Research Paper Series

No. 32

Agency Problem with Auditing in a Real Options Model

Takashi Shibata and Michi Nishihara

May 2007

Graduate School of Social Sciences, Tokyo Metropolitan University

Agency Problem with Auditing in a Real Options Model*

Takashi Shibata[a†] and Michi Nishihara[b‡]

[a] *Graduate School of Social Science, Tokyo Metropolitan University.*
[b] *Center for the Study of Finance and Insurance, Osaka University.*

(First Version: May 15, 2007)

Abstract: This paper examines investment timing by the manager in a decentralized firm in the presence of asymmetric information. In particular, we extend the agency problem in a real options model to incorporate an audit technology which allows the owner, at a cost, to verify private information. The implied investment triggers include those in three related papers: standard full information model (e.g., McDonald and Siegel, 1986); Grenadier and Wang (2005); Shibata (2007). An increase in the penalty for the manager's false report *always* reduces inefficiency in the investment triggers, while it does *not necessarily* reduce inefficiency in the total social welfare. Most importantly, however, the full information investment triggers and total social welfare can be approximated arbitrarily closely by making the penalty sufficiently large.

Keywords: Real Options; Asymmetric Information; Agency Conflicts; Bonus-incentive; Audit; Penalty.

JEL classification: G13; D82.

---

*We would like to thank Masao Fukushima, Chiaki Hara, and Masaaki Kijima for helpful comments.
†Correspondence author. Email: `tshibata@tmu.ac.jp`
‡Email: `nishihara@sigmath.es.osaka-u.ac.jp`

1 Introduction

In most modern corporations, shareholders delegate the investment decision to managers, taking advantage of managers' special skills and expertise. In this situation, asymmetric information is likely between them. Asymmetric information is a situation where a portion of the underlying state variable is privately observed by the managers, while it is not observed by the owners. The managers with private information have an incentive to provide a false report and then divert free cash flow to themselves. Thus, asymmetric information leads to agency conflicts.

The real options model has become a standard framework for investment timing decisions in corporate finance. An excellent overview of the standard real options approach is found in Dixit and Pindyck (1994) and Trigeorgis (1996). In the standard real options model, however, there are no agency conflicts between an owner and a manager, because the firm is assumed to be managed by the owner.[1]

Several studies have begun the task of incorporating agency conflicts in the real options model. Bjerksund and Stensland (2000), Mæland (2002), and Grenadier and Wang (2005) develop models of investment timing in the presence of agency conflicts arising from asymmetric information between the owner and the manager.[2] In such a situation, the owner must design a contract to provide incentives for the manager to truthfully reveal private information, in that the owner gives a bonus-incentive that is contingent on the investment timing. The implied investment timings then differ significantly from those in the standard full information real options model. Although these strategies turn out to be suboptimal, they will reduce the owner's losses arising from asymmetric information. Without any incentive mechanism that induces the manager to truthfully reveal private information, the owner suffers further distortions.

To our best knowledge, there has been little examination of contracts other than the bonus-incentive in a real options model under asymmetric information. The owner may increase his/her own value by designing other mechanisms to induce the manager to truthfully reveal private information. One important way is to use an audit technology that can detect the manager's untruthful report and provide some penalty after detection

[1] Recently, the standard real options model has been extended in various ways. For example, Weeds (2002), Lambrecht and Perraudin (2003), and Nishihara and Fukushima (2007) consider investment timing by taking into account strategic interactions. Bernardo and Chowdhry (2002) and Shibata (2006) analyze investment decisions under incomplete information.

[2] While these papers focus on the agency conflicts between owners and managers, a similar problem exists between stockholders and bondholders. Mello and Parsons (1992), Mauer and Triantis (1994), Leland (1998), Mauer and Ott (2000), Morellec (2001), and Mauer and Sarkar (2005) examine the agency problem between stockholders and bondholders using the real options approach.

of a false report.[3] That is, giving the bonus-incentive to the manager can be regarded as the "carrot," while auditing and fining the manager can be regarded as the "stick." Thus, designing the optimal contract with both the bonus-incentive and audit technology is natural. In most modern corporations, the audit system is set so that shareholders may inspect managers' behavior. Shibata (2007, hereafter, S model) considers the model of investment timing developed by Grenadier and Wang (2005, hereafter, GW model) to incorporate audit technology, and shows that it *always* leads to an increase in the owner's value, but *not always* leads to an increase in total social welfare. Thus, the GW and S models are defined as the *agency problem without auditing* and the *agency problem with auditing under the limited-liability constraints on penalties*, respectively. In the S model, the *limited-liability* constraints on penalties are assumed when the manager is fined by detecting a false announcement. However, in equilibrium the manager is not fined because the manager truthfully reveals private information. Therefore, what is of great interest is to investigate how the agency problem influences investment timing, the owners' value, and total social welfare, by removing the *limited-liability* constraints on penalties.

In this paper, we consider investment timing by the manager in a decentralized firm in the presence of asymmetric information. As explained above, the *limited-liability* constraints on penalties are assumed in an S model that has extended the model developed by the GW model to incorporate audit technology. We investigate the *agency problem with auditing* by removing the *limited-liability* constraints on penalties. Thus, we simply call our model the *agency problem with auditing*.

In the *agency problem with auditing*, the implied investment triggers can be derived in three feasible regions because the nature of the investment triggers depends on the magnitude of the penalty. Three feasible regions are the *bonus-incentive only region*, the *joint (combination) bonus-incentive and auditing region*, and the *auditing only region*. Then, the implied investment triggers include those in three related problems, i.e., the *full information problem* (standard real options model, e.g., McDonald and Siegel, 1986), the *agency problem without auditing* (GW model), and the *agency problem with auditing under the limited-liability constraints on penalties* (S model). In particular, the implied investment trigger in the *bonus-incentive only region* is equivalent to the one in the *agency problem without auditing* (GW model). The implied investment trigger in the *joint (combination) bonus-incentive and auditing region* includes the one in the *agency problem with auditing under the limited-liability constraints on penalties* (S model). Most importantly, the full information investment trigger can be approximated by making the penalty for the manager's false report sufficiently large. That is, the implied investment trigger in the

[3]See Townsend (1979), Baron and Besanko (1984), and Laffont and Tirole (1986).

auditing only region converges to the one in the *full information problem*, as the penalty is increased without limit. Consequently, the implied investment triggers in the *agency problem with auditing* include those in three related problems.

In the *agency problem with auditing*, we analyze inefficiency in investment timings and total social loss arising from asymmetric information. On the one hand, an increase in the penalty for a manager's false report decreases the investment triggers. On the other hand, an increase in the penalty does *not necessarily* decrease the total social loss while it *always* increases the owner's value. These results imply that an owner's (individual) rationality does not necessarily leads to total social rationality. As a result, an increase in the penalty does not necessarily reduce inefficiency in total social welfare while it *always* reduces inefficiency in investment timing.

We structure the remainder of the paper as follows. Section 2 describes the related papers of our model. Considering the problems of the three related papers as a benchmark before analyzing our problem is useful. Section 3 provides the solution to our model, i.e., the agency problem with auditing. We then discuss the properties of the solution. In Section 4, we investigate the model implications. Section 5 concludes.

2 Related papers

In this section, we begin with a description of the setup in the agency problem. We then briefly review three related papers. As a benchmark, we provide the solution to the *full information problem*, the *agency problem without auditing*, and the *agency problem with auditing under limited-liability constraints on penalties*. These problems are the same as those in the McDonald and Siegel (standard real options), the Grenadier and Wang, and the Shibata models, respectively.

2.1 Model setup

The owner of a firm has the option to invest in a single project. We assume that the owner (principal) delegates the investment decision to a manager (agent). Throughout our analysis, we suppose that capital markets are frictionless, and agents are risk neutral and can borrow and lend freely at a constant interest rate, $r > 0$.

We assume that the project revenue $(X_t)_{t\geq 0}$ follows a geometric Brownian motion, i.e.,

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}z_t, \quad X_0 = x, \tag{1}$$

where $(z_t)_{t\geq 0}$ denotes the standard Brownian motion, and where the mean growth rate μ, as well as the volatility σ, are positive constants. For convergence, we assume that $r > \mu$.

We assume that the cost expenditure to undertake the investment, which we denote by I, is completely sunk. The cost expenditure, I, could take one of two possible values: I_1 or I_2 with $I_2 > I_1$, where $I_i > 0$ for all $i \in \{1, 2\}$. We denote $\Delta I = I_2 - I_1$. We assume that I_1 represents a "lower cost" expenditure and I_2 represents a "higher cost" expenditure. The probability of drawing I_1 equals q, an exogenous variable.

We assume that the project revenue, X_t, is observed by both the owner and the manager. However, the cost expenditure is privately observed only by the manager. Immediately after making a contract with the owner at time zero, the manager observes whether the cost expenditure is of "lower cost" or "higher cost." On the other hand, the owner cannot observe the true value of I. Therefore, the owner must induce the manager to truthfully reveal private information at the time when the manager undertakes the investment. While the manager could attempt to report I_2 when the true value is I_1, in equilibrium the manager will report the true value to the owner at the time when the manager undertakes the investment.

Although the owner cannot contract on the cost expenditure privately observed by the manager, the owner can contract on the observable component, project value, X_t. Contingent on the trigger to undertake the investment $x(I_i)$ when $I = I_i$, the owner must design the optimal contract both to provide the bonus-incentive and to audit the manager in order to truthfully reveal private information.[4]

2.2 Full information problem (standard real options model)

As a benchmark, the first problem is the *full information problem*. We consider the optimization problem when the owner observes the true value of I. This problem is equivalent to the problem in which there is no delegation of the investment decision because the manager has no information advantage. Thus, this solution turns out to be the *full information no-agency* solution.

Let $V(x; I_i)$ denote the project value function for $I = I_i$ for each i ($i \in \{1, 2\}$). The value, $V(x; I_i)$, is formulated as:

$$V(x; I_i) = \sup_{\tau_i} \mathbb{E}^x \left[\mathrm{e}^{-r\tau_i}(X_{\tau_i} - I_i) \right], \quad i \in \{1, 2\}, \tag{2}$$

for $\tau_i > 0$ at time zero $t = 0$. Here, $\mathbb{E}^x[\cdot]$ denotes the expectation operator given that

[4]The assumption that a portion of the project value is privately observed only by one person (here, the manager) and not observed by the other (here, the owner) is quite common in the asymmetric information literature. An excellent overview of the analysis of asymmetric information situations is found in Fudenberg and Tirole (1991), Mas-Colell et al. (1995), Laffont and Martimort (2002), and Salanié (2005).

$X_0 = x$, and τ_i is the time that the investment is exercised at the trigger $x_i = x(I_i)$ for each i ($i \in \{1,2\}$), i.e., $\tau_i := \inf\{t \geq 0; X_t = x_i\}$. In this paper, it is assumed that the current state value $X_0 = x$ is sufficiently low so that the investment is not undertaken immediately. Mathematically, we assume that $\tau_i > 0$ and $x < x_i$ for all i ($i \in \{1,2\}$). The value function for $I = I_i$ are

$$V(x; I_i) = \left(\frac{x}{x_i}\right)^{\beta} (x_i - I_i), \tag{3}$$

where β is defined by

$$\beta = \frac{1}{\sigma^2}\Big(- \Big(\mu - \frac{1}{2}\sigma^2\Big) + \sqrt{\Big(\mu - \frac{1}{2}\sigma^2\Big)^2 + 2r\sigma^2}\Big) > 1. \tag{4}$$

In the *full information problem*, the owner solves the following optimization problem:

$$\max_{x_1, x_2} \quad q\left(\frac{x}{x_1}\right)^{\beta} (x_1 - I_1) + (1-q)\left(\frac{x}{x_2}\right)^{\beta} (x_2 - I_2), \tag{5}$$

where $x < x_i$ for all i ($i \in \{1,2\}$). The solution can be summarized as

$$(x_1^*, x_2^*) = \left(\frac{\beta}{\beta - 1} I_1, \quad \frac{\beta}{\beta - 1} I_2\right).$$

Let superscript "asterisk" refer to the *full information no-agency* optimum. Substituting these solutions into the objective function gives the *full information no-agency* total social value $\pi^*(x)$, i.e.,

$$\pi^*(x) = q\left(\frac{x}{x_1^*}\right)^{\beta} (x_1^* - I_1) + (1-q)\left(\frac{x}{x_2^*}\right)^{\beta} (x_2^* - I_2). \tag{6}$$

2.3 Agency problem without auditing (Grenadier and Wang, 2005)

As a benchmark, the second problem is the *agency problem without auditing* in that the owner provides incentives for the manager with only the bonus-incentive. This setting is the same as in the GW model.[5] Thus, let superscript "GW" refer to the optimum in this setting.

In the *agency problem without auditing*, the owner designs a contract at time zero that commits the owner to give *only* the bonus-incentive to the manager at the time of investment. Renegotiation is not allowed.[6]

[5] This setting is exactly the same as the *hidden information only region* in the GW model (see Subsection 3.3). Although they consider hidden information as well as hidden action, hidden action does not influence the investment timing. Therefore, we focus on only hidden information that causes inefficiency in investment timing.

[6] While commitment may cause *ex post* inefficiency in investment timing, it increases the *ex ante* owner's option value.

Since there are only two possible values of I, there can be at most two pairs consisting of two components (investment trigger and bonus-incentive) that will be chosen by the manager.[7] Thus, the contract is modeled as a mechanism,

$$\mathcal{M}^{\mathrm{GW}} = (x(\tilde{I}), w(\tilde{I}); \tilde{I} \in \{I_1, I_2\}),$$

which may be contingent on a reported $\tilde{I}$. Because the revelation principle ensures that the manager reveals a true I as private information, we make no distinction between a reported $\tilde{I}$ and a true I.[8] Thus, we drop the suffix "tilde" on the reported $\tilde{I}$ and simply write the reported expenditure as I.

As explained in Appendix A.2., the optimal contract, $\mathcal{M}^{\mathrm{GW}} = (x_i^{\mathrm{GW}}, w_i^{\mathrm{GW}}; i \in \{1,2\})$, turns out to be

$$\begin{aligned}
(x_1^{\mathrm{GW}}, w_1^{\mathrm{GW}}) &= \left(x_1^*,\ \left(\frac{x_1^*}{x_2^{\mathrm{GW}}}\right)^{\beta} \Delta I\right), \\
(x_2^{\mathrm{GW}}, w_2^{\mathrm{GW}}) &= \left(\frac{\beta}{\beta-1}(I_2 + \frac{q}{1-q}\Delta I),\ 0\right).
\end{aligned}$$

It is important to note that $x_1^{\mathrm{GW}} = x_1^*$, $x_2^{\mathrm{GW}} > x_2^*$, $w_1^{\mathrm{GW}} > 0$, and $w_2^{\mathrm{GW}} = 0$. Furthermore, the total social value $\pi^{\mathrm{GW}}(x)$ is equal to

$$\pi^{\mathrm{GW}}(x) = q\left(\frac{x}{x_1^*}\right)^{\beta}(x_1^* - I_1) + (1-q)\left(\frac{x}{x_2^{\mathrm{GW}}}\right)^{\beta}\left(x_2^{\mathrm{GW}} - I_2\right), \tag{7}$$

where $x < x_1^*$. Importantly, agency conflict leads to a decrease in the total social value, i.e., $\pi^*(x) > \pi^{\mathrm{GW}}(x)$.

2.4 Agency problem with auditing under limited-liability constraints on penalties (Shibata, 2007)

As a benchmark, the final problem is the *agency problem with auditing under limited-liability constraints on penalties* when the manager is fined. This model is the same as in Shibata (2007). Thus, let superscript "S" stand for the optimum in this setting.

The audit technology allows the owner, at a cost, to verify the state announced by the manager, and to fine the manager for cheating when a false report is detected. Let us assume that the owner has an audit technology and that the manager's true value can be

[7] We need not examine the possibility of a *pooling equilibrium* in which only one investment trigger/bonus-incentive pair is offered. This is because a *pooling equilibrium* is always dominated by a *separating equilibrium* with two investment trigger/bonus-incentive pairs.

[8] See Fudenberg and Tirole (1991), Mas-Colell et al.(1995), and Salanié (2005) for the revelation principle.

observed with probability p if the owner incurs a cost $c(p)$ with $c(0) = 0$, $c' > 0$, $c'' > 0$, and $\lim_{p \uparrow 1} c(p) = +\infty$.[9]

Then, the contract in the S model is modeled as a mechanism,

$$\mathcal{M}^{\mathrm{S}} = (x(\tilde{I}), w(\tilde{I}), p(\tilde{I}), P(\tilde{I}, I); I, \tilde{I} \in \{I_1, I_2\}, I \neq \tilde{I}).$$

which may be contingent on a reported $\tilde{I}$. Here, $p(\tilde{I})$ is the probability of auditing, and $P(I, \tilde{I})$ is the penalty if the manager's reported value $\tilde{I}$ differs from its observed true value I ($I \in \{I_1, I_2\}, \tilde{I} \in \{I_1, I_2\}, I \neq \tilde{I}$). For notational simplicity, we drop the reported $\tilde{I}$ and simply write the function as $P(I) := P(I, \tilde{I})$ for all I ($I \in \{I_1, I_2\}, \tilde{I} \in \{I_1, I_2\}, I \neq \tilde{I}$).[10] Furthermore, we need not examine the possibility of a *pooling equilibrium*.[11]

As shown in Appendix A.2., the optimal contract $\mathcal{M}^{\mathrm{S}} = (x_i^{\mathrm{S}}, w_i^{\mathrm{S}}, p_i^{\mathrm{S}}, P_i^{\mathrm{S}}; i \in \{1, 2\})$ is obtained as follows. If $c'(0) \leq \frac{q}{1-q}\Delta I$, $\mathcal{M}^{\mathrm{S}}$ turns out to be:

$$\begin{aligned}
(x_1^{\mathrm{S}}, w_1^{\mathrm{S}}, p_1^{\mathrm{S}}, P_1^{\mathrm{S}}) &= \left(x_1^*,\ \left(\frac{x_1^*}{x_2^{\mathrm{S}}}\right)^{\beta} \Delta I \cdot (1 - p_2^{\mathrm{S}}),\ 0,\ \Delta I \right), \\
(x_2^{\mathrm{S}}, w_2^{\mathrm{S}}, p_2^{\mathrm{S}}, P_2^{\mathrm{S}}) &= \left(\frac{\beta}{\beta - 1}\Big(I_2 + c(p_2^{\mathrm{S}}) + \frac{q}{1-q}(1 - p_2^{\mathrm{S}}) \cdot \Delta I\Big),\ 0,\ c'^{-1}(\frac{q}{1-q}\Delta I),\ 0 \right).
\end{aligned}$$

Otherwise, $\mathcal{M}^{\mathrm{S}}$ is equal to the one which $p_2^{\mathrm{S}} = 0$ is substituted. Interestingly, we obtain $x_1^{\mathrm{S}} = x_1^*$, $x_2^* < x_2^{\mathrm{S}} \leq x_2^{\mathrm{GW}}$, $w_1^{\mathrm{GW}} \geq w_1^{\mathrm{S}} > 0$, and $w_2^{\mathrm{S}} = 0$. The total social value $\pi^{\mathrm{S}}(x)$ becomes

$$\pi^{\mathrm{S}}(x) = q\Big(\frac{x}{x_1^*}\Big)^{\beta}(x^* - I_1) + (1 - q)\Big(\frac{x}{x_2^{\mathrm{S}}}\Big)^{\beta}(x_2^{\mathrm{S}} - I_2 - c(p_2^{\mathrm{S}})), \tag{8}$$

where $x < x_1^*$. It is straightforward to obtain $\pi^*(x) > \pi^{\mathrm{S}}(x)$.

We have reviewed three related problems to our model. In the next section, we consider the original problem, and compare our problem with three related problems.

3 Model

In this section, we consider the *agency problem with auditing* in which the *limited-liability* constraints on penalties are removed in Shibata (2007) as described in the previous section. We then provide the solution to the problem that we simply call the *agency problem with auditing.* Finally, we discuss some properties of the solution to our problem.

[9]These assumptions are intuitively reasonable. The first assumption is that there is no cost incurred if the owner does not use the audit technology. The second and third assumptions imply that $c(p)$ is strictly increasing and convex in p. The final assumption is that complete auditing incurs a huge cost that the owner cannot afford.

[10]Because the revelation principle ensures that the manager reveals a true I as private information, we make no distinction between a reported $\tilde{I}$ and a true I, except for $P(I, \tilde{I})$.

[11]This is because a *pooling equilibrium* is always dominated by a *separating equilibrium.*

3.1 Agency problem with auditing

Although the *limited-liability* constraints on penalties are not satisfied, the manager is not fined in equilibrium because the manager truthfully reveals private information. Thus, we remove the *limited-liability* constraints on penalties. As a result, the owner may increase the *ex ante* owner's value as well as the total social value by removing these constraints on penalties. We assume that the owner has exactly the same audit technology as in Shibata (2007). However, the penalty $P > 0$ is given exogenously.[12] Recall that the audit technology enables the owner to observe the manager's true type whenever the owner audits the manager's type. Thus, the probability of an audit is equal to the probability of detection.

The possibility of an audit mechanism significantly enlarges the set of incentive-feasible schemes. An incentive scheme in the *agency problem with auditing* includes an investment trigger $x(\tilde{I})$, a bonus-incentive $w(\tilde{I})$, and a probability of an audit $p(\tilde{I})$ which may be contingent on a reported $\tilde{I}$. Thus, the contract in this problem is modeled as a mechanism,[13]

$$\mathcal{M}^{\mathrm{A}} = (x(\tilde{I}), w(\tilde{I}), p(\tilde{I}); \tilde{I} \in \{I_1, I_2\}).$$

Let superscript "A" refer to the optimum in the *agency problem with auditing.* Again, since the revelation principle ensures that the manager truthfully reveals a true I as private information, we make no distinction between a reported $\tilde{I}$ and a true I. For notational simplicity, we drop the "tilde" on $\tilde{I}$.

Then, the *agency problem with auditing* is to maximize the owner's option value through choice of the mechanism $\mathcal{M}^{\mathrm{A}}$, i.e.,

$$\begin{aligned} \max_{x_1,x_2,w_1,w_2,p_1,p_2} \quad & q\Big(\frac{x}{x_1}\Big)^{\beta}(x_1 - I_1 - w_1 - c(p_1)) \\ & + (1-q)\Big(\frac{x}{x_2}\Big)^{\beta}(x_2 - I_2 - w_2 - c(p_2)), \end{aligned} \tag{9}$$

subject to

$$\Big(\frac{x}{x_1}\Big)^{\beta} w_1 \geq \Big(\frac{x}{x_2}\Big)^{\beta}(w_2 + \Delta I - p_2 P), \tag{10}$$

$$\Big(\frac{x}{x_1}\Big)^{\beta} w_2 \geq \Big(\frac{x}{x_2}\Big)^{\beta}(w_1 - \Delta I - p_1 P), \tag{11}$$

[12] This assumption does not lose generality. If the penalty is endogenous, the optimal penalty for the owner is equal to the maximum value that the owner can impose. Thus, our model can be regarded as the maximum value of the penalty is fixed although the penalty is decided endogenously.

[13] Exactly as in the two agency problems described in the previous section, we need not examine the possibility of a *pooling equilibrium.* This is because a *pooling equilibrium* is always dominated by a *separating equilibrium.*

$$w_i \geq 0, \quad i \in \{1, 2\}, \tag{12}$$

$$q\Big(\frac{x}{x_1}\Big)^{\beta} w_1 + (1-q)\Big(\frac{x}{x_2}\Big)^{\beta} w_2 \geq 0, \tag{13}$$

$$1 \geq p_i \geq 0, \quad i \in \{1, 2\}. \tag{14}$$

Here, the objective function (9) is the *ex ante* owner's option value.

Constraints (10) and (11) are the *ex post* incentive-compatibility constraints for the manager under states I_1 and I_2, respectively. Consider, for example, constraint (10). The manager's payoff in state I_1 is $(x/x_1)^{\beta} w_1$ if he/she tells the truth, but it is $(x/x_1)^{\beta}(w_2 + \Delta I - p_2 P_1)$ if he/she instead claims that it is state I_2. Thus, he/she tells the truth if (10) is satisfied. Constraint (11) follows similarly. Constraint (11) will be shown not to be binding. Thus, only constraint (10) will be relevant to our discussion.

Constraints (12) and (13) are the *ex post* limited-liability constraints and the *ex ante* participation constraint, respectively. These three constraints are exactly the same as those in the GW setting. The nonnegative bonus-incentives w_1 and w_2 ensure that the manager makes an agreement about employment. For example, if $w_2 < 0$, then the manager would refuse the contract on learning that $I = I_2$. In addition, it is straightforward to show that the limited-liability constraints (12) imply the participation constraint (13). Thus, only constraint $w_1 \geq 0$ will be relevant to our discussion.

Constraint (14) is obvious, where p_i is the probability of an audit.

In summary, there are seven inequality constraints (10) to (14) on the *agency problem with auditing.*

3.2 A simplified agency problem with auditing

Although the optimization problem is subject to seven inequality constraints, we can simplify the problem in the following three steps. First, (13) is automatically satisfied. This is because (12) implies (13). Second, the manager in state I_2 does not have the incentive to tell a lie as a manager in state I_1. This is because the manager in state I_2 suffers a loss in such a false announcement. Thus, (11) is automatically satisfied, $p_1^{\mathrm{A}} = 0$ and $w_2^{\mathrm{A}} = 0$ are obtained in optimum. Finally, $p_2 \leq 1$ in (14) is automatically satisfied. This statement is shown by $\lim_{p_2 \uparrow 1} c(p_2) = +\infty$ and $c'(p_2) > 0$ for any p_2.

As a result, the simplified optimization problem is as follows:

$$\max_{x_1, x_2, w_1, p_2} q\Big(\frac{x}{x_1}\Big)^{\beta}(x_1 - I_1 - w_1) + (1-q)\Big(\frac{x}{x_2}\Big)^{\beta}(x_2 - I_2 - c(p_2)), \tag{15}$$

subject to

$$\Big(\frac{x}{x_1}\Big)^{\beta} w_1 \;\; \geq \;\; \Big(\frac{x}{x_2}\Big)^{\beta}(\Delta I - p_2 P), \quad w_1 \geq 0, \quad p_2 \geq 0, \tag{16}$$

where $x < x_i$ for all i ($i \in \{1, 2\}$).

3.3 Optimal solution

We first define the three feasible regions that serve to determine the characteristics of the solution. The nature of the solution depends on the magnitude of the penalty P. The contract can be derived in three possible regions: the *bonus-incentive only region*, the *joint (combination) bonus-incentive and auditing region*, and the *auditing only region.* Let superscript "Ab", "Ac", and "Aa" refer to the optimum for the three feasible regions, respectively.

As shown in Appendix A.1., we can obtain the following results.

Proposition 1 *Suppose that the penalty is finite. In the agency problem with auditing, the optimal contract* $\mathcal{M}^{\mathrm{A}} = (x_1^{\mathrm{A}}, w_1^{\mathrm{A}}, p_1^{\mathrm{A}}, x_2^{\mathrm{A}}, w_2^{\mathrm{A}}, p_2^{\mathrm{A}})$ *is as follows:*
bonus-incentive only region (Ab): $0 \leq P < \frac{1-q}{q} c'(0)$

$$\begin{aligned}(x_1^{\mathrm{Ab}}, w_1^{\mathrm{Ab}}, p_1^{\mathrm{Ab}}) &= \left(x_1^*, \quad w_1^{\mathrm{GW}}, \quad 0\right), \\ (x_2^{\mathrm{Ab}}, w_2^{\mathrm{Ab}}, p_2^{\mathrm{Ab}}) &= \left(x_2^{\mathrm{GW}}, \quad 0, \quad 0\right).\end{aligned}$$

joint bonus-incentive and auditing region (Ac): $\frac{1-q}{q} c'(0) \leq P < \max\{\Delta I, \ \frac{1-q}{q} c'(\frac{\Delta I}{P})\}$

$$\begin{aligned}(x_1^{\mathrm{Ac}}, w_1^{\mathrm{Ac}}, p_1^{\mathrm{Ac}}) &= \left(x_1^*, \quad \left(\frac{x_1^*}{x_2^{\mathrm{Ac}}}\right)^{\beta} (\Delta I - p_2^{\mathrm{Ac}} P), \quad 0\right), \\ (x_2^{\mathrm{Ac}}, w_2^{\mathrm{Ac}}, p_2^{\mathrm{Ac}}) &= \left(\frac{\beta}{\beta - 1} (I_2 + c(p_2^{\mathrm{Ac}}) + \frac{q}{1-q} \cdot (\Delta I - p_2^{\mathrm{Ac}} P)), \quad 0, \quad c'^{-1}(\frac{q}{1-q} P)\right).\end{aligned}$$

auditing only region (Aa): $P \geq \max\{\Delta I, \ \frac{1-q}{q} c'(\frac{\Delta I}{P})\}$

$$\begin{aligned}(x_1^{\mathrm{Aa}}, w_1^{\mathrm{Aa}}, p_1^{\mathrm{Aa}}) &= (x_1^*, \quad 0, \quad 0), \\ (x_2^{\mathrm{Aa}}, w_2^{\mathrm{Aa}}, p_2^{\mathrm{Aa}}) &= \left(\frac{\beta}{\beta - 1} \left(I_2 + c(p_2^{\mathrm{Aa}})\right), \quad 0, \quad \frac{\Delta I}{P}\right).\end{aligned}$$

Here, in the *joint region*, x_2^{Ac} is decided through p_2^{Ac}, and w_1^{Ac} is decided through x_2^{Ac}. Similarly, in the *auditing only region*, x_2^{Aa} is decided through p_2^{Aa}. Proposition 1 implies that our model corresponds to two papers of three related papers described in the previous section.

Remark 1 *The solution in the bonus-incentive only region is the exactly the same as in Grenadier and Wang (2005). The solution in the joint region includes that in Shibata (2007).*

The first statement is obvious. As for the second statement, $P = \Delta I$ is endogenously decided in the S model.[14] Thus, the solution for $P = \Delta I$ in the *joint region* turns out to be in the S model.

We then discuss the properties of the solution to the *agency problem with auditing.*

Corollary 1 *Suppose that the penalty is finite. The optimal contract has the following properties:*

$$\begin{array}{lll} x_1^* = x_1^{\mathrm{A}} = x_1^{\mathrm{GW}}, & 0 \le w_1^{\mathrm{A}} \le w_1^{\mathrm{GW}}, & p_1^{\mathrm{A}} = 0 \\ x_2^* < x_2^{\mathrm{A}} \le x_2^{\mathrm{GW}}, & w_2^{\mathrm{A}} = 0, & 0 \le p_2^{\mathrm{A}} < 1, \end{array}$$

for any A *(*$\mathrm{A} \in \{\mathrm{Ab}, \mathrm{Ac}, \mathrm{Aa}\}$*). In particular, we have*

$$x_2^* < x_2^{\mathrm{Aa}} < x_2^{\mathrm{Ac}} < x_2^{\mathrm{Ab}} = x_2^{\mathrm{GW}}, \quad 0 = w_1^{\mathrm{Aa}} < w_1^{\mathrm{Ac}} < w_1^{\mathrm{Ab}} = w_1^{\mathrm{GW}}.$$

Moreover, x_2^{Ac} is concave with P, while x_2^{Aa} is convex with P. The bonus-incentive w_1^{Ac} is concave with P. The probability p_2^{Ac} is increasing and convex with P, while p_2^{Aa} is decreasing and convex with P.

Corollary 1 implies that there are five important properties. The first property of the solution is that $x_1^{\mathrm{A}} = x_1^*$ and $x_2^* < x_2^{\mathrm{A}} \le x_2^{\mathrm{GW}}$ for any A ($\mathrm{A} \in \{\mathrm{Aa}, \mathrm{Ab}, \mathrm{Ac}\}$). It is less costly for the owner to distort x_2^{A} away from x_2^* than to distort x_1^{A} away from x_1^* in equilibrium.

The second property of the solution is that $x_2^{\mathrm{A}} < x_2^{\mathrm{GW}}$ and $w_1^{\mathrm{A}} < w_1^{\mathrm{GW}}$ if $p_2^{\mathrm{A}} > 0$, and that $x_2^{\mathrm{A}} = x_2^{\mathrm{GW}}$ and $w_1^{\mathrm{A}} = w_1^{\mathrm{GW}}$ otherwise. In other words, a decrease in x_2^{A} is equivalent to the decrease in w_1^{A}.

The third property of the solution is that $(\Delta I - p_2^{\mathrm{A}} P) > w_1^{\mathrm{A}} \ge 0$. Here, $(\Delta I - p_2^{\mathrm{A}} P)$ can be regarded as the *information rent* for the manager in state I_1. The owner gives the manager in state I_1 a portion of the *information rent.* Importantly, note that the *information rent* is decreasing with P. This result corresponds to the remarkable result for unlimited penalties in Subsection 3.4.

The fourth property of the solution is that p_2^{A} is unimodal with P (see Figure 3.). The reason is that p_2^{Ac} is increasing and concave with P, while p_2^{Aa} is decreasing and convex with P. The first statement is straightforward because $c(p)$ is increasing and convex with p. The second statement is shown by $p_2^{\mathrm{Aa}} = \frac{\Delta I}{P}$ in optimum.

The final property of the solution is that an increase in the penalty P changes the contract $\mathcal{M}^{\mathrm{A}}$ from the *bonus-incentive only region* to the *auditing only region* via the *joint*

[14] This implies that the owner raises the penalty as much as possible in case of a detected lie by the manager in state I_1. This property is called as the "Maximal Penalty Principle." We will show that our model obtains the same property. This is because an increase in the penalty *always* increases the owner's option value. See Proposition 3 or Figure 4.

region. This property is intuitive as follows. For example, as P is larger, the auditing technology is more available. Then, the owner tends to take into consideration the audit technology rather than bonus-incentives.

3.4 Optimal value

For all three feasible regions, substituting the solutions into the owner's and manager's option values, $\pi_{\mathrm{o}}^{\mathrm{A}}$ and $\pi_{\mathrm{m}}^{\mathrm{A}}$, respectively, yields

$$\begin{aligned} \pi_{\mathrm{o}}^{\mathrm{A}}(x) &= q\Big(\frac{x}{x_1^*}\Big)^{\beta}(x_1^* - I_1) \\ &\quad + (1-q)\Big(\frac{x}{x_2^{\mathrm{A}}}\Big)^{\beta}\Big(x_2^{\mathrm{A}} - I_2 - c(p_2^{\mathrm{A}}) - \frac{q}{1-q}(\Delta I - p_2^{\mathrm{A}}P)\Big), \end{aligned} \tag{17}$$

$$\pi_{\mathrm{m}}^{\mathrm{A}}(x) = q\Big(\frac{x}{x_2^{\mathrm{A}}}\Big)^{\beta}(\Delta I - p_2^{\mathrm{A}}P), \tag{18}$$

for any A (A $\in \{\mathrm{Ab}, \mathrm{Ac}, \mathrm{Ab}\}$). Because $\pi^{\mathrm{A}}(x) = \pi_{\mathrm{o}}^{\mathrm{A}}(x) + \pi_{\mathrm{m}}^{\mathrm{A}}(x)$, the total social value $\pi^{\mathrm{A}}(x)$ is

$$\pi^{\mathrm{A}}(x) = q\Big(\frac{x}{x_1^*}\Big)^{\beta}(x_1^* - I_1) + (1-q)\Big(\frac{x}{x_2^{\mathrm{A}}}\Big)^{\beta}\Big(x_2^{\mathrm{A}} - I_2 - c(p_2^{\mathrm{A}})\Big), \tag{19}$$

for any A (A $\in \{\mathrm{Ab}, \mathrm{Ac}, \mathrm{Ab}\}$). Obviously, we recognize that inefficiency is caused by the term in state I_2.

Proposition 2 *Suppose that the penalty is finite. Then we have*

$$\pi_{\mathrm{o}}^{\mathrm{A}}(x) \geq \pi_{\mathrm{o}}^{\mathrm{GW}}(x), \quad \pi_{\mathrm{m}}^{\mathrm{A}}(x) \leq \pi_{\mathrm{m}}^{\mathrm{GW}}(x), \quad \mathrm{A} \in \{\mathrm{Ab}, \mathrm{Ac}, \mathrm{Aa}\},$$

In particular, the optimal value has the following properties:

$$\begin{aligned} &\pi_{\mathrm{o}}^{\mathrm{Aa}}(x) \geq \pi_{\mathrm{o}}^{\mathrm{Ac}}(x) \geq \pi_{\mathrm{o}}^{\mathrm{Ab}}(x) = \pi_{\mathrm{o}}^{\mathrm{GW}}(x), \\ 0 = {} &\pi_{\mathrm{m}}^{\mathrm{Aa}}(x) \leq \pi_{\mathrm{m}}^{\mathrm{Ac}}(x) \leq \pi_{\mathrm{m}}^{\mathrm{Ab}}(x) = \pi_{\mathrm{m}}^{\mathrm{GW}}(x). \end{aligned}$$

Moreover, we obtain

$$\pi^*(x) > \pi^{\mathrm{A}}(x), \quad \mathrm{A} \in \{\mathrm{Ab}, \mathrm{Ac}, \mathrm{Aa}\}.$$

Proposition 2 implies that there are two important properties. One is that $\pi_{\mathrm{o}}^{\mathrm{A}}(x)$ is monotone increasing with P, while $\pi_{\mathrm{m}}^{\mathrm{A}}(x)$ is monotone decreasing with P (see Figures 4 and 5). The other is that asymmetric information *always* leads to a decrease in total social value for any finite penalty P.

In order to measure the "inefficiency" arising from asymmetric information, we define *total social loss* as $L^{\mathrm{A}}(x)$[15]

$$L^{\mathrm{A}}(x) = \pi^*(x) - \pi^{\mathrm{A}}(x) \geq 0, \quad \mathrm{A} \in \{\mathrm{Ab}, \mathrm{Ac.Aa}\}.$$

Here, $L^{\mathrm{A}}(x)$ is strictly positive for any finite penalty P. Obviously from the definition, the property of total social value $\pi^{\mathrm{A}}(x)$ is equivalent to that of total social loss $L^{\mathrm{A}}(x)$. We will discuss $L^{\mathrm{A}}(x)$ rather than $\pi^{\mathrm{A}}(x)$ when we investigate inefficiency in total social welfare.

3.5 Unlimited penalties

Although unlimited penalties are of theoretical interest (penalties are limited in practice), we examine how the solution, the value, and the loss are changed as the penalty P is sufficiently large.

Proposition 3 *The full information solution, value, and loss, are approximated closely as the penalty is increased without limit. As $P \uparrow +\infty$, we have*

$$x_2^{\mathrm{Aa}} \downarrow x_2^*, \quad p_2^{\mathrm{Aa}} \downarrow 0, \quad \pi^{\mathrm{Aa}}(x) \downarrow \pi^*(x),$$

and

$$L^{\mathrm{Aa}}(x) \downarrow 0.$$

These results are the same as those in Baron and Besanko (1984, Proposition 4) and imply the following results.

Remark 2 *The solution in the auditing only region in our model converges to that in the standard real options model as the penalty P becomes sufficiently larger.*

Remarks 1 and 2 imply that the solutions in our model include those in the three related papers, McDonald and Siegel (1986), Grenadier and Wang (2005), and Shibata (2007).

4 Model implications

In this section, we analyze several of the more important implications of the model. Subsection 4.1 investigates the effects of auditing on the solutions and values. Subsection 4.2 examines the stock price reaction to the information released via the manager's investment decision. Subsection 4.3 demonstrates the "asset substitution" between the owner and the manager caused by an increase in the volatility of project value, the key parameter in a real options model.

[15]This definition is exactly the same as in Grenadier and Wang (2005, Subsection 4.3).

4.1 Effects of auditing

In this subsection, we consider the effects of auditing by using numerical examples. We define the auditing cost function $c(p_i)$ as

$$c(p_i) = \alpha \frac{p_i}{1 - p_i}, \quad i \in \{1, 2\}. \tag{20}$$

Here, the parameter α is interpreted as a measure of "efficiency" for the auditing cost function. Suppose that parameters are $q = 0.5$, $\sigma = 0.2$, $r = 0.07$, $\mu = 0.03$, $I_1 = 50$, $I_2 = 80$, and $\alpha = 20$.

Figure 1 depicts the investment trigger x_2^{A} with respect to the penalty P. An increase in P changes the trigger x_2^{A} from the *bonus-incentive only region* (Ab) to the *auditing only region* (Aa) via the *joint bonus-incentive and auditing region* (Ac). In particular, the *bonus-incentive only region* (Ab) is on $0 \leq P \leq 20$, the *joint region* (Ac) is on $20 \leq P \leq 66.45$, and the *auditing only region* (Aa) is on $P > 66.45$. Also, we can see that x_2^{A} is monotone decreasing with P. Naturally, triggers x_2^* and x_2^{GW} does not depend on P.

[Insert Figure 1 about here]

Figure 2 demonstrates the bonus-incentive w_1^{A} with respect to P. Note that w_1^{A} is decreasing with P. Also, we consider the comparative statics with respect to α ($\alpha \in \{1, 5, 20\}$). Under $\alpha = 1$, the *bonus-incentive only region* (Ab) is on $0 \leq P \leq 1$, the *joint region* (Ac) is on $1 \leq P \leq 36$, and the *auditing only region* (Aa) is on $P \geq 36$. Under $\alpha = 5$, *bonus-incentive only region* (Ab) is on $0 \leq P \leq 5$, the *joint region* (Ac) is on $5 \leq P \leq 45$, and the *auditing only region* (Aa) is on $P \geq 45$. An increase in α increases w_1^{A}. This implies that the bonus-incentive to the manager is increasing as the auditing cost function is becoming more inefficient.

[Insert Figure 2 about here]

Figure 3 depicts the probability of auditing p_2^{A} with respect to P. Note that p_2^{A} is unimodal with P. In particular, as explained earlier, p_2^{Ac} is increasing and concave with P, while p_2^{Aa} is decreasing and convex with P. More interestingly, an increase in α decreases p_2^{A}. This implies that the probability of auditing is decreasing as the auditing cost function is getting more inefficient. This result is equivalent to the result that an increase in α increases w_1^{A}. Thus, an increase in inefficiency of the auditing cost function enables the owner to take into consideration the bonus-incentive rather than the audit technology.

[Insert Figure 3 about here]

Figure 4 depicts the owner's option value $\pi_{\mathrm{o}}^{\mathrm{A}}(x)$ with respect to P. The value $\pi_{\mathrm{o}}^{\mathrm{A}}(x)$ is monotone increasing with P. This property corresponds to the "Maximal Penalty Principle."[16] Also, an increase in α decreases $\pi_{\mathrm{o}}^{\mathrm{A}}(x)$. This result is obvious because a decrease in inefficiency of the auditing cost function increases the cost expenditure that the owner incurs to use the audit technology.

[Insert Figure 4 about here]

Figure 5 demonstrates the manager's option value $\pi_{\mathrm{m}}^{\mathrm{A}}(x)$ with respect to P. The value $\pi_{\mathrm{m}}^{\mathrm{A}}(x)$ is monotone decreasing with P. The reason is that an increase in P decreases the *information rent* for the manager in state I_1, which leads to the decrease in the bonus-incentive. Moreover, an increase in α decreases $\pi_{\mathrm{m}}^{\mathrm{A}}(x)$. As explained earlier, the owner tends to take in consideration the bonus-incentive more than the audit technology if the auditing cost function is becoming more inefficient.

[Insert Figure 5 about here]

Figures 4 and 5 imply that an increase in P leads to "asset substitution" between the owner and the manager. Wealth is transferred from the manager to the owner by an increase in P.

Figure 6 depicts the total social loss $L^{\mathrm{A}}(x)$ with respect to P. The most important result is that $L^{\mathrm{A}}(x)$ is *not* monotone decreasing with P. On the one hand, $L^{\mathrm{Ab}}(x)$ is constant with P, while $L^{\mathrm{Aa}}(x)$ is *always* decreasing with P. On the other hand, $L^{\mathrm{Ac}}(x)$ is increasing or decreasing with P. Thus, an increase in P does *not necessarily* lead to a decrease in $L^{\mathrm{A}}(x)$. However, recall that an increase in P *always* increases the owner's option value. Consequently, an owner's (individual) rationality does *not necessarily* lead to total social rationality.

Moreover, an decrease in the parameter α does *not necessarily* decrease $L^{\mathrm{A}}(x)$. Consider, for example, $P = 30$. Then, $L^{\mathrm{A}}(x)$ under $\alpha = 20$ is smaller than under $\alpha = 5$. Thus, a reduction in inefficiency of the auditing cost *always* brings a benefit to the owner, while it does *not necessarily* bring a benefit to total social welfare.

[Insert Figure 6 about here]

Finally, in Figures 1 to 6, we see that all the solutions and values, x_2^{A}, w_1^{A}, p_2^{A}, $\pi_{\mathrm{o}}^{\mathrm{A}}(x)$, $\pi_{\mathrm{m}}^{\mathrm{A}}(x)$, and $L^{\mathrm{A}}(x)$, converge to the *full information* optimums, as $P \uparrow \infty$. Moreover, the smaller α is, the more quick the *full information* optimums are approximated.

[16]See Laffont and Martimort (2002) for "Maximal Penalty Principle" in detail.

4.2 Stock price reaction

In this subsection, we investigate the stock price reaction to the manager's investment decision. The stock price is the owner's option value that is given in (17). This discussion is the same in Grenadier and Wang (2005).

Prior to the point at which x reaches the trigger x_1^*, the market does not know the true value of I. The market believes that $I = I_1$ with probability q and $I = I_2$ with probability $1 - q$.

Once x hits the trigger x_1^*, the private information is fully revealed. The manager's investment decision signals $I = I_1$ to the market. If the manager undertakes the investment at x_1^*, then the stock price instantly jumps upward to

$$\pi_{\mathrm{o}}^{\mathrm{A}}(x_1^*) \quad = \quad x_1^* - I_1 - w_1^{\mathrm{A}}, \quad (\mathrm{A} \in \{\mathrm{Ab}, \mathrm{Ac}, \mathrm{Aa}\}). \tag{21}$$

Otherwise, then the market recognizes $I = I_2$. Then the stock price instantly jumps downward to

$$\pi_{\mathrm{o}}^{\mathrm{A}}(x_1^*) \quad = \quad \left(\frac{x_1^*}{x_2^{\mathrm{A}}}\right)^{\beta} (x_2^{\mathrm{A}} - I_2 - c(p_2^{\mathrm{A}})), \quad (\mathrm{A} \in \{\mathrm{Ab}, \mathrm{Ac}, \mathrm{Aa}\}). \tag{22}$$

Figure 7 demonstrates the stock price reaction to investment. Here, we suppose that $\alpha = 5$, and $P = 40$.[17] Then the stock price is 66.31 just prior to $x_1^* = 128.43$. If the manager undertakes the investment at x_1^*, the stock price jumps upward to 76.93. Otherwise, the stock price jumps downward to 54.19.

[Insert Figure 7 about here]

Clearly, we conclude that the jump size of the stock price reaction is increasing with P. The intuition behind this result is that an increase in P *always* decreases the *information rent* for the manager in state I_1, $\Delta I - p_2^{\mathrm{A}} P$. This result is new and not shown by Grenadier and Wang. This is because the *information rent* for the manager in state I_1 is constant in the GW model.

Furthermore, an increase in σ increases the stock price reaction, while a decrease in α increases the stock price reaction.[18]

4.3 Asset substitution

We begin with investigation of the effects on the owner's and manager's option values, $\pi_{\mathrm{o}}^{\mathrm{A}}(x)$ and $\pi_{\mathrm{m}}^{\mathrm{A}}(x)$, with respect to the volatility of project uncertainty, σ.

[17]The other parameters are the same as in the previous examples.

[18]The reason is straightforward. That is, an increase in σ increases $\pi_{\mathrm{o}}^{\mathrm{A}}(x)$, while a decrease in α increases $\pi_{\mathrm{o}}^{\mathrm{A}}(x)$.

Remark 3 *In the agency problem under asymmetric information, an increase in* σ *increases* $\pi_{\mathrm{o}}^{\mathrm{A}}(x)$*, while it has an ambiguous effect on* $\pi_{\mathrm{m}}^{\mathrm{A}}(x)$*.*

More interestingly, an increase in σ may give rise to "asset substitution" in the *agency problem.* If the underlying current state $x < x_2^{\mathrm{A}}$ is relatively high, in that,

$$\left|\log\left(\frac{x}{x_2^{\mathrm{A}}}\right)\right| < \left|\frac{1}{\beta - 1}\right|, \tag{23}$$

then an increase in σ decreases $\pi_{\mathrm{m}}^{\mathrm{A}}(x)$. Thus, an increase in σ shifts wealth from the manager to the owner. This possibility to transfer wealth is known as "asset substitution." Figure 8 depicts the effect on $\pi_{\mathrm{o}}^{\mathrm{A}}(x)$ and $\pi_{\mathrm{m}}^{\mathrm{A}}(x)$ with respect to σ.[19] If the value of the right-hand side in (23) does not exceed the value of the left-hand side in (23), an increase in σ increases $\pi_{\mathrm{m}}^{\mathrm{A}}(x)$. Otherwise, an increase in σ decreases $\pi_{\mathrm{m}}^{\mathrm{A}}(x)$. Thus, we find the wealth transfer known as "asset substitution" if σ is more than 0.2342.

[Insert Figure 8 about here]

Whether an increase in σ leads either an increase or a decreases in $\pi^{\mathrm{A}}(x)$ is an interesting question. This is because $\pi_{\mathrm{m}}^{\mathrm{A}}(x) = \pi_{\mathrm{o}}^{\mathrm{A}}(x) + \pi_{\mathrm{m}}^{\mathrm{A}}(x)$.

Remark 4 *In the agency problem under asymmetric information, an increase in* σ *does not necessarily increase* $\pi^{\mathrm{A}}(x)$*.*

Recall that an increase in σ *always* increases the value in the standard real options model (see Dixit and Pindyck, 1994, Chapter 5). Remark 4 implies that the impact of σ under asymmetric information may be different from under full information. The reason is that the solution that maximizes the owner's value does *not* correspond to the solution that maximizes the total social value.[20]

5 Concluding remarks

This paper extends the agency problem in a real options model to incorporate auditing technology. The implied investment triggers are derived in three regions: the *bonus-incentive only region*, the *joint region*, and the *auditing only region*. Thus, the implied

[19] In this numerical example, we suppose that $x = 50$. To depict both values in the same diagram, constants 1/6 and 5 are multiplied by the owner's and manager's option values, respectively.

[20] Mathematically, the solution in the *full information problem* is to maximize total social value. In contrast, the solution in the *agency problem* under asymmetric information is to maximize the owner's value, *not* total social value. We cannot apply the *envelope theorem* under asymmetric information. This difference arises from the above result. See Mas-Colell et al. (1995) for the *envelope theorem*.

investment triggers in these three regions include those in the Grenadier and Wang, the Shibata and standard full information real options (e.g., McDonald and Siegel, 1986) models, respectively. In particular, the full information investment trigger is approximated arbitrarily closely by making the penalty sufficiently large. We also investigate inefficiency in the implied investment triggers and total social welfare. An increase in the penalty *always* reduces inefficiency in investment triggers, while it does *not necessarily* reduce inefficiency in total social welfare.

Appendices

A.1 Proof of Lemma and Proposition

Proof of Proposition 1

The Lagrangian can be formulated as:

$$\begin{aligned}\mathcal{L} &= qx_1^{-\beta}(x_1 - I_1 - w_1) + (1-q)x_2^{-\beta}(x_2 - I_2 - c(p_2)) \\ &\quad + \lambda_1 \left(x_1^{-\beta} w_1 - x_2^{-\beta}(\Delta I - p_2 P)\right) + \lambda_2 w_1 + \lambda_3 p_2,\end{aligned}$$

where λ_i $(i \in \{1,2,3\})$ denotes the multiplier on the constraints. The Kuhn-Turker conditions are

$$\frac{\partial \mathcal{L}}{\partial x_1} = qx_1^{-\beta}\left(1 - \beta x_1^{-\beta}(x_1 - I_1 - w_1)\right) - \lambda_1 x_1^{-\beta} w_1 \beta x_1^{-1} = 0, \tag{A.1}$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = (1-q)x_2^{-\beta}\left(1 - \beta x_2^{-\beta}(x_2 - I_2 - c(p_2))\right) + \lambda_1 x_2^{-\beta}(\Delta I - p_2 P)\beta x_2^{-1} = 0, \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial w_1} = -(q - \lambda_1)x_1^{-\beta} + \lambda_2 = 0, \tag{A.3}$$

$$\frac{\partial \mathcal{L}}{\partial p_2} = -\left((1-q)c'(p_2) - \lambda_1 P\right)x_2^{-\beta} + \lambda_3 = 0, \tag{A.4}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = x_1^{-\beta} w_1 - x_2^{-\beta}(\Delta I - p_2 P) = 0, \tag{A.5}$$

and

$$\lambda_2 w_1 = \lambda_3 p_2 = 0, \quad \lambda_i \geq 0 \quad (i \in \{1,2,3\}). \tag{A.6}$$

The solution depends on whether or not λ_2 and λ_3 are equal to zero. First, suppose that $\lambda_2 > 0$ and $\lambda_3 > 0$. Then we have $w_1 = 0$ and $p_2 = 0$. These imply $\Delta I = 0$, which contradicts $\Delta I > 0$. Thus, at least one of λ_2 and λ_3 must be binding. Second, suppose that $\lambda_2 = 0$ and $\lambda_3 > 0$. Then we have $\lambda_1 = q$. Obviously we have the solution in the *bonus-incentive only region* with $\frac{1-q}{q}c'(0) > P \geq 0$. Third, suppose that $\lambda_2 = \lambda_3 = 0$.

Then we obtain $w_1 > 0$ and $p_2 > 0$. It is straightforward to obtain the solution in the *joint bonus-incentive and auditing region.* Finally, suppose that $\lambda_2 > 0$ and $\lambda_3 = 0$. Then we have $P > \frac{1-q}{q}c'(\frac{\Delta I}{P})$, and $P > \Delta I$ due to $p_2 = \frac{\Delta I}{P} < 1$. Thus, we have the solution in the *auditing only region* with $P \leq \max\{\frac{1-q}{q}c'(\frac{\Delta I}{P}), \frac{\Delta I}{P}\}$. □

Proof of Corollary 1

Here, we prove that $x_2^* < x_2^{\mathrm{Aa}} < x_2^{\mathrm{Ac}} < x_2^{\mathrm{Ab}}$. First, it is clear to obtain $x_2^* < x_2^{\mathrm{Aa}}$ from

$$x_2^{\mathrm{Aa}} = x_2^* + \frac{\beta}{\beta - 1}c(p_2^{\mathrm{Aa}}),$$

and $c(p_2^{\mathrm{Aa}}) > 0$. Second, $x_2^{\mathrm{Aa}} < x_2^{\mathrm{Ac}}$ can be proved because of

$$x_2^{\mathrm{Ac}} = x_2^{\mathrm{Aa}} + \frac{\beta}{\beta - 1}\frac{q}{1 - q}(\Delta I - p_2^{\mathrm{Ac}}P),$$

and $(\Delta I - p_2^{\mathrm{Ac}}P) > 0$. Finally, we prove $x_2^{\mathrm{Ac}} < x_2^{\mathrm{Ab}}$. The trigger x_2^{Ac} is equal to

$$x_2^{\mathrm{Ac}} = x_2^{\mathrm{Ab}} + \frac{\beta}{\beta - 1}\left(c(p_2^{\mathrm{Ac}}) - c'(p_2^{\mathrm{Ac}}) \cdot p_2^{\mathrm{Ac}}\right). \tag{A.7}$$

Here, we have used $c'(p_2^{\mathrm{Ac}}) = \frac{q}{1-q}P$ in optimum. Since $c(p)$ is strictly increasing and convex with p, the second term is negative. Thus, the proof is complete. □

A.2 Related papers

In Appendix A.2, we formulate the optimization problem in the two related papers, Grenadier and Wang (2005) and Shibata (2007).

A.2.1 Grenadier and Wang model

The *agency problem without auditing* (GW model) is to maximize the owner's option value through choice of the mechanism $\mathcal{M}^{\mathrm{GW}}$:

$$\max_{x_1, x_2, w_1, w_2} \quad q\Big(\frac{x}{x_1}\Big)^{\beta}(x_1 - I_1 - w_1) + (1 - q)\Big(\frac{x}{x_2}\Big)^{\beta}(x_2 - I_2 - w_2), \tag{A.8}$$

subject to

$$\Big(\frac{x}{x_1}\Big)^{\beta} w_1 \geq \Big(\frac{x}{x_2}\Big)^{\beta}(w_2 + \Delta I), \tag{A.9}$$

$$\Big(\frac{x}{x_1}\Big)^{\beta} w_2 \geq \Big(\frac{x}{x_2}\Big)^{\beta}(w_1 - \Delta I), \tag{A.10}$$

$$w_i \geq 0, \quad i \in \{1, 2\}, \tag{A.11}$$

$$q\Big(\frac{x}{x_1}\Big)^{\beta} w_1 + (1 - q)\Big(\frac{x}{x_2}\Big)^{\beta} w_2 \geq 0. \tag{A.12}$$

Here, the objective function (A.8) is the *ex ante* owner's option value, which we denote as $\pi_{\rm o}(x)$. Constraints (A.9) and (A.10) are the *ex post* incentive-compatibility constraints. Constraints (A.11) are the *ex post* limited-liability constraints. Constraint (A.12) is the *ex ante* participation constraint, in which the left-hand side is the *ex ante* manager's option value, which we denote as $\pi_{\rm m}(x)$.

In optimum, we can show that only two of the five constraints (A.9) to (A.12) are binding, i.e., $w_1^{\rm GW} = (x_1^*/x_2^{\rm GW})^\beta \Delta I$ and $w_2^{\rm GW} = 0$. Thus, we can simplify the problem as follows:

$$\max_{x_1,x_2} \quad q\Big(\frac{x}{x_1}\Big)^\beta (x_1 - I_1) + (1-q)\Big(\frac{x}{x_2}\Big)^\beta (x_2 - I_2 - \frac{q}{1-q}\Delta I). \tag{A.13}$$

Equation (A.13) means that the owner's value is reduced by the term $\frac{q}{1-q}\Delta I$, compared with the *full information problem* defined by (5). Obviously, the optimal contract $\mathcal{M}^{\rm GW}$ is obtained as in Subsection 2.3.

Substituting $\mathcal{M}^{\rm GW}$ into the owner's and manager's option value functions gives

$$\pi_{\rm o}^{\rm GW}(x) = q\Big(\frac{x}{x_1^*}\Big)^\beta (x_1^* - I_1) + (1-q)\Big(\frac{x}{x_2^{\rm GW}}\Big)^\beta \Big(x_2^{\rm GW} - I_2 - \frac{q}{1-q}\Delta I\Big), \tag{A.14}$$

$$\pi_{\rm m}^{\rm GW}(x) = q\Big(\frac{x}{x_2^{\rm GW}}\Big)^\beta \Delta I, \tag{A.15}$$

where $x < x_1^*$. By the definition, the sum of these values is the total social value, i.e., $\pi^{\rm GW}(x) = \pi_{\rm o}^{\rm GW}(x) + \pi_{\rm m}^{\rm GW}(x)$.

A.2.2 Shibata model

The *agency problem with auditing under limited-liability constraints on penalties* (S model) is to maximize the owner's option value through choice of the mechanism $\mathcal{M}^{\rm S}$:

$$\max_{x_1,x_2,w_1,w_2,p_1,p_2,P_1,P_2} \quad q\Big(\frac{x}{x_1}\Big)^\beta (x_1 - I_1 - w_1 - c(p_1)) \tag{A.16}$$
$$+(1-q)\Big(\frac{x}{x_2}\Big)^\beta (x_2 - I_2 - w_2 - c(p_2)),$$

subject to

$$\Big(\frac{x}{x_1}\Big)^\beta w_1 \geq \Big(\frac{x}{x_2}\Big)^\beta (w_2 + \Delta I - p_2 P_1), \tag{A.17}$$

$$\Big(\frac{x}{x_1}\Big)^\beta w_2 \geq \Big(\frac{x}{x_2}\Big)^\beta (w_1 - \Delta I - p_1 P_2), \tag{A.18}$$

$$w_i \geq 0, \quad i \in \{1,2\}, \tag{A.19}$$

$$q\Big(\frac{x}{x_1}\Big)^\beta w_1 + (1-q)\Big(\frac{x}{x_2}\Big)^\beta w_2 \geq 0, \tag{A.20}$$

$$P_1 \leq (w_2 + \Delta I) \tag{A.21}$$

$$P_2 \geq (w_1 - \Delta I), \tag{A.22}$$

$$1 \geq p_i \geq 0, \quad i \in \{1,2\}. \tag{A.23}$$

Here, (A.17) and (A.18) are the *ex post* incentive-compatibility constraints. Constraints (A.20) and (A.19) are exactly the same as in the GW model. Constraints (A.21) and (A.22) are the *ex post* limited-liability constraints on penalties.

Then, we obtain $w_2^{\mathrm{S}} = 0$, $p_1^{\mathrm{S}} = 0$, and $P_2^{\mathrm{S}} = 0$ in optimum, and reduce the number of constraints to only three: (A.17), (A.21), and $p_2 \geq 0$ in (A.23). Because the first two constraints (A.17) and (A.21) are binding in equilibrium, substituting (A.21) into (A.17) gives

$$\left(\frac{x}{x_1^{\mathrm{S}}}\right)^{\beta} w_1^{\mathrm{S}} = \left(\frac{x}{x_2^{\mathrm{S}}}\right)^{\beta} \Delta I(1 - p_2^{\mathrm{S}}). \tag{A.24}$$

Here, the bonus-incentive w_1^{S} must satisfy equation (A.24) in optimum.

Hence, we can simplify the owner's optimization problem as follows:

$$\max_{x_1, x_2, p_2} \quad q\left(\frac{x}{x_1}\right)^{\beta}(x_1 - I_1) + (1-q)\left(\frac{x}{x_2}\right)^{\beta}\left(x_2 - I_2 - \frac{q}{1-q}\Delta I(1-p_2) - c(p_2)\right), \tag{A.25}$$

subject to $p_2 \geq 0$.

We compare (A.25) with (5) and (A.13), respectively. Equation (A.25), on the one hand, is reduced by the term $\frac{q}{1-q}\Delta I(1-p_2) + c(p_2)$, compared with (5) in the *full information problem*. This term is interpreted as the "inefficiency cost" in the S model. Equation (A.25), on the other hand, is increased by $\frac{q}{1-q}\Delta I \cdot p_2$, at a cost $c(p_2)$, compared with (A.13). As a result, we obtain that the incentive scheme in the S model is the only incentive scheme to decrease the "inefficiency cost" in the GW model. Naturally, the owner decides whether to use the audit technology, taking into account the trade-off between its costs and benefits. As shown in Subsection 2.4., we obtained the solution $\mathcal{M}^{\mathrm{S}}$ in the S model.

Furthermore, the owner's and manager's values, $\pi_{\mathrm{o}}^{\mathrm{S}}(x)$ and $\pi_{\mathrm{m}}^{\mathrm{S}}(x)$, become

$$\pi_{\mathrm{o}}^{\mathrm{S}}(x) = q\left(\frac{x}{x_1^*}\right)^{\beta}(x^* - I_1) + (1-q)\left(\frac{x}{x_2^{\mathrm{S}}}\right)^{\beta}\left(x_2^{\mathrm{S}} - I_2 - \frac{q}{1-q}\Delta I(1-p_2^{\mathrm{S}}) - c(p_2^{\mathrm{S}})\right), \tag{A.26}$$

$$\pi_{\mathrm{m}}^{\mathrm{S}}(x) = q\left(\frac{x}{x_2^{\mathrm{S}}}\right)^{\beta}\frac{q}{1-q}\Delta I(1-p_2^{\mathrm{S}}), \tag{A.27}$$

where $x < x_1^*$. The sum of these values is the total social value $\pi^{\mathrm{S}}(x)$ defined by (8).

References

[1] Baron, D., and Besanko, D. 1984. "Regulation, Asymmetric Information, and Auditing." *RAND Journal of Economics*, Vol.15, pp.447–470.

[2] Bernardo, A. E., and Chowdhry, E. B. 2002. "Resources, Real Options, and Corporate Strategy." *Journal of Financial Economics*, Vol.63, pp.211–234.

[3] Bjerksund, P., and Stensland, G. 2000. "A Self-enforced Dynamic Contract for Processing of Natural Resources. In Brennan, M.J., Trigeorgis, L., eds., *Project Flexibility, Agency, and Competition.* Oxford University Press, Oxford, (2000), pp.109–127.

[4] Dixit, A.K., and Pindyck, R.S. 1994. *Investment Under Uncertainty.* Princeton University Press, Princeton, NJ, 1994.

[5] Fudenberg, D., and Tirole, J. 1991. *Game Theory.* MIT Press, Cambridge, 1991.

[6] Grenadier, S.R., and Wang, N. 2005. "Investment Timing, Agency, and Information." *Journal of Financial Economics*, Vol.75 (2005), pp.493–533.

[7] Laffont, J., and Martimort, D. 2002. *The Theory of Incentives: The Principal-Agent Model.* Princeton University Press, Princeton, NJ.

[8] Laffont, J., and Tirole, J. 1986. "Using Cost Observation to Regulated Firms." *Journal of Political Economy*, Vol.94, pp.614–641.

[9] Lambrecht B., and Perraudin, W. 2003. "Real Options and Preemption under Incomplete Information." *Journal of Economic Dynamics and Control*, Vol.27, pp.619–643.

[10] Leland, H.E. 1998. "Agency Costs, Risk Management, and Capital Structure." *Journal of Finance*, Vol.53, pp.1213–1243.

[11] Mæland, J. 2002. "Valuation of Irreversible Investments and Agency Problems." Forthcoming in Trigeorgis, L. eds., *Innovation, Organization and Strategy*, Cambridge University Press, Cambridge.

[12] Mas-Colell, A., Whinston, M., and Green, J.R. 1995. *Microeconomic Theory.* Oxford Student Press, Oxford.

[13] Mauer D.C., and Ott, S.H. 2000. "Agency Costs, Underinvestment, and Optimal Capital Structure: the Effect of Growth Options to Expand." In Brennan, M.J., and Trigeorgis, L., eds., *Project Flexibility, Agency, and Competition.* Oxford University Press, Oxford, pp151–180.

[14] Mauer, D.C., and Sarkar, S. 2005. "Real Options, Agency Conflicts, and Optimal Capital Structure." *Journal of Banking & Finance*, Vol.29, pp.1405–1428.

[15] Mauer, D.C., and Triantis, A. 1994. "Interactions of Corporate Financing and Investment Decisions: A Dynamic Framework." *Journal of Finance*, Vol.49, pp1253–1277.

[16] McDonald, R., and Siegel, D. R. 1986. "The Value of Waiting to Invest." *Quarterly Journal of Economics*, Vol.101, pp707–727.

[17] Mello, A. and Parsons, J.J. 1992. "Measuring the Agency Costs of Debt." *Journal of Finance*, Vol.47, pp.1887–1904.

[18] Morellec, E. 2001. "Asset Liquidity, Capital Structure, and Secured Debt." *Journal of Financial Economics*, Vol.61, pp.173–206.

[19] Nishihara, M., and Fukushima, M. 2007. "Evaluation of Firm's Loss Due to Incomplete Information in Real Investment Decision." Forthcoming in *European Journal of Operational Research.*

[20] Salanié, B. 2005. *The Economics of Contracts.* 2nd Edition. MIT Press, Cambridge.

[21] Shibata, T. 2006. "The Impacts of Uncertainties in the Real Options Model under Incomplete Information." Forthcoming in *European Journal of Operational Research.*

[22] Shibata, T. 2007. "Investment Timing, Asymmetric Information, and Audit Structure: A Real Options Framework." Working paper, No.30, Tokyo Metropolitan University, Tokyo.

[23] Townsend, R. 1979. "Optimal Contracts and Competitive Markets with Costly State Verification." *Journal of Economic Theory*, Vol.21, pp.265–293.

[24] Trigeorgis, L. 1986. *Real Options–Managerial Flexibility and Strategy in Resource Allocation.* MIT Press, Cambridge.

[25] Weeds, H. 2002. "Strategic Delay in a Real Options Model of R&D Competition." *Review of Economic Studies*, Vol.69, pp729–747.

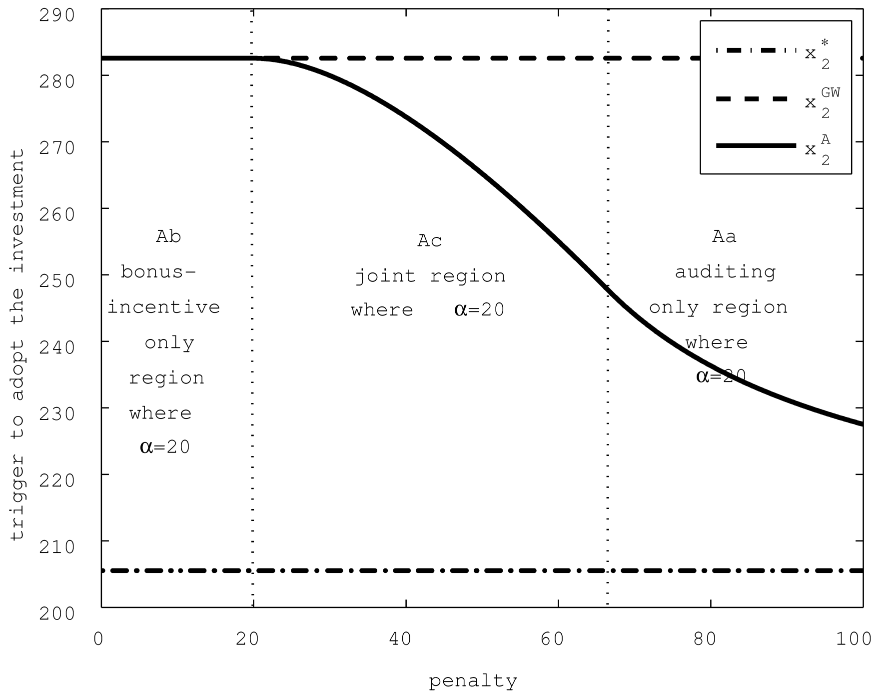

Figure 1: Trigger to adopt investment with respect to penalty

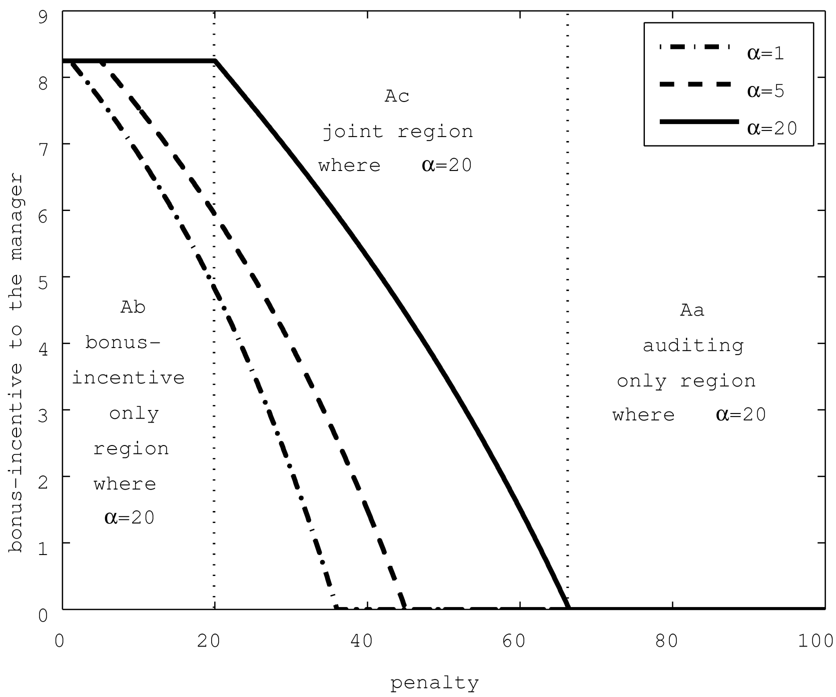

Figure 2: Bonus-incentive with respect to penalty

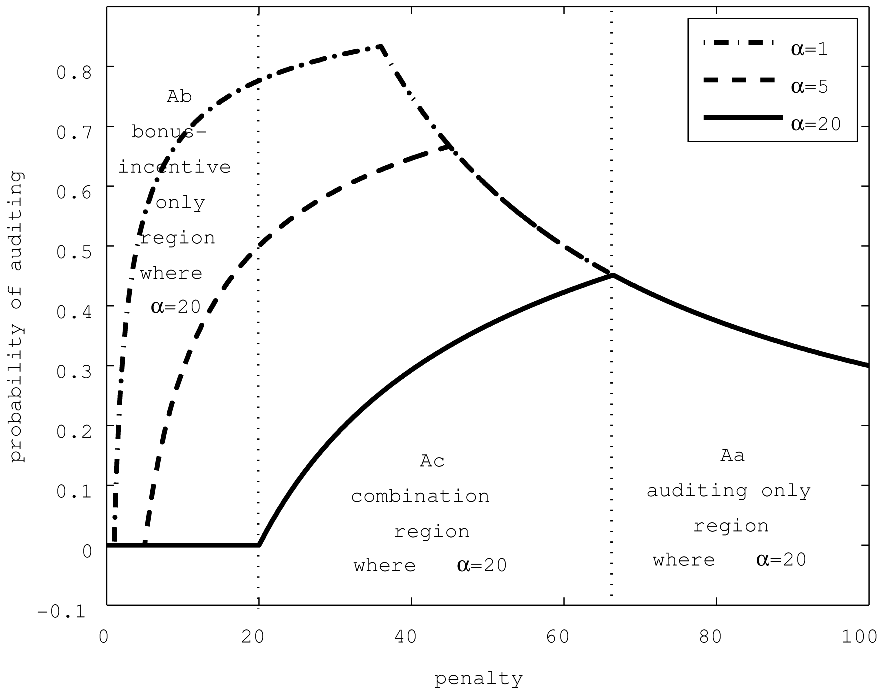

Figure 3: Probability of auditing with respect to penalty

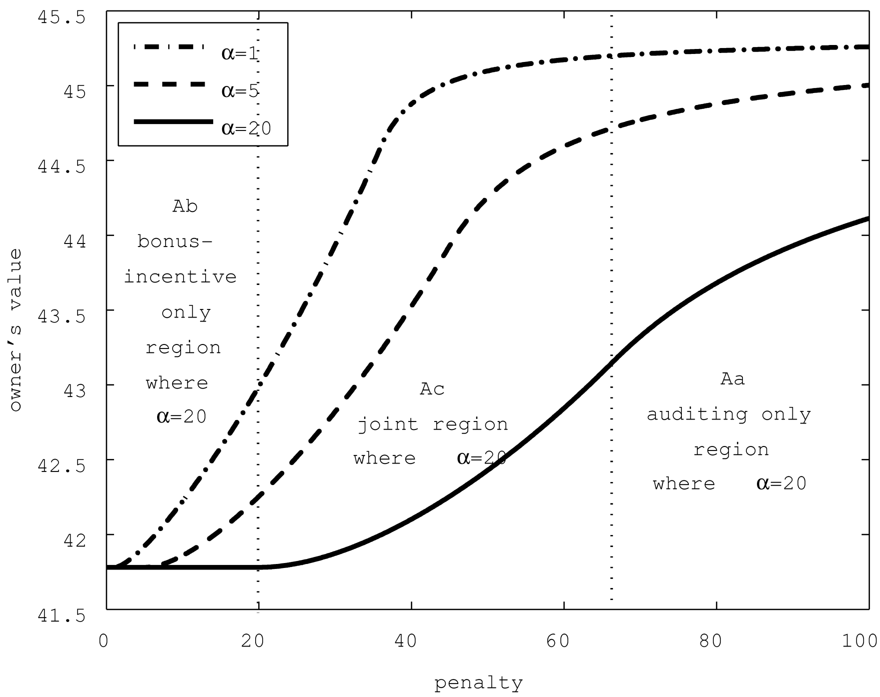

Figure 4: Owner's value with respect to penalty

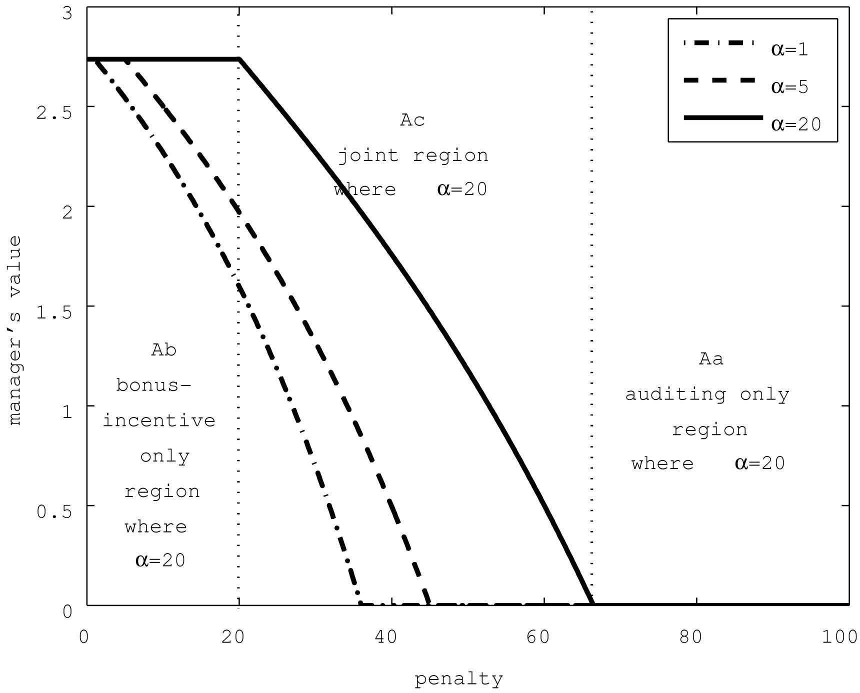

Figure 5: Manager's value with respect to penalty

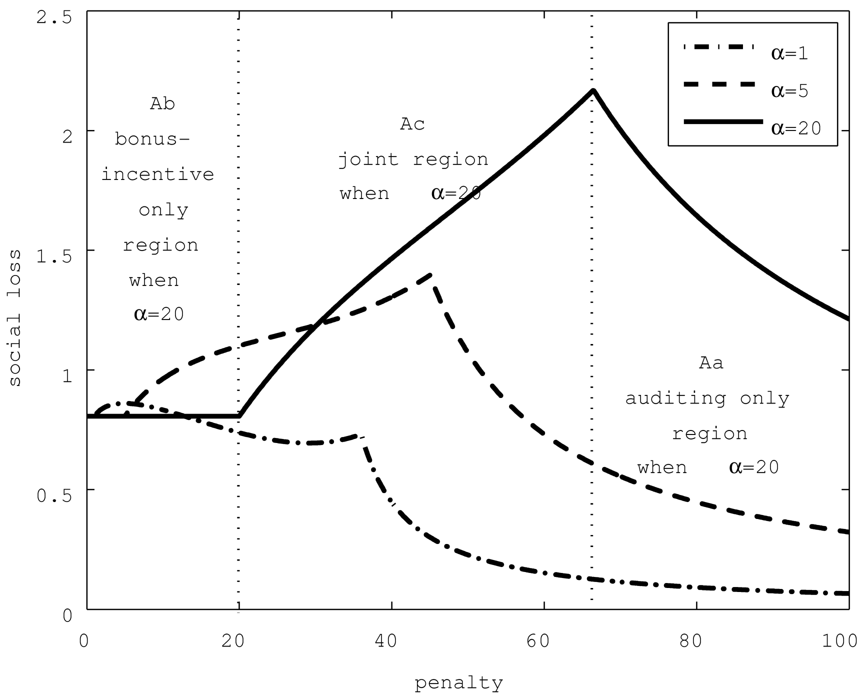

Figure 6: Total social loss with respect to penalty

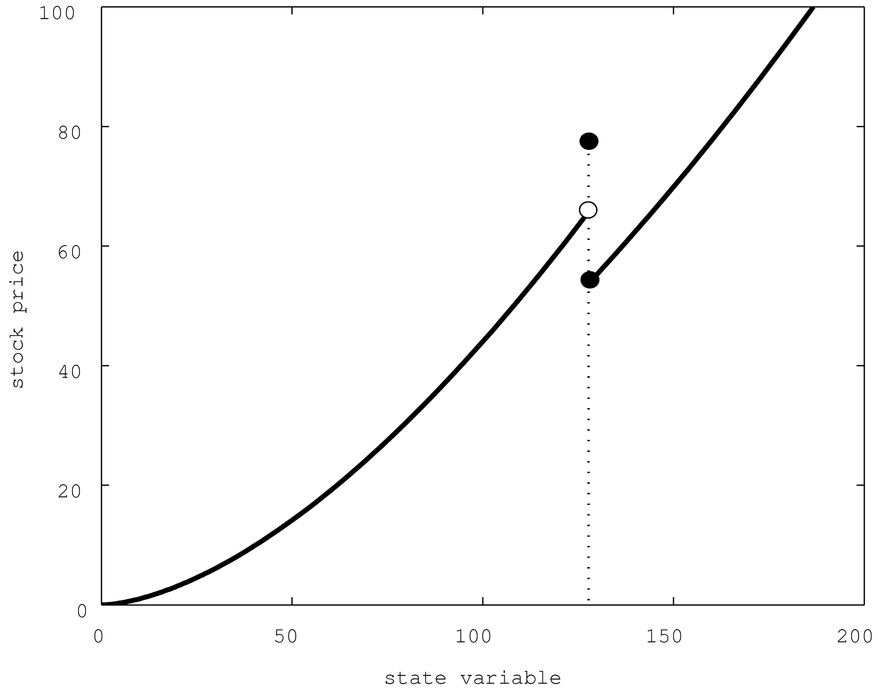

Figure 7: Stock price reaction to investment

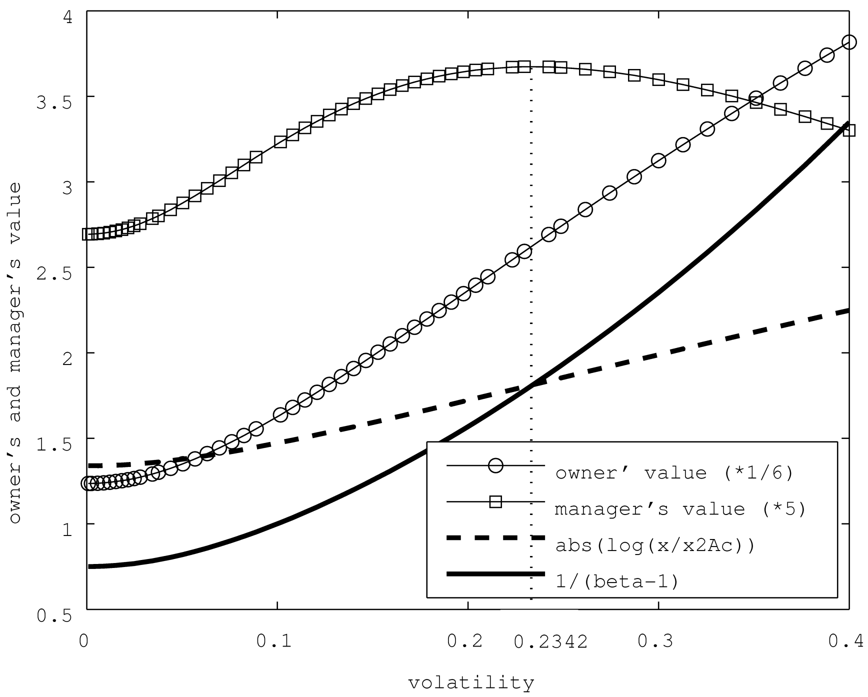

Figure 8: Owner's and manager's value with respect to volatility