Research Paper Series

No. 33

The Relation Between Non-Bossiness and Monotonicity

Hideki Mizukami† and Takuma Wakayama‡

July 2007

†Faculty of Economics, University of Toyama, 3190 Gofuku, Toyama, 930-8555, JAPAN; mizukami@eco.u-toyama.ac.jp
‡Faculty of Urban Liberal Arts, Tokyo Metropolitan University, 1-1 Minami-Osawa, Hachioji, Tokyo 192-0397, JAPAN; t_waka@center.tmu.ac.jp

The Relation between Non-Bossiness and Monotonicity*

HIDEKI MIZUKAMI
Faculty of Economics
University of Toyama
mizukami@eco.u-toyama.ac.jp

TAKUMA WAKAYAMA†
Faculty of Urban Liberal Arts
Tokyo Metropolitan University
t_waka@center.tmu.ac.jp

July 3, 2007

Abstract

It is well-known that in some economic environments, non-bossiness and monotonicity are related to each other. In this paper, we have provided a new domain-richness condition, called *weak monotonic closedness*, on which non-bossiness in conjunction with individual monotonicity is equivalent to monotonicity. Moreover, we have obtained characterization results in several types of economies by applying our main result. The characterization results imply that many interesting social choice functions satisfy non-bossiness.

Keywords: Non-Bossiness, Individual Monotonicity, Monotonicity, Weak Monotonic Closedness.

JEL Classification Numbers: D51, D71, D78.

*This paper was previously entitled "Relationships between Non-Bossiness and Nash Implementability." We are grateful to an associate editor and two anonymous referees for their very helpful comments and suggestions. We are also grateful to Shigehiro Serizawa, Koji Takamiya, and seminar participants at the 2005 RIMS Workshop "Mathematics of Optimization: Methods and Practical Solutions" for their valuable comments.

†Correspondent: Faculty of Urban Liberal Arts, Tokyo Metropolitan University, 1-1 Minami-Osawa, Hachioji, Tokyo 192-0397, JAPAN.

1 Introduction

Mechanism design literature deals with a considerable number of allocation rules (or direct revelation mechanisms). The following example pertains to one characteristic of the allocation rules in a private goods economy: there is an agent, called a *boss*, who can change another agent's consumption bundle by changing her preferences without changing her own bundle. Allocation rules showing this characteristic were called *bossy* by Satterthwaite and Sonnenschein (1981); however, the concept of bossy allocation rules was already known, since the well-known Vickrey-Clarke-Groves type of allocation rules (Vickrey (1961), Clarke (1971), and Groves (1973)) had bossy characteristics. In this way, bossy allocation rules could be regarded as acceptable if the Vickrey-Clarke-Groves type of allocation rules appeared attractive. Nevertheless, Satterthwaite and Sonnenschein (1981) regarded bossy allocation rules as undesirable at least in terms of simplicity of design (see Satterthwaite and Sonnenschein (1981) for details). Therefore, they introduced the notion of *non-bossiness*, which requires that there should be no boss.

Ever since its inception, non-bossiness has been widely used in the literature on strategy-proofness. This is because in various types of economies, non-bossiness is "collusion-proof"— in the sense that when combined with strategy-proofness, non-bossiness implies coalitional strategy-proofness. However, coalitional strategy-proofness is generally too demanding, because it prevents not only self-enforcing coalitional manipulations but also non-self-enforcing coalitional manipulations. This indicates that there is no need to rule out coalitional manipulations that are not self-enforcing, unless an additional assumption that agents can sign binding agreements is imposed. Hence, when coupled with strategy-proofness, non-bossiness appears too strong without the additional assumption.[1] However, most of the literature has neglected to explain the reasonableness and desirability of non-bossiness per se. Therefore, thus far, this issue appears to be unresolved.

It has been shown that non-bossiness and *monotonicity* are related to each other in some economic environments such as pure exchange economies with the domain of classical preferences (e.g., Barberà and Jackson (1995)) and housing markets with the domain of strict preferences (e.g., Takamiya (2001)). So, we provide a new domain-richness condition including the above economic environments, called *weak monotonic closedness*. Our main result is that on weakly monotonically closed domains, (weak) monotonicity is equivalent to the conjunction of non-bossiness and individual (weak) monotonicity. That is, we show the "decomposition theorem," which says that monotonicity can be decomposed into non-bossiness and individual monotonicity in several economies. This finding, combined with that by Maskin (1999), implies the desirability of non-bossiness per se in the light of Nash implementability: non-bossiness is a necessary condition for Nash implementation.

We also apply the above decomposition theorem to generalized indivisible goods economies, pure exchange economies, and public good economies. Several studies have been made on characterizing monotonic social choice functions in many environments. Since many environments satisfy weak monotonic closedness, it is found from our decomposition theorem that in several environments, monotonic social choice functions satisfy non-bossiness. This implies that many interesting social choice functions satisfy non-bossiness.

[1] It may not be necessary for us to be concerned with manipulation by very large coalitions, because it is difficult to coordinate the actions of agents in such coalitions, as pointed out by Schummer (2000) and Serizawa (2006). Therefore, non-bossiness combined with strategy-proofness might still appear strong, even if the additional assumption is imposed.

2 Notation and Definitions

Let $N := \{1,2,\ldots,n\}$ be the set of *agents*, where $2 \leq n < +\infty$. Let $Z_i := X_i \times Y$ be the *consumption space* for agent $i \in N$, where X_i is an arbitrary non-empty set and Y is an arbitrary set, which may be empty. Let $A \subseteq X_1 \times X_2 \times \cdots \times X_n \times Y$ be the set of *feasible allocations*. Given $a \in A$, let $a_i = (x_i, y) \in X_i \times Y$ denote agent i's *consumption bundle*.

Each agent $i \in N$ has *preferences* over Z_i, which are represented by a complete and transitive binary relation R_i. The strict preference relation associated with R_i is denoted by P_i. Let $\mathscr{R}_i$ denote the set of possible preferences for agent $i \in N$. A *domain* is denoted by $\mathscr{R} := \mathscr{R}_1 \times \mathscr{R}_2 \times \cdots \times \mathscr{R}_n$. A *preference profile* is a list $R = (R_1, R_2, \ldots, R_n) \in \mathscr{R}$.

Let $LC_i(a;R_i) := \{b \in A \colon a_i \, R_i \, b_i\}$ be agent i's *lower contour set* of $a \in A$ at $R_i \in \mathscr{R}_i$. Let $UC_i(a;R_i) := \{b \in A \colon b_i \, R_i \, a_i\}$ be agent i's *upper contour set* of $a \in A$ at $R_i \in \mathscr{R}_i$.

A *social choice function* is a single-valued function $f \colon \mathscr{R} \to A$ that assigns a feasible allocation $a \in A$ to each preference profile $R \in \mathscr{R}$. Let f_i denote agent i's consumption bundle assigned by f. Given a social choice function f and a preference profile $R \in \mathscr{R}$, we write $f_i(R) = (f^{x_i}(R), f^y(R)) \in X_i \times Y$.

Now we introduce a domain-richness condition. A domain $\mathscr{R}$ is *weakly monotonically closed* if, for all $i \in N$, all $R_i, R_i' \in \mathscr{R}_i$, and all $a, b \in A$ with $a_i = b_i$, there exists $\bar{R}_i \in \mathscr{R}_i$ such that $LC_i(a;R_i) \subseteq LC_i(a;\bar{R}_i)$ and $LC_i(b;R_i') \subseteq LC_i(b;\bar{R}_i)$. Note that every rich domain in the sense of Dasgupta et al. (1979) is weakly monotonically closed, but not vice versa.[2] As shown by Dasgupta et al. (1979), for example, the domain of all quasi-linear preferences over public goods and transfers is not rich, but is weakly monotonically closed.

Next, we introduce the key axiom of this paper, called *non-bossiness* (Satterthwaite and Sonnenschein (1981)). Non-bossiness requires that if an agent changes her preferences but her consumption bundle is unchanged, then the bundle of each agent should be unchanged.

Definition 1 (Non-Bossiness). A social choice function f satisfies *non-bossiness* if, for all $R \in \mathscr{R}$, all $i \in N$, and all $R_i' \in \mathscr{R}_i$, if $f_i(R) = f_i(R_i', R_{-i})$, then $f(R) = f(R_i', R_{-i})$.

Finally, we introduce several notions of monotonicity.

Definition 2 (Several Notions of Monotonicity). We say that $R_i' \in \mathscr{R}_i$ is a *strictly monotonic transformation* of R_i at a if (i) $UC_i(a;R_i') \subseteq UC_i(a;R_i)$ and (ii) $a_i' \, P_i \, a_i$ for all $a' \in UC_i(a;R_i')$ with $a_i' \neq a_i$. Let $M(a;R_i)$ be the set of strictly monotonic transformation of R_i at a.

- *Monotonicity*: A social choice function f satisfies *monotonicity* if, for all $R, R' \in \mathscr{R}$, if $LC_i(f(R);R_i) \subseteq LC_i(f(R);R_i')$ for all $i \in N$, then $f(R') = f(R)$.

- *Individual Monotonicity*: A social choice function f satisfies *individual monotonicity* if, for all $R \in \mathscr{R}$, all $i \in N$, and all $R_i' \in \mathscr{R}_i$, if $LC_i(f(R);R_i) \subseteq LC_i(f(R);R_i')$, then $f_i(R_i', R_{-i}) = f_i(R)$.

- *Weak Monotonicity*: A social choice function f satisfies *weak monotonicity* if, for all $R, R' \in \mathscr{R}$, if $R_i' \in M(f(R);R_i)$ for all $i \in N$, then $f(R') = f(R)$.

- *Individual Weak Monotonicity*: A social choice function f satisfies *individual weak monotonicity* if, for all $R \in \mathscr{R}$, all $i \in N$, and all $R_i \in \mathscr{R}_i$, if $R_i' \in M(f(R);R_i)$, then $f_i(R_i', R_{-i}) = f_i(R)$.

[2] Rich domains in the sense of Dasgupta et al. (1979) are called *monotonically closed domains* in Maskin (1985).

Remark 1. The following facts follow from definitions.

(i) Individual monotonicity is weaker than monotonicity.

(ii) Individual weak monotonicity is weaker than weak monotonicity.

(iii) Weak monotonicity is weaker than monotonicity.

(iv) Individual weak monotonicity is weaker than individual monotonicity.

3 Main Results

Theorem 1 below provides the relationship between non-bossiness and monotonicity, which says that when the domain is weakly monotonically closed, monotonicity can be decomposed into non-bossiness and individual monotonicity.

Theorem 1. *Suppose that $\mathscr{R}$ is weakly monotonically closed. Then, a social choice function satisfies monotonicity if and only if it satisfies both non-bossiness and individual monotonicity.*

Proof. (The *only if* part): It follows from Remark 1 that monotonicity implies individual monotonicity. Thus, it is sufficient to show that monotonicity implies non-bossiness. Pick any $R \in \mathscr{R}$, any $i \in N$, and any $R'_i \in \mathscr{R}_i$ such that $f_i(R) = f_i(R'_i, R_{-i})$. We want to show $f(R) = f(R'_i, R_{-i})$.

Since $\mathscr{R}$ is weakly monotonically closed, we can choose $\bar{R}_i \in \mathscr{R}_i$ such that $LC_i(f(R);R_i) \subseteq LC_i(f(R);\bar{R}_i)$ and $LC_i(f(R'_i,R_{-i});R'_i) \subseteq LC_i(f(R'_i,R_{-i});\bar{R}_i)$. Since $LC_j(f(R);R_j) \subseteq LC_j(f(R);R_j)$ and $LC_j(f(R'_i,R_{-i});R_j) \subseteq LC_j(f(R'_i,R_{-i});R_j)$ for all $j \neq i$, monotonicity implies $f(\bar{R}_i,R_{-i}) = f(R)$ and $f(\bar{R}_i,R_{-i}) = f(R'_i,R_{-i})$, respectively. Thus, $f(R) = f(\bar{R}_i,R_{-i}) = f(R'_i,R_{-i})$.

(The *if* part): Pick any $R, R' \in \mathscr{R}$ such that $LC_i(f(R);R_i) \subseteq LC_i(f(R);R'_i)$ for all $i \in N$. We want to show $f(R') = f(R)$.

Step 1: $\boldsymbol{f(R'_i,R_{-i}) = f(R)}$. Pick any $i \in N$. Since $LC_i(f(R);R_i) \subseteq LC_i(f(R);R'_i)$, individual monotonicity implies $f_i(R'_i,R_{-i}) = f_i(R)$. So, we have $f(R'_i,R_{-i}) = f(R)$ by non-bossiness.

Step 2: $\boldsymbol{f(R'_i,R'_j,R_{-i,j}) = f(R'_i,R_{-i})}$. Pick any $j \in N \setminus \{i\}$. Since $f(R) = f(R'_i,R_{-i})$ by Step 1, $LC_j(f(R'_i,R_{-i});R_j) \subseteq LC_j(f(R'_i,R_{-i});R'_j)$. So, individual monotonicity implies $f_j(R'_i,R'_j,R_{-i,j}) = f_j(R'_i,R_{-i})$. Therefore, we obtain $f(R'_i,R'_j,R_{-i,j}) = f(R'_i,R_{-i})$ by non-bossiness.

Iteration of these steps for remaining agents in N yields $f(R') = f(R)$. $\square$

Remark 2. Weak monotonic closedness is required only to show that monotonicity implies non-bossiness.

It is easy to show that Theorem 1 remains true if we replace monotonicity and individual monotonicity with weak monotonicity and individual weak monotonicity, respectively.

Theorem 2. *Suppose that $\mathscr{R}$ is weakly monotonically closed. Then, a social choice function satisfies weak monotonicity if and only if it satisfies both non-bossiness and individual weak monotonicity.*

4 Applications

4.1 Nash Implementation

In this subsection, we examine the relationships between non-bossiness and Nash implementability.[3] Maskin (1999) showed that monotonicity is necessary and almost sufficient for Nash implementation (See Maskin (1999) for the definition of Nash implementation). When coupled with the results of Maskin (1999), Theorem 1 leads to the following corollaries.

Corollary 1. *Suppose that $\mathscr{R}$ is weakly monotonically closed. Then, if a social choice function is Nash implementable, then it satisfies both non-bossiness and individual monotonicity.*

Corollary 2. *Suppose that $n \geq 3$. Then, if a social choice function satisfies non-bossiness, individual monotonicity, and no veto power,[4] then it is Nash implementable.*

Corollary 1 implies that every social choice function that violates non-bossiness or individual monotonicity is never Nash implementable, provided that $\mathscr{R}$ is weakly monotonically closed. So, bossy social choice functions (e.g., the "second-price auction" (Vickrey (1961)), the "pivotal mechanism" (Clarke (1971)), the "inversely dictatorial rule" (Zhou (1991)), etc.) are never Nash implementable. Corollary 2 provides a convenient way to check Nash implementability of social choice functions, since non-bossiness is an easy-to-check condition.

Corollaries 1 and 2 together indicate that non-bossiness has close relationships to Nash implementability, in the sense that non-bossiness is a necessary condition for Nash implementation and is part of the sufficient condition for Nash implementation. These relationships tell us that non-bossiness per se is desirable from the point of view of Nash implementability. This desirability of non-bossiness per se seems important in terms of requiring no additional assumption, which is in contrast to the desirability mentioned in the introduction.

4.2 Generalized Indivisible Goods Economies

In this subsection, we consider generalized indivisible goods economies introduced by Sönmez (1999).[5] For all $i \in N$, let e_i be the set of indivisible goods that agent i initially owns. Let $K = \bigcup_{i \in N} e_i$ and $Y = \emptyset$. For all $i \in N$, the consumption space for agent $i \in N$ is $Z_i = X_i = 2^K$. Let

$$A \subseteq \left\{(x_1, x_2, \ldots, x_n) \in \left(2^K\right)^n : \text{for all } k \in K, \#\{i \in N \colon k \in x_i\} = 1\right\} \subset X_1 \times X_2 \times \cdots \times X_n$$

be the set of feasible allocations.[6] Assume $e = (e_1, e_2, \ldots, e_n) \in A$. This general model is an extension of well-known types of allocation problems which have been studied (e.g., housing markets (Shapley and Scarf (1974)), marriage problems (Gale and Shapley (1962)), indivisible goods exchange economies, etc.).

We now consider the domain of strict preferences. This domain is an important example of weakly monotonically closed domains. Then, the following is a direct corollary of Theorem 1.

[3] In housing markets (Shapley and Scarf (1974)) with the domain of strict preferences, Takamiya (2001) has already shown that non-bossiness has relationships to Nash implementability. In other environments, however, the relationship of non-bossiness to Nash implementability is not yet known.

[4] A social choice function f satisfies *no veto power* if, for all $R \in \mathscr{R}$, all $a \in A$, and all $i \in N$, if $a_j \; R_j \; b_j$ for all $b \in A$ and all $j \neq i$, then $f(R) = a$.

[5] We are grateful to an anonymous referee who suggested this application.

[6] Given a set B, let $\#B$ be the cardinality of the set B.

Corollary 3. *In generalized indivisible goods economies with the domain of strict preferences, a social choice function satisfies monotonicity if and only if it satisfies both non-bossiness and individual monotonicity.*

Corollary 3 generalizes the result of Takamiya (2001) who showed the same equivalence theorem in housing markets with the domain of strict preferences.

Takamiya (2003) and Sönmez (1999) showed that in generalized indivisible goods economies with the domain of strict preferences, monotonicity is equivalent to coalitional strategy-proofness, and whenever the *core correspondence*[7] is nonempty-valued, if a social choice function satisfies *strategy-proofness*,[8] *individual rationality*,[9] and *Pareto efficiency*,[10] then it is a selection from the core correspondence and the core correspondence must be single-valued, respectively. When coupled with the results of Sönmez (1999) and Takamiya (2003), Corollary 3 leads to the following corollary.

Corollary 4. *Consider generalized indivisible goods economies with the domain of strict preferences. Whenever the core correspondence is nonempty-valued, if a social choice function satisfies non-bossiness, individual monotonicity, individual rationality, and Pareto efficiency, then it is a selection from the core correspondence, and the core correspondence is single-valued.*

We obtain the converse of Corollary 4 by Corollary 3 and the result of Takamiya (2003), which shows that in generalized indivisible goods economies with the domain of strict preferences, if the core correspondence is single-valued, then the unique selection from the core correspondence satisfies monotonicity.

Corollary 5. *In generalized indivisible goods economies with the domain of strict preferences, if the core correspondence is single-valued, then the unique selection from the core correspondence satisfies both non-bossiness and individual monotonicity.*

Corollaries 4 and 5 tell us that in housing markets with the domain of strict preferences, the "strong core solution" is the only social choice function which satisfies non-bossiness, individual monotonicity, individual rationality, and Pareto efficiency.

4.3 Pure Exchange Economies

There are ℓ private goods. For all $i \in N$, $X_i = \mathbb{R}^{\ell}_{+}$. Let $e = (e_1, e_2, \ldots, e_n) \in \mathbb{R}^{\ell \cdot n}_{+}$ be the endowment vector, where $e_i \in \mathbb{R}^{\ell}_{+}$ is agent i's endowment. Let $Y = \emptyset$. Thus, for all $i \in N$, the consumption space for agent $i \in N$ is $Z_i = X_i = \mathbb{R}^{\ell}_{+}$. The set of feasible allocations is $A = \left\{a = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^{\ell \cdot n}_{+} \colon \sum_{i \in N} x_i = \sum_{i \in N} e_i\right\}$. A preference relation R_i is *classical* if it is continuous on Z_i, strictly monotone on the interior of Z_i, and strictly convex on the interior of Z_i. The following is a direct corollary of Theorem 2.

Corollary 6. *In pure exchange economies with the domain of classical preferences, a social choice function satisfies weak monotonicity if and only if it satisfies both non-bossiness and individual weak monotonicity.*

[7]Let $a, b \in A$, $R \in \mathscr{R}$, and $S \subseteq N$. Then *a dominates b via S under R* if (i) $\bigcup_{i \in S} a_i \subseteq \bigcup_{i \in S} e_i$; (ii) $a_i \; R_i \; b_i$ for all $i \in S$; and (iii) $a_j \; P_j \; b_j$ for some $j \in S$. The *core* is the set of all allocations which are not dominated by any other allocations. The *core correspondence* is the correspondence that assigns the set of allocations in the core to each preference profile.

[8]A social choice function f satisfies *strategy-proofness* if, for all $R \in \mathscr{R}$ and all $i \in N$, there is no $R'_i \in \mathscr{R}_i$ such that $f_i(R'_i, R_{-i}) \; P_i \; f_i(R)$.

[9]A social choice function f satisfies *individual rationality* if, for all $R \in \mathscr{R}$ and all $i \in N$, $f_i(R) \; R_i \; e_i$.

[10]A social choice function f satisfies *Pareto efficiency* if, for all $R \in \mathscr{R}$, there is no other allocation a such that $a_i \; R_i \; f_i(R)$ for all $i \in N$ and $a_j \; P_j \; f_j(R)$ for some $j \in N$.

This result improves upon the well-known result of Barberà and Jackson (1995), which states that non-bossiness together with strategy-proofness implies weak monotonicity in pure exchange economies with the domain of classical preferences, because individual weak monotonicity is much weaker than strategy-proofness. An example of social choice functions satisfying non-bossiness and individual weak monotonicity in pure exchange economies with the domain of classical preferences is the "voluntary trading rule" (Barberà (2001)).

4.4 Public Good Economies

There are one private good and one public good. For all $i \in N$, let $X_i = \mathbb{R}_+$ be i's private good consumption space and $e_i \in \mathbb{R}_+$ i's endowment. Let $Y = \mathbb{R}_+$ be the public good consumption space. Thus, for all $i \in N$, the consumption space for agent $i \in N$ is $Z_i = X_i \times Y = \mathbb{R}_+ \times \mathbb{R}_+$. The set of feasible allocations is $A = \left\{a = (x_1, x_2, \ldots, x_n, y) \in \mathbb{R}^n_+ \times \mathbb{R}_+ \colon C(y) \leq \sum_{i \in N}(e_i - x_i)\right\}$, where $C \colon \mathbb{R}_+ \to \mathbb{R}_+$ is an increasing function. A preference relation R_i is *quasi-linear* if there is a function $v_i \colon \mathbb{R}_+ \to \mathbb{R}$ such that her preference relation can be represented by the function assigning to each bundle $(x_i, y) \in Z_i$ the value $u_i(x_i, y) = v_i(y) + x_i$. We denote by $\mathscr{R}^Q$ the *domain of quasi-linear preferences*. Note that the the domain of quasi-linear preferences is weakly monotonically closed, but not monotonically closed.

In this subsection, we study how agents share the cost of the public good. We now define *preference independent cost share rules* which were first introduced by Serizawa (2006).

Definition 3 (Preference Independent Cost Share Rules). A social choice function f is a *preference independent cost share rule* if there is a list of norm cost share functions $(t_1, t_2, \ldots, t_n)$ such that (i) for all $i \in N$, $t_i \colon Y \to \mathbb{R}_+$, (ii) for all $R \in \mathscr{R}$, $C(f^y(R)) \leq \sum_{i \in N} t_i(f^y(R))$, and (iii) for all $i \in N$ and all $R \in \mathscr{R}$, $f^{x_i}(R) = e_i - t_i(f^y(R))$.

Under a preference independent cost share rule, each agent has her own *norm cost share function* which assigns her cost share to each level of the public good. Preference independent cost share rules require agents to share the cost of the public good according to predetermined their norm cost share functions. "Equal cost share rules" and "proportional cost share rules" are examples of preference independent cost share rules.

Theorem 3. *Consider public good economies where the domain is either classical or quasi-linear. If a social choice function satisfies non-bossiness and individual weak monotonicity, then it is a preference independent cost share rule.*

Proof. Suppose that f is not a preference independent cost share rule. Without loss of generality, we consider $\mathscr{R} = \mathscr{R}^Q$. Let $R, R' \in \mathscr{R}^Q$ be such that $f^y(R) = f^y(R')$ but $f^{x_i}(R) \neq f^{x_i}(R')$ for some $i \in N$.

Step 1: $\boldsymbol{f(R''_j, R_{-j}) = f(R)}$ and $\boldsymbol{f(R''_j, R'_{-j}) = f(R')}$. Pick any $j \in N$. Since $f^y(R) = f^y(R')$, there exists $R''_j \in \mathscr{R}^Q_j$ such that $R''_j \in M(f(R); R_j) \cap M(f(R'); R'_j)$. By individual weak monotonicity, $f_j(R''_j, R_{-j}) = f_j(R)$ and $f_j(R''_j, R'_{-j}) = f_j(R')$. By non-bossiness, $f(R''_j, R_{-j}) = f(R)$ and $f(R''_j, R'_{-j}) = f(R')$.

Step 2: $\boldsymbol{f(R''_j, R''_k, R_{-j,k}) = f(R''_j, R_{-j})}$ and $\boldsymbol{f(R''_j, R''_k, R'_{-j,k}) = f(R''_j, R'_{-j})}$. Pick any $k \in N \setminus \{j\}$. Since $f^y(R''_j, R_{-j}) = f^y(R) = f^y(R') = f^y(R''_j, R'_{-j})$ by Step 1, there exists $R''_k \in \mathscr{R}^Q_k$ such that $R''_k \in M(f(R''_j, R_{-j}); R_k) \cap M(f(R''_j, R'_{-j}); R'_k)$. Then by individual weak monotonicity, $f_k(R''_j, R''_k, R_{-j,k}) = f_k(R''_j, R_{-j})$ and $f_k(R''_j, R''_k, R'_{-j,k}) = f_k(R''_j, R'_{-j})$. By non-bossiness, $f(R''_j, R''_k, R_{-j,k}) = f(R''_j, R_{-j})$ and $f(R''_j, R''_k, R'_{-j,k}) = f(R''_j, R'_{-j})$.

Iterating the above steps for further agents in N provides $f(R) = f(R'') = f(R')$, which is a contradiction. □

Serizawa (2006) showed that when the domain is either classical (see Subsection 4.3 for the definition of classical preferences) or quasi-linear, if a social choice function is effectively pairwise strategy-proof, then it is a non-bossy preference independent cost share rule. Since non-bossiness and individual weak monotonicity are weaker than effective pairwise strategy-proofness, Theorem 3 strengthens Theorem 1 of Serizawa (2006). However, there exists a preference independent cost share rule that violates weak monotonicity, that is, violates either non-bossiness or individual weak monotonicity. Example 2 of Serizawa (2006) illustrates the rule.[11]

5 Conclusion

In this paper, we provided the new notion of a rich domain, called weakly monotonically closed domains, on which non-bossiness combined with individual monotonicity would be equivalent to monotonicity. That is, on weakly monotonically closed domains, monotonicity can be decomposed into non-bossiness and individual monotonicity.

By applying this decomposition theorem, we examined the desirability of non-bossiness per se in the framework of Nash implementation. As pointed out by Satterthwaite and Sonnenschein (1981), non-bossiness is automatically satisfied in pure public goods economies, that is, in economies with non-excludability and non-rivalness. This means that bossiness is characteristic to economies with excludability or rivalness, such as private goods economies, excludable public goods economies, and the commons. Therefore, one negative aspect of imposing non-bossiness is that it rules out the social choice functions that are inherent to economies with excludability or rivalness to identify such economies with pure public goods economies. After taking this into account, Corollary 1 showed the impossibility of Nash implementation in economies with excludability or rivalness. That is, the corollary indicated that in economies with excludability or rivalness, it would be impossible to implement bossy social choice functions in Nash equilibria, which embody the characteristic inherent to those economies.

[11] Although Serizawa (2006) did not show that the rule violates weak monotonicity, it is easy to check the fact.

References

BARBERÀ, S. (2001): "An Introduction to Strategy-Proof Social Choice Functions," *Social Choice and Welfare*, 18, 619–653.

BARBERÀ, S. AND M. O. JACKSON (1995): "Strategy-Proof Exchange," *Econometrica*, 63, 51–87.

CLARKE, E. H. (1971): "Multipart Pricing of Public Goods," *Public Choice*, 11, 17–33.

DASGUPTA, P., P. J. HAMMOND, AND E. MASKIN (1979): "The Implementation of Social Choice Rules: Some General Results on Incentive Compatibility," *Review of Economic Studies*, 46, 185–216.

GALE, D. AND L. S. SHAPLEY (1962): "College Admissions and the Stability of Marriage," *American Mathematical Monthly*, 69, 9–15.

GROVES, T. (1973): "Incentives in Teams," *Econometrica*, 41, 617–631.

MASKIN, E. (1985): "The Theory of Implementation in Nash Equilibrium: A Survey," in *Social Goals and Social Organization*, ed. by L. Hurwicz, D. Schmeidler, and H. Sonnenschein, Cambridge University Press, 173–204.

——— (1999): "Nash Equilibrium and Welfare Optimality," *Review of Economic Studies*, 66, 23–38.

SATTERTHWAITE, M. A. AND H. SONNENSCHEIN (1981): "Strategy-Proof Allocation Mechanisms at Differentiable Points," *Review of Economic Studies*, 48, 587–597.

SCHUMMER, J. (2000): "Manipulation through Bribes," *Journal of Economic Theory*, 91, 180–198.

SERIZAWA, S. (2006): "Pairwise Strategy-Proofness and Self-Enforcing Manipulation," *Social Choice and Welfare*, 26, 305–331.

SHAPLEY, L. S. AND H. SCARF (1974): "On Cores and Indivisibility," *Journal of Mathematical Economics*, 1, 23–37.

SÖNMEZ, T. (1999): "Strategy-Proofness and Essentially Single-Valued Cores," *Econometrica*, 67, 677–689.

TAKAMIYA, K. (2001): "Coalition Strategy-Proofness and Monotonicity in Shapley-Scarf Housing Markets," *Mathematical Social Sciences*, 41, 201–213.

——— (2003): "On Strategy-Proofness and Essentially Single-Valued Cores: A Converse Result," *Social Choice and Welfare*, 20, 77–83.

VICKREY, W. (1961): "Counterspeculation, Auctions, and Competitive Sealed Tenders," *Journal of Finance*, 16, 8–37.

ZHOU, L. (1991): "Inefficiency of Strategy-Proof Allocation Mechanisms in Pure Exchange Economies," *Social Choice and Welfare*, 8, 247–254.