac{1}{q''(\xi^{\mathrm{p}})\theta(\xi^{\mathrm{p}}) + q'(\xi^{\mathrm{p}})\theta'(\xi^{\mathrm{p}})} < 0. \tag{24}$$

Here, we have used the second-order condition. Therefore, a decrease in b increases the optimal management effort ξ^{p}. Intuitively, the more efficient the cost function is, the larger the management effort is.

4.4 Optimal contract and values

In this subsection, the optimal values are obtained by substituting the optimal actions into the value functions. The government's and firm's values, $\pi_{\mathrm{g}}^{\mathrm{p}}(x)$ and $\pi_{\mathrm{f}}^{\mathrm{p}}(x)$, are

$$\pi_{\mathrm{g}}^{\mathrm{p}}(x) = q(\xi^{\mathrm{p}})\left(\frac{x}{x_1^*}\right)^{\beta}(x_1^* - I_1) + (1 - q(\xi^{\mathrm{p}}))\left(\frac{x}{x_2^{**}(\xi^{\mathrm{p}})}\right)^{\beta}\Big(x_2^{**}(\xi^{\mathrm{p}}) - I_2^{**}(\xi^{\mathrm{p}})\Big) - b\xi^{\mathrm{p}}, \tag{25}$$

and

$$\pi_{\mathrm{f}}^{\mathrm{p}}(x) = q(\xi^{\mathrm{p}})\left(\frac{x}{x_2^{**}(\xi^{\mathrm{p}})}\right)^{\beta}\Big(\Delta I - p_2^{**}(\xi^{\mathrm{p}})P\Big). \tag{26}$$

By the definition, the total social value is given by the sum of these two values, $\pi^{\mathrm{p}}(x) = \pi_{\mathrm{g}}^{\mathrm{p}}(x) + \pi_{\mathrm{f}}^{\mathrm{p}}(x)$. As a result, we have the following.

[19] This condition is *not* the necessary condition *but* the sufficient condition, in that we have the inequality (23).

[20] However, whether or not the inequity $\xi^{\mathrm{p}} > \xi^{\mathrm{n}}$ holds in all the cases is ambiguous.

Lemma 2 *In the subgame after privatization, the optimal contract $\mathcal{M}^{\mathrm{p}}$ is*

$$\mathcal{M}^{\mathrm{p}} = \{\xi^{\mathrm{p}},\, (x_1^*,\, w_1^{**}(\xi^{\mathrm{p}}),\, 0),\, (x_2^{**}(\xi^{\mathrm{p}}),\, 0,\, p_2^{**}(\xi^{\mathrm{p}}))\}. \tag{27}$$

The total social value, $\pi^{\mathrm{p}}(x) = \pi_{\mathrm{g}}^{\mathrm{p}}(x) + \pi_{\mathrm{f}}^{\mathrm{p}}(x)$, is

$$\begin{aligned} \pi^{\mathrm{p}}(x) &= q(\xi^{\mathrm{p}})\Big(\frac{x}{x_1^*}\Big)^{\beta}(x_1^* - I_1) \\ &\quad +(1 - q(\xi^{\mathrm{p}}))\Big(\frac{x}{x_2^{**}(\xi^{\mathrm{p}})}\Big)^{\beta}(x_2^{**}(\xi^{\mathrm{p}}) - I_2 - c(p_2^{**}(\xi^{\mathrm{p}}))) - b\xi^{\mathrm{p}}. \end{aligned} \tag{28}$$

4.5 Unlimited penalties

Although unlimited penalties are of theoretical interest in the subgame after privatization (penalties are limited in practice), we examine how the optimal actions and values are changed as the penalty increases.

Corollary 1 *The optimal actions and values obtained in the subgame after nationalization are approximated closely as the penalty is increased without limit in the subgame after privatization. As $P \uparrow +\infty$, we have*

$$\xi^{\mathrm{p}} \to \xi^{\mathrm{n}}, \quad x_2^{\mathrm{p}}(\xi^{\mathrm{p}}) \to x_2^{\mathrm{n}}, \quad s_1^{\mathrm{p}}(\xi^{\mathrm{p}}) \to 0, \quad p_2^{\mathrm{p}}(\xi^{\mathrm{p}}) \to 0,$$

and

$$\pi_{\mathrm{g}}^{\mathrm{p}}(x) \uparrow \pi_{\mathrm{g}}^{\mathrm{n}}(x) = \pi^{\mathrm{n}}(x), \quad \pi_{\mathrm{f}}^{\mathrm{p}}(x) \downarrow 0, \quad \pi^{\mathrm{p}}(x) \to \pi^{\mathrm{n}}(x).$$

These results are the same as those in Baron and Besanko (1984, proposition 4), Nishihara and Shibata (2007), and Shibata and Nishihara (2007, proposition 3).

We have obtained the optimal contracts and values under both nationalization and privatization. In the next section, we compare the optimal contracts, $\mathcal{M}^{\mathrm{p}}$ and $\mathcal{M}^{\mathrm{n}}$, and values, $\pi^{\mathrm{p}}(x)$ and $\pi^{\mathrm{n}}(x)$.

5 Nationalization versus Privatization

In this section, we first discuss efficiency in allocation and productivity between nationalization and privatization. We then compare efficiency in the total social welfare between them. Also, in this section, we use numerical examples in order to understand the differences between the two organizational modes.

5.1 Efficiencies in allocation and productivity

Lemmas 1 and 2 lead to the following results.

Proposition 1 *Assume that the penalty is finite. We have* $x_1^* = x_1^{\mathrm{n}}(\xi^{\mathrm{n}}) = x_1^{\mathrm{p}}(\xi^{\mathrm{p}})$*,* $x_2^* = x_2^{\mathrm{n}}(\xi^{\mathrm{n}}) < x_2^{\mathrm{p}}(\xi^{\mathrm{p}})$*. Also, we have* $\xi^{\mathrm{n}} < \xi^{\mathrm{p}}$ *in many cases.*

Proposition 1 implies that there are trade-offs of efficiencies in allocation and productivity between nationalization and privatization in many cases. In particular, $x_1^* = x_1^{\mathrm{n}}(\xi^{\mathrm{n}}) = x_1^{\mathrm{p}}(\xi^{\mathrm{p}})$ and $x_2^* = x_2^{\mathrm{n}}(\xi^{\mathrm{n}}) < x_2^{\mathrm{p}}(\xi^{\mathrm{p}})$ imply that efficiency in allocation under nationalization is *always* higher than under privatization, while $\xi^{\mathrm{n}} < \xi^{\mathrm{p}}$ implies that efficiency in productivity under nationalization is lower than under privatization in many cases. In other words, the nationalized firm *always* enhances efficiency in allocation although it reduces efficiency in productivity *in many cases*, while the privatized firm enhances efficiency in productivity *in many cases* although it *always* reduces efficiency in allocation. As a result, the government should nationalize (privatize) the firm if it attaches importance to efficiency in allocation (productivity) rather than in productivity (allocation) in many cases.

Now we investigate the optimal actions between nationalization and privatization in greater detail by using numerical examples. In order to do so, we define the cost function for an audit $c(p_i)$ and the probability $q(\xi)$ as

$$c(p_i(\cdot)) = \alpha \frac{p_i(\cdot)}{1 - p_i(\cdot)}, \quad i \in \{1, 2\}, \tag{29}$$

and

$$q(\xi) = (1 - \mathrm{e}^{-\lambda \xi}), \tag{30}$$

respectively. Here, the parameters α in (29) and λ in (30) are some positive constants. Suppose that parameters are $\sigma = 0.2$, $r = 0.07$, $\mu = 0.03$, $I_1 = 50$, $I_2 = 80$, $b = 1$, $\alpha = 20$, and $\lambda = 0.215$.

Figure 2 depicts the *ex ante* actions, ξ^{n} and ξ^{p}, with respect to the penalty P. The effort ξ^{n} is constant with P, while ξ^{p} is decreasing with P. Under privatization, the *subsidy-only region* is on $0 \leq P \leq 9.5$, the *joint region* is on $9.5 \leq P \leq 46$, and the *audit-only region* is on $P \geq 46$. In particular, ξ^{p} is constant with P on the *subsidy-only region*, ξ^{p} is increasing with P on the *joint region*, and ξ^{p} is decreasing and convex with P on the *audit-only region.* Most importantly, we observe that ξ^{p} is *always* larger than ξ^{n} in this numerical example.

[Insert Figure 2 about here]

Figures 3 to 5 depict the *ex post* actions $(x_2^{\mathrm{p}}(\xi^{\mathrm{p}}), w_1^{\mathrm{p}}(\xi^{\mathrm{p}}), p_2^{\mathrm{p}}(\xi^{\mathrm{p}}))$ with respect to P.[21] Figure 3 demonstrates the investment triggers x_2^{n} and $x_2^{\mathrm{p}}(\xi^{\mathrm{p}})$. Naturally, $x_2^{\mathrm{n}}(\xi^{\mathrm{n}}) = x_2^*$ does not depend on P, while $x_2^{\mathrm{p}}(\xi^{\mathrm{p}})$ depends on P. In particular, $x_2^{\mathrm{p}}(\xi^{\mathrm{p}})$ is constant with P in the *subsidy-only region*, $x_2^{\mathrm{p}}(\xi^{\mathrm{p}})$ is unimodal with P in the *joint region*, and $x_2^{\mathrm{p}}(\xi^{\mathrm{p}})$ is decreasing and convex with P in the *audit-only region*.

[Insert Figure 3 about here]

Figure 4 demonstrates the subsidy $s_1^{\mathrm{p}}(\xi^{\mathrm{p}})$. Note that $s_1^{\mathrm{p}}(\xi^{\mathrm{p}})$ is decreasing with P. In particular, $s_1^{\mathrm{p}}(\xi^{\mathrm{p}})$ is constant with P in the *subsidy* and *audit-only regions*, while $s_1^{\mathrm{p}}(\xi^{\mathrm{p}})$ is decreasing with P in the *joint region*.

[Insert Figure 4 about here]

Figure 5 depicts the probability of an audit $p_2^{\mathrm{p}}(\xi^{\mathrm{p}})$. Note that $p_2^{\mathrm{p}}(\xi^{\mathrm{p}})$ is unimodal with P. In particular, $p_2^{\mathrm{p}}(\xi^{\mathrm{p}})$ remains zero in the *subsidy-only region*, $p_2^{\mathrm{p}}(\xi^{\mathrm{p}})$ is increasing and concave with P in the *joint region*, and $p_2^{\mathrm{p}}(\xi^{\mathrm{p}})$ is decreasing and convex with P in the *audit-only region*.

[Insert Figure 5 about here]

Figures 6 and 7 depicts the government's and firm's values, $\pi_{\mathrm{g}}^{\mathrm{p}}(x)$ and $\pi_{\mathrm{f}}^{\mathrm{p}}(x)$. In Figure 6, the value $\pi_{\mathrm{g}}^{\mathrm{p}}(x)$ is monotone increasing with P. This property corresponds to the "Maximal Penalty Principle."[22] In Figure 7, the value $\pi_{\mathrm{f}}^{\mathrm{p}}(x)$ is monotone decreasing with P. The reason is that an increase in P decreases the *information rent* for the manager in state I_1, which leads to a decrease in the subsidy. Figures 6 and 7 imply that an increase in P leads to "asset substitution" between the government and the firm. The wealth is transferred from the firm to the government by an increase in P.

[Insert Figure 6 about here]

[Insert Figure 7 about here]

Finally, in Figures 2 to 7, we can see that all the actions and values under privatization converge to those under nationalization as the penalty is increased without limit, i.e., $\xi^{\mathrm{p}} \to \xi^{\mathrm{n}}$, $x_2^{\mathrm{p}}(\xi^{\mathrm{p}}) \to x_2^{\mathrm{n}}$, $s_1^{\mathrm{p}}(\xi^{\mathrm{p}}) \to 0$, $p_2^{\mathrm{p}}(\xi^{\mathrm{p}}) \to 0$, $\pi_{\mathrm{g}}^{\mathrm{p}} \uparrow \pi_{\mathrm{g}}^{\mathrm{n}}(x)$, and $\pi_{\mathrm{f}}^{\mathrm{p}}(x) \downarrow 0$ as $P \uparrow +\infty$.

[21] Here, we omit $(x_1^{\mathrm{p}}, s_2^{\mathrm{p}}, p_1^{\mathrm{p}})$ because we have $x_1^{\mathrm{p}} = x_1^{\mathrm{n}}$, $s_2^{\mathrm{p}} = 0$, and $p_1^{\mathrm{p}} = 0$.

[22] See Laffont and Martimort (2002) for the "Maximal Penalty Principle" in detail.

5.2 Comparisons of the total social values

The government may attach importance to efficiency in both allocation and productivity. In such a situation, it is difficult for the government to choose whether to nationalize or privatize the firm. Then, it is natural that the government should choose one of the alternatives by comparing the total social values under both nationalization and privatization.

We have assumed a *symmetric* cost structure for management effort in that the cost functions are the same for nationalization and privatization. Here, we also consider an *asymmetric* cost structure in that the cost functions are different for the two institutional modes. In this section, we investigate the total social values under both a *symmetric* and an *asymmetric* cost structures. Then, the government chooses whether to nationalize or privatize the firm, according to the magnitude of the total social value.

Under a *symmetric* cost structure for management effort, we have the following result.

Proposition 2 *Under a symmetric cost structure, the government always prefers nationalization to privatization, in that* $\pi^{\mathrm{n}}(x) > \pi^{\mathrm{p}}(x)$ *for finite penalty* P *and* $\pi^{\mathrm{p}}(x) \to \pi^{\mathrm{n}}(x)$ *as* $P \uparrow \infty$.

Proposition 2 implies that privatization is *not* preferred to nationalization from the viewpoint of the total social value under a *symmetric* cost structure.

Figure 8 shows the total social values under a *symmetric* cost structure for management effort, $\pi^{\mathrm{n}}(x)$ and $\pi^{\mathrm{p}}(x)$. Here, we set the parameter $b = 1$ under both nationalization and privatization.[23] Naturally, we see $\pi^{\mathrm{n}}(x) > \pi^{\mathrm{p}}(x)$ for finite penalty P, and $\pi^{\mathrm{p}}(x) \to \pi^{\mathrm{n}}(x)$ as P goes to infinity. The most interesting thing is that $\pi^{\mathrm{p}}(x)$ is *not* monotone increasing with P, although $\pi^{\mathrm{p}}_{\mathrm{g}}(x)$ and $\pi^{\mathrm{p}}_{\mathrm{f}}(x)$ are monotone increasing and decreasing with P, respectively (see Figures 6 and 7). Consequently, we conclude that an government's (individual) rationality does *not necessarily* lead to the total social rationality.

[Insert Figure 8 about here]

Under an *asymmetric* cost structure for management effort, we assume $b = 1$ under nationalization and $b \in (0, 1)$ under privatization. This assumption is reasonable because the cost disadvantage under nationalization offsets the information advantage under nationalization. In such a situation, it is important to note that the total social value under privatization may be larger than under nationalization. As a result, we have

Proposition 3 *Under an asymmetric cost structure, there exists some* $\hat{P}$ *such that* $\pi^{\mathrm{p}}(x) \geq \pi^{\mathrm{n}}(x)$ *for all* $P \geq \hat{P}$. *The government prefers privatization to nationalization if and only if* $P \geq \hat{P}$.

[23] The other parameters are the same as in the previous numerical examples (Figures 2 to 7).

Proposition 3 implies that privatization may be preferred to nationalization from the viewpoint of the total social value under an *asymmetric* cost structure.[24]

Figure 9 demonstrates $\pi^{\mathrm{n}}(x)$ and $\pi^{\mathrm{p}}(x)$ under an *asymmetric* cost structure for management effort. Here, we set the parameters, $b = 1$ under nationalization and $b = 0.75$ under privatization. Then, the *subsidy-only region* is on $0 \le P \le 3.1$, the *joint region* is on $3.1 \le P \le 42.1$, and the *audit-only region* is on $P \ge 42.1$. The most important result is that $\pi^{\mathrm{p}}(x)$ may be larger than $\pi^{\mathrm{n}}(x)$ under an *asymmetric* cost structure. In particular, on $P \ge \hat{P} = 51.8$, $\pi^{\mathrm{p}}(x)$ is strictly larger than $\pi^{\mathrm{n}}(x)$.

[Insert Figure 9 about here]

6 Concluding Remarks

This paper examines investment timing strategy under both nationalization and privatization. We find that the investment timing under privatization is *always* later than under nationalization, while the management effort under privatization is larger than under nationalization *in almost all cases*. We conclude that efficiency in allocation under nationalization is higher than under privatization, while efficiency in productivity under nationalization is lower than under privatization. These results are exactly the same as in Schmidt (1996a, 1996b) and fit well with the stylized facts. However, the model is very simple in many respects. An important aim for future research is to consider efficiency in allocation and productivity by imposing more complicated institutional settings.

Appendix

Derivation of ex post optimal contracts under privatization

Here, we derive the *ex post* actions under *privatization* $(x_i(\xi), s_i(\xi), p_i(\xi))$ in $\mathcal{M}^{\mathrm{p}}$ $(i \in \{1, 2\})$. For notational simplicity, in this appendix, we drop the parameter ξ and simply write (x_i, s_i, p_i).

In the optimization problem for the *ex post* actions under *privatization*, the Lagrangian can be formulated as:

$$\begin{aligned} \mathcal{L} &= q x_1^{-\beta}(x_1 - I_1 - s_1) + (1-q) x_2^{-\beta}(x_2 - I_2 - c(p_2)) \\ &\quad + \lambda_1 \left(x_1^{-\beta} s_1 - x_2^{-\beta}(\Delta I - p_2 P) \right) + \lambda_2 s_1 + \lambda_3 p_2, \end{aligned}$$

[24]Recall that this result is never obtained under a *symmetric* cost structure.

where λ_i $(i \in \{1,2,3\})$ denotes the multiplier on the constraints. The Kuhn-Turker conditions are

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x_1} &= q x_1^{-\beta}\Big(1-\beta x_1^{-\beta}(x_1-I_1-s_1)\Big)-\lambda_1 x_1^{-\beta}s_1\beta x_1^{-1}=0, &\text{(A.1)}\\
\frac{\partial \mathcal{L}}{\partial x_2} &= (1-q)x_2^{-\beta}\Big(1-\beta x_2^{-\beta}(x_2-I_2-c(p_2))\Big)+\lambda_1 x_2^{-\beta}(\Delta I-p_2P)\beta x_2^{-1}=0, &\text{(A.2)}\\
\frac{\partial \mathcal{L}}{\partial s_1} &= -(q-\lambda_1)x_1^{-\beta}+\lambda_2=0, &\text{(A.3)}\\
\frac{\partial \mathcal{L}}{\partial p_2} &= -\Big((1-q)c'(p_2)-\lambda_1 P\Big)x_2^{-\beta}+\lambda_3=0, &\text{(A.4)}\\
\frac{\partial \mathcal{L}}{\partial \lambda_1} &= x_1^{-\beta}s_1-x_2^{-\beta}(\Delta I-p_2P)=0, &\text{(A.5)}
\end{aligned}$$

and

$$\lambda_2 s_1=\lambda_3 p_2=0, \quad \lambda_i\geq 0 \quad (i\in\{1,2,3\}). \tag{A.6}$$

The solution depends on whether or not λ_2 and λ_3 are equal to zero. First, suppose that $\lambda_2>0$ and $\lambda_3>0$. Then we have $s_1=0$ and $p_2=0$. These imply $\Delta I=0$, which contradicts $\Delta I>0$. Therefore, at least one of λ_2 and λ_3 must be binding. Second, suppose that $\lambda_2=0$ and $\lambda_3>0$. Then we have $\lambda_1=q$. Obviously we have the solution in the *subsidy-only region* with $\frac{1-q}{q}c'(0)>P\geq 0$. Third, suppose that $\lambda_2=\lambda_3=0$. Then we obtain $s_1>0$ and $p_2>0$. It is straightforward to obtain the solution in the *joint region*. Finally, suppose that $\lambda_2>0$ and $\lambda_3=0$. Then we have $P>\frac{1-q}{q}c'(\frac{\Delta I}{P})$, and $P>\Delta I$ because of $p_2=\frac{\Delta I}{P}<1$. Therefore, we have the solution in the *audit-only region* with $P\leq \max\{\frac{1-q}{q}c'(\frac{\Delta I}{P}),\frac{\Delta I}{P}\}$. □

References

[1] Baron, D., and Besanko, D., (1984), Regulation, asymmetric information, and auditing. *RAND Journal of Economics*, 15, 447–470.

[2] Corneo, G., and Rob, R., (2003), Working in public and private firms. *Journal of Public Economics*, 87, 1335–1352.

[3] De Fraja, G., (1993), Productive efficiency in public and private firms. *Journal of Public Economics*, 50, 15–30.

[4] Dixit, A.K., and Pindyck, R.S. (1994), *Investment under uncertainty.* Princeton University Press, Princeton, NJ.

[5] Fudenberg, D., and Tirole, J., (1991), *Game theory.* MIT Press, Cambridge.

[6] Grenadier, S.R., and Wang, N., (2005), Investment timing, agency, and information. *Journal of Financial Economics*, 75, 493–533.

[7] Laffont, J., and Martimort, D., (2002), *The theory of incentives: the principal-agent model*. Princeton University Press, Princeton, NJ.

[8] Laffont, J., and Tirole, J., (1991), Privatization and incentives. *Journal of Law, Economics, and Organization*, 7, 84–105.

[9] Laffont, J., and Tirole, J., (1993), Privatization and incentives, in *A theory of incentive in procurement and regulation*. MIT Press, Cambridge, MA.

[10] Mas-Colell, A., Whinston, M., and Green, J.R., (1995), *Microeconomic theory*. Oxford Student Press, Oxford.

[11] Mæland, J., (2002), Valuation of irreversible investments and agency problems, forthcoming in Trigeorgis, L. (ed.), *Innovation, Organization and Strategy*. Cambridge University Press.

[12] McDonald, R., and Siegel, D. R., (1986), The value of waiting to invest. *Quarterly Journal of Economics*, 101, 707–727.

[13] Megginson, W., Nash, R., Van, M., (1994), The financial and operating performance of newly privatized firms: an international empirical analysis. *Journal of Finance*, 49, 403–452.

[14] Nishihara, M., and Shibata, T., (2007), The agency problem between the owner and the manager in real investment: the bonus-audit relationship. forthcoming in *Operations Research Letters*.

[15] Sappington, D, and Stiglitz, J., (1987), Privatization, information and incentives, *Journal of Policy Analysis and Management*, 6, 567–582.

[16] Shapiro, C., Willig, R, (1990), Economic rationales for the scope of privatization. in Suleiman E. and Waterbury, J. (ed.), *The political economy of public sector reform and privatization*. Westview Press. Boulder, CO.

[17] Salanié, B., (2005), *The economics of contracts*. 2nd Edition. MIT Press.

[18] Schmidt, K., (1996a), The costs and benefits of privatization: an incomplete contracts approach. *Journal of Law, Economics, and Organization*, 12, 1–24.

[19] Schmidt, K., (1996b), Incomplete contracts and privatization. *European Economic Review*, 40, 569–579.

[20] Shibata, T., (2007), Investment timing, asymmetric information, and audit structure: a real options framework. Working paper, No.30, Tokyo Metropolitan University, Tokyo.

[21] Shibata, T. and Nishihara, M, (2007), Agency problem with auditing in a real options model. Working paper, No. 32, Tokyo Metropolitan University, Tokyo.

[22] Shleifer, A., (1998), State versus private ownership. *Journal of Economic Perspectives*, 12, 133–150.

[23] Vickers, J. and Yarrow, G., (1991), Economic perspectives on privatization. *Journal of Economic Perspectives*, 5, 111-132.

[24] Villalonga, B., (2000), Privatization and efficiency: differentiating ownership effects from political, organizational, and dynamic effects. *Journal of Economic Behavior and Organization*, 42, 43–74.

[25] Williamson, O., (1985), *The economic institutions of capitalism.* Free Press, New York, NY.

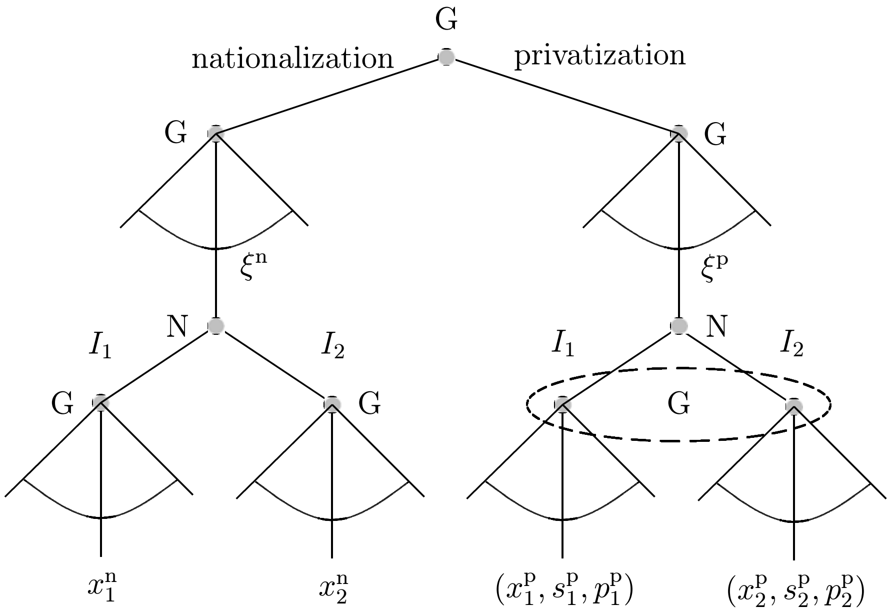

Figure 1: Structure of the Game

The government's actions in the subgames after nationalization and privatization imply the contracts between the government and the firm. The contract in the subgame after *nationalization* is $\mathcal{M}^{\mathrm{n}} = \{\xi, x_i(\xi); i = 1, 2\}$. The contract in the subgame after *privatization* is $\mathcal{M}^{\mathrm{p}} = \{\xi, x_i(\xi), s_i(\xi), p_i(\xi); i = 1, 2\}$. Here, let "G" and "N" refer to government and nature, respectively.

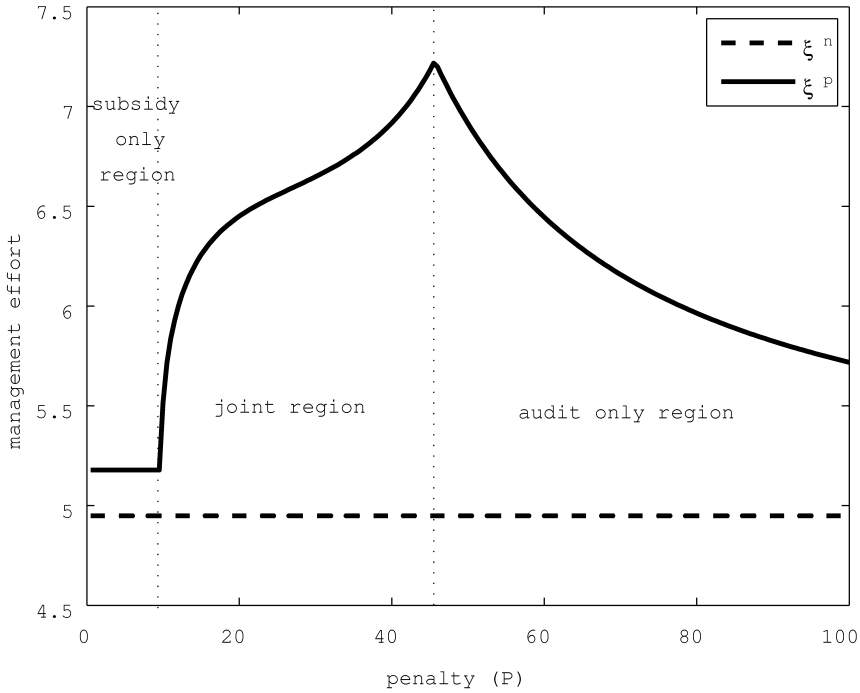

Figure 2: Management Efforts

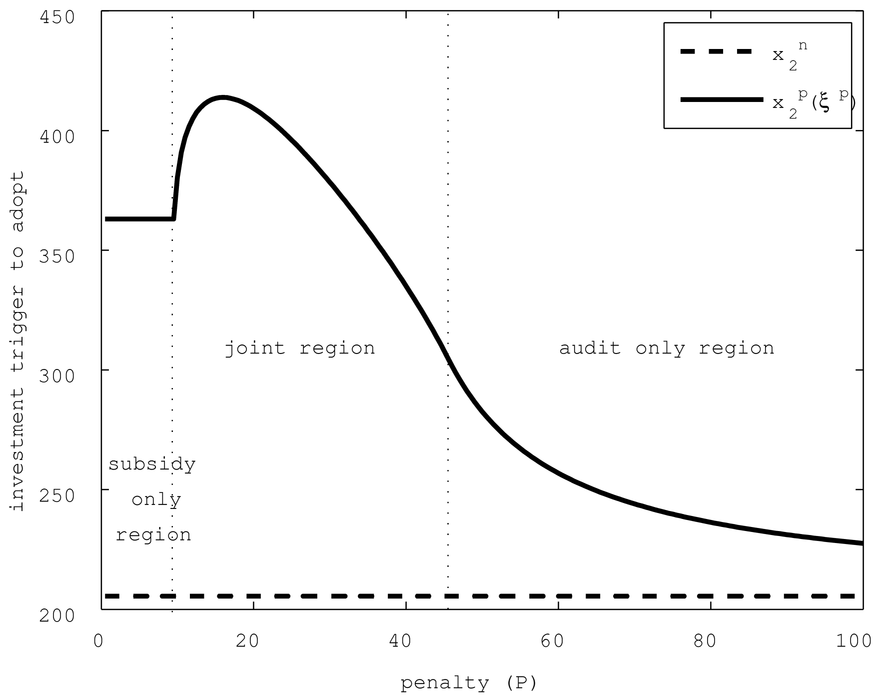

Figure 3: Investment Triggers

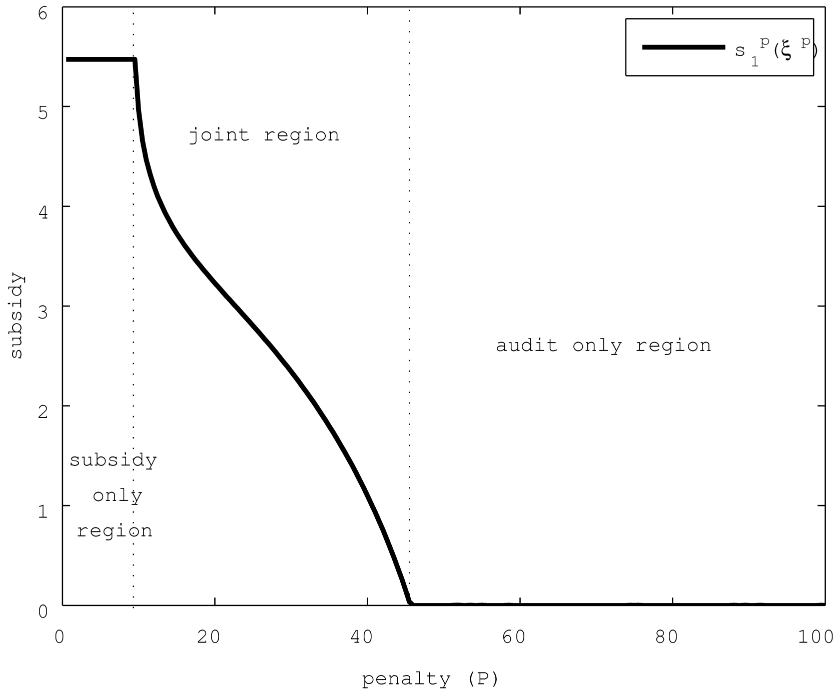

Figure 4: Subsidy

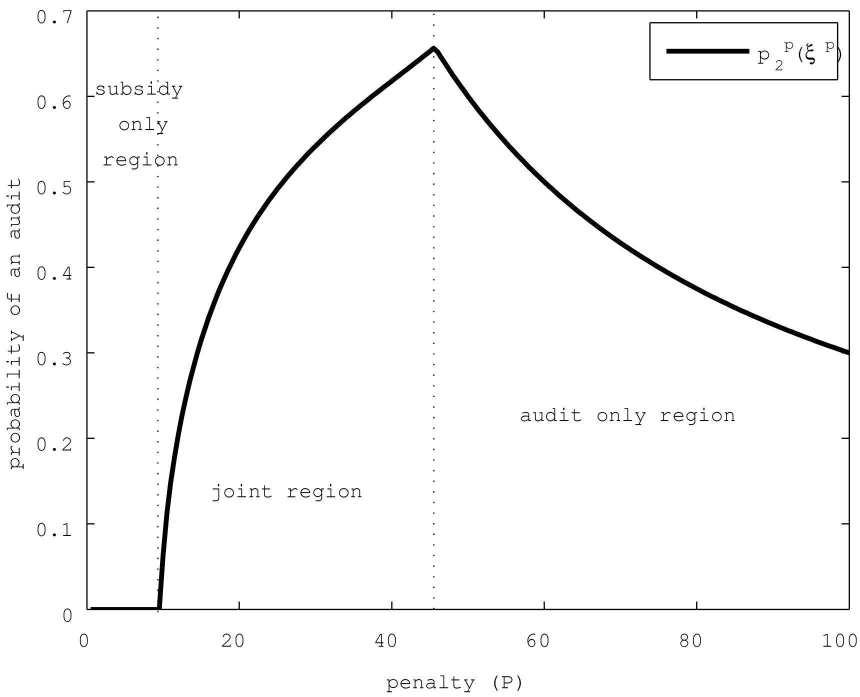

Figure 5: Probability of an Audit

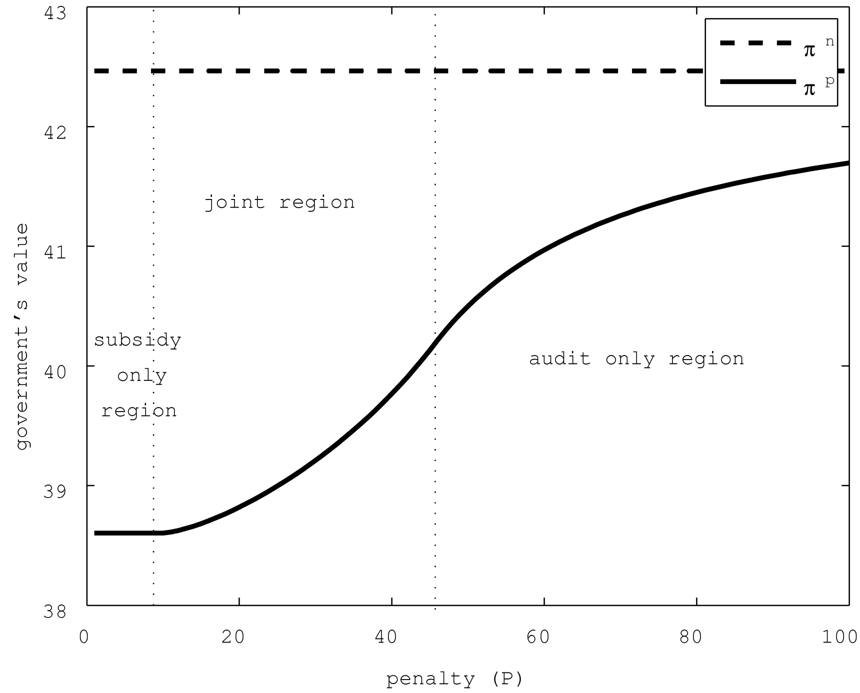

Figure 6: Government's Value

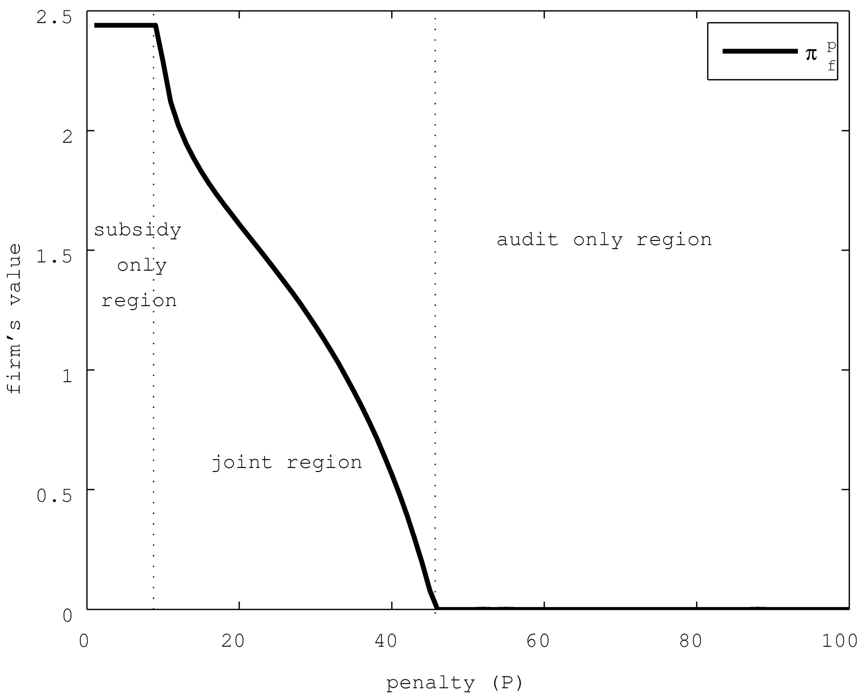

Figure 7: Firm's Value

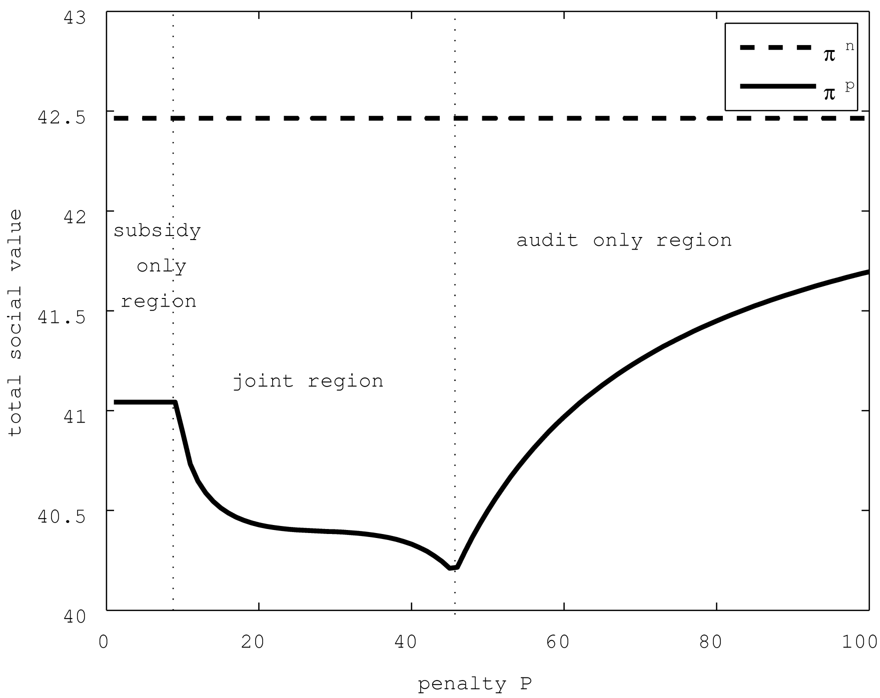

Figure 8: Total Social Value for Investment under Symmetric Cost Structure for Management Effort

The values $\pi^{\mathrm{n}}(x)$ and $\pi^{\mathrm{p}}(x)$ are depicted under the same parameter $b = 1$.

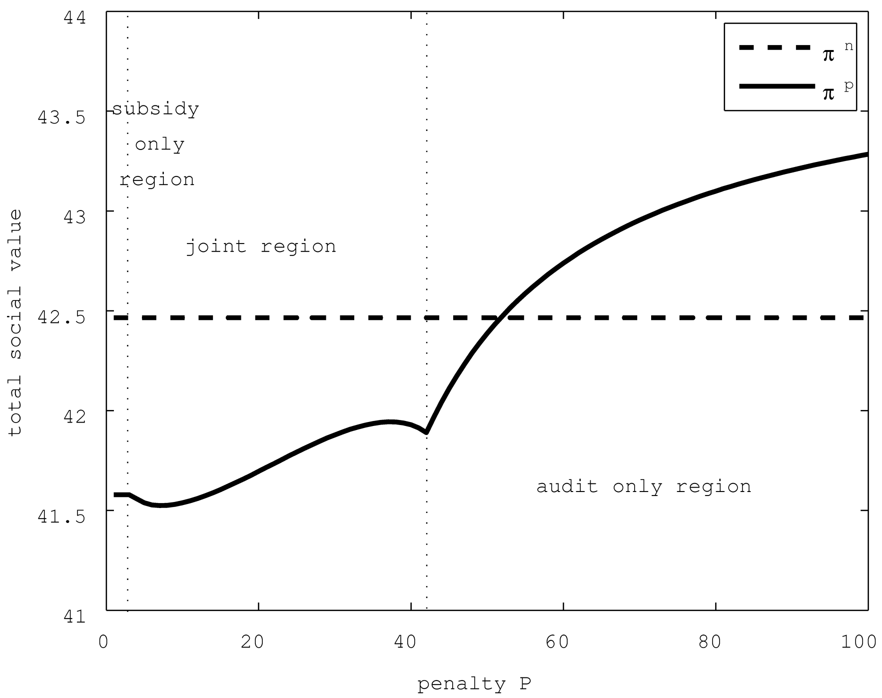

Figure 9: Total Social Value for Investment under Asymmetric Cost Structure for Management Effort

The value $\pi^{\mathrm{n}}(x)$ is depicted under $b = 1$, while $\pi^{\mathrm{p}}(x)$ is depicted under $b = 0.75$.