Research Paper Series

No. 17

A Maximal Domain for the Existence of Bribe-proof Rules

Takuma Wakayama[†]

November, 2006

[†]Faculty of Urban Liberal Arts, Tokyo Metropolitan University, 1-1 Minami-Osawa, Hachiouji, Tokyo 192-0397, JAPAN; t_waka@center.tmu.ac.jp

A Maximal Domain for the Existence of Bribe-proof Rules*

Takuma Wakayama[†]

Faculty of Urban Liberal Arts, Tokyo Metropolitan University

t_waka@center.tmu.ac.jp

November 29, 2006

Abstract

This paper considers the problem of allocating an amount of a perfectly divisible good among agents. First, we prove that when the amount of the good is fixed, the uniform rule is the only rule satisfying *bribe-proofness* (Schummer, 2000) and *symmetry* on the single-peaked domain. Next, we examine how large a preference domain can be to allow for the existence of *bribe-proof* and *symmetric* rules when the amount of the good is a variable. We demonstrate that the convex domain is the unique maximal domain including the single-peaked one for *bribe-proofness* and *symmetry* when the amount of the good is a variable.

Keywords: Bribe-proofness, Convex preferences, Symmetry.

JEL Classification Numbers: C72, D71, D78.

*An earlier version of this paper was presented at the 2005 Autumn Meeting of the Japanese Economic Association in Tokyo. I thank Hideki Mizukami, Shinji Ohseto, and especially Shigehiro Serizawa for their valuable suggestions and comments. I also thank Hiroo Sasaki, Shohei Takagi, Yoshikatsu Tatamitani, and participants of the above meeting for their helpful comments.

[†]Faculty of Urban Liberal Arts, Tokyo Metropolitan University, 1-1 Minami-Osawa, Hachiouji, Tokyo 192-0397, JAPAN.

1 Introduction

We are considering economies characterized as "allotment economies" where there exists an amount of one perfectly divisible good that is not freely disposable, each agent has a preference relation on the consumption levels of the good, and the amount of the good is allocated among agents.[1] When the amount of the good is fixed, an "economy" is described by a preference profile. A set of preference profiles is called a "preference domain." A "rule" is a mapping defined on a preference domain, which assigns a feasible allocation of the good to each economy.

Sprumont (1991) introduced the axiomatic analysis of this class of problems using the "single-peaked domain."[2] The axioms he considered are *strategy-proofness*, *Pareto-efficiency*, and *anonymity*. *Strategy-proofness* is the requirement that each agent cannot be better off by her own misrepresentations regardless of the other agents' representations. *Pareto-efficiency* requires that agents cannot obtain better consumption levels by redistributing their original consumption levels. *Anonymity* is the requirement that whenever the preferences of two agents are switched, their consumption levels are also switched. Sprumont (1991) demonstrated that the "uniform rule" (Benassy, 1982) is the unique *strategy-proof*, *Pareto-efficient*, and *anonymous* rule on the single-peaked domain. Ching (1994) strengthened Sprumont's result by replacing *anonymity* with a weaker axiom, *symmetry*; this axiom states that if two agents' preferences are the same, their welfare from allocation should be the same.

A *strategy-proof* rule is not necessarily immune from strategic behavior by a coalition of agents. If a rule is immune from strategic behavior by any kind of coalition, then the rule is said to be *coalitionally strategy-proof*. The larger the coalition, the more difficult it is for the coalition to manipulate. Thus, society does not need to be concerned about manipulations from large coalitions. Hence, *coalitional strategy-proofness* is quite a strong requirement. Nevertheless, a collusion between two agents is relatively easy. Thus, it is plausible that an agent can be bribed by another agent to misrepresent her preferences. We are interested in rules that eliminate the possibility of this type of manipulation. Such rules are said to be *bribe-proof*. The concept of *bribe-proofness* was first introduced by Schummer (2000).[3]

[1] The model has been given several interpretations. See Sprumont (1991) and Barberà et al. (1997).

[2] "Single-peakedness" denotes that each agent has a most preferred consumption level and above or below this preferred consumption level welfare is decreasing. The "single-peaked domain" is the set of all single-peaked preference profiles.

[3] Serizawa (2006) proposed a similar concept of *bribe-proofness*. Called *effective pairwise*

Schummer (2000) and Mizukami (2003) studied *bribe-proofness* in the public good economies with quasi-linear preferences, and proved that it is essentially impossible to design a non-trivial *bribe-proof* rule on many domains. In contrast to their negative results, we demonstrate that in allotment economies, there is a non-trivial *bribe-proof* rule on a *naturally* restricted domain. We assume that the perfectly divisible good is freely transferable among agents. As a result, the condition of *bribe-proofness* is natural. In this paper, we provide a new characterization of the uniform rule on the single-peaked domain by employing *bribe-proofness*. We show that the uniform rule on the single-peaked domain is a unique *bribe-proof* and *symmetric* rule (Theorem 1).

We then ask how much the single-peaked domain can be enlarged to maintain the existence of *bribe-proof* and *symmetric* rules. Ching and Serizawa (1998) studied a similar question in allotment economies.[4] The model of Ching and Serizawa (1998) is different from Sprumont (1991) in that rules have the amount of the good to be allocated as a variable, and each economy is characterized by a pair of one preference profile and the amount of the good. We adapt the same setting as Ching and Serizawa (1998) in identifying maximal domains including the single-peaked one for *bribe-proofness* and *symmetry*. They showed that the "single-plateaued domain"[5] is the unique maximal domain including the single-peaked one for *strategy-proofness*, *Pareto-efficiency*, and *symmetry*. *Bribe-proofness* implies *strategy-proofness* and *Pareto-efficiency* on the single-peaked domain. Thus, one might conjecture that the single-plateaued domain is also the unique maximal domain including the single-peaked one for *bribe-proofness* and *symmetry*. However, this conjecture is incorrect. We demonstrate that the "convex domain,"[6] which is strictly larger than the single-plateaued domain, is the unique maximal domain including the single-peaked one for *bribe-proofness* and *symmetry* (Theorem 2).

---

strategy-proofness, it requires that the rule must be immune to unilateral manipulation and self-enforcing pairwise manipulation in the sense that no pair of agents has an incentive to betray the partner. He provided an alternative characterization of the uniform rule by employing *effective pairwise strategy-proofness*. In Appendix B, we examine the relationship between *bribe-proofness* and *effective pairwise strategy-proofness*.

[4]In other environments, a number of articles have identified maximal preference domains allowing for *strategy-proof* (or *coalitionally strategy-proof*) social choice functions. A partial list of such articles includes Barberà et al. (1991), Serizawa (1995), Berga and Serizawa (2000) for voting environments, and Ehlers (2002) for house allocation problems.

[5]"Single-plateaued preferences" are variants of single-peaked preferences for which the sets of most preferred consumption levels are intervals. The "single-plateaued domain" is the set of all single-plateaued preference profiles.

[6]A preference is "convex" if its upper contour set of any consumption level is convex. The "convex domain" is the set of all convex preference profiles.

Massò and Neme (2006) studied a problem similar to this paper. They investigated the set of *strongly bribe-proof* rules in allotment economies. *Strong bribe-proofness* is the property that no *coalition* of agents can compensate another agent to misrepresent her preferences, making all the members of the coalition better off after an appropriate redistribution. They first considered the possibility of manipulations through bribes in allotment economies. As mentioned earlier, collusion is especially difficult for a large coalition. However, it is relatively easy for pairs of agents to coordinate their preference revelations and to arrange transfers. Thus, in this paper, we weaken *strong bribe-proofness* by restricting collusion to pairwise one.

The paper proceeds as follows: The next section provides notation and definitions. Section 3 demonstrates the main results. Section 4 concludes with remaining problems. Appendix A contains proofs omitted from the main text. Appendix B examines the relationship between *bribe-proofness* and *effective pairwise strategy-proofness*. Appendix C establishes the independence of axioms in Theorem 1.

2 Preliminaries

We consider the problem of allocating one perfectly divisible private good among agents. We denote the set of *agents* by $N = \{1, 2, \ldots, n\}$, where $2 \leq n < +\infty$. Let $M \in \mathbb{R}_{++}$ be the amount of the good that has to be distributed among agents. Note that free disposal of the good is not allowed.

Let $\overline{\mathbb{R}}_+ = \mathbb{R}_+ \cup \{+\infty\}$. Each agent $i \in N$ has a preference relation R_i over $\overline{\mathbb{R}}_+$. By I_i, we denote the indifference relation associated with R_i; i.e., for each $x, y \in \overline{\mathbb{R}}_+$, xI_iy if and only if xR_iy and yR_ix. By P_i, we denote the strict preference relation associated with R_i; i.e., for each $x, y \in \overline{\mathbb{R}}_+$, xP_iy if and only if xR_iy and not yR_ix. We assume that preferences are continuous; i.e., for each $x \in \overline{\mathbb{R}}_{++}$, the sets $\{y \in \overline{\mathbb{R}}_+ : yR_ix\}$ and $\{y \in \overline{\mathbb{R}}_+ : xR_iy\}$ are closed. Let $\mathscr{R}$ be the set of all continuous preferences. Given $R_i \in \mathscr{R}$, let $p(R_i) = \{x \in \overline{\mathbb{R}}_+ : xR_iy \text{ for each } y \in \overline{\mathbb{R}}_+\}$ be the set of preferred consumption levels for R_i. Let $\underline{p}(R_i) = \inf p(R_i)$ and $\overline{p}(R_i) = \sup p(R_i)$. If $p(R_i)$ is a singleton, then we slightly abuse notation and denote by $p(R_i)$ the single element. Let $\mathscr{R}^n$ be the set of *preference profile* $R = (R_1, R_2, \ldots, R_n)$, where $R_i \in \mathscr{R}$ for each $i \in N$. We often denote $N \setminus \{i\}$ by "$-i$." With this notation, (R'_i, R_{-i}) is the preference profile where agent i's preference relation is R'_i, and the preference relation of agent $j \neq i$ is R_j. We name a pair $(R, M) \in \mathscr{R}^n \times \mathbb{R}_{++}$ an *economy*.

Let $Z(M) = \left\{z = (z_1, z_2, \ldots, z_n) \in \mathbb{R}^n_+ : \sum_{i \in N} z_i = M\right\}$ be the set of *allocations* for $(R, M) \in \mathscr{R}^n \times \mathbb{R}_{++}$, where agent $i \in N$ receives z_i. A subset $\mathscr{R}_a^n$ of $\mathscr{R}^n$ represents a *domain.* A *rule* is a function $\varphi\colon \mathscr{R}_a^n \times \mathbb{R}_{++} \to \mathbb{R}_+^n$ associating with each economy $(R, M) \in \mathscr{R}_a^n \times \mathbb{R}_{++}$ an allocation $\varphi(R) \in Z(M)$. We denote by $\varphi_i(R, M)$ the amount of the good allocated to agent $i \in N$. When we want to emphasize the domain of a rule, we name it a rule on $\mathscr{R}_a^n \times \mathbb{R}_{++}$.

In this paper, we focus on preferences with the following restrictions. A preference relation $R_i \in \mathscr{R}$ is *single-peaked* if $p(R_i)$ is a singleton and for each $x, y \in \overline{\mathbb{R}}_+$, we have xP_iy whenever $y < x \leq p(R_i)$ or $p(R_i) \leq x < y$. Let $\mathscr{R}_S^n \subseteq \mathscr{R}^n$ be the set of all single-peaked preference profiles. We name it the *single-peaked domain.* A preference relation $R_i \in \mathscr{R}$ is *single-plateaued* if $p(R_i)$ is an interval $[\underline{p}(R_i), \overline{p}(R_i)]$ and for each $x, y \in \overline{\mathbb{R}}_+$ such that $y < x \leq \underline{p}(R_i)$ and $\overline{p}(R_i) \leq x < y$, xP_iy. Let $\mathscr{R}_{SP}^n \subseteq \mathscr{R}^n$ be the set of all single-plateaued preference profiles. We name it the *single-plateaued domain.* A preference relation $R_i \in \mathscr{R}$ is *convex* if $p(R_i)$ is an interval $[\underline{p}(R_i), \overline{p}(R_i)]$ and for each $x, y \in \overline{\mathbb{R}}_+$ such that $y < x \leq \underline{p}(R_i)$ and $\overline{p}(R_i) \leq x < y$, xR_iy. Let $\mathscr{R}_C^n \subseteq \mathscr{R}^n$ be the set of all convex preference profiles. We name it the *convex domain.* Note that $\mathscr{R}_S^n \subset \mathscr{R}_{SP}^n \subset \mathscr{R}_C^n$.

We now introduce several axioms. *Strategy-proofness* is an incentive compatibility property, which requires that no agent should be able to misrepresent her preferences in a way that results in a direct gain to her, irrespective of the other agents' representations. *Bribe-proofness*, which was first introduced by Schummer (2000), requires each agent to have no incentive to bribe another agent to misrepresent in order to jointly benefit, irrespective of what the other agents represent.[7]

Strategy-proofness: For each $(R, M) \in \mathscr{R}_a^n \times \mathbb{R}_{++}$, each $i \in N$, and each $R_i' \in \mathscr{R}_a$, $\varphi_i(R, M)R_i\varphi_i(R_i', R_{-i}, M)$.

Bribe-proofness: For each $(R, M) \in \mathscr{R}_a^n \times \mathbb{R}_{++}$ and each $i, j \in N$, there exist no $R_i' \in \mathscr{R}_a$ and $b \in \mathbb{R}$ such that $(\varphi_i(R_i', R_{-i}, M) + b)P_i\varphi_i(R, M)$ and $(\varphi_j(R_i', R_{-i}, M) - b)P_j\varphi_j(R, M)$.

Remark 1. By choosing $i = j$ and $b = 0$, *bribe-proofness* implies *strategy-proofness.*

Next, we introduce the standard requirement of efficiency, *Pareto-efficiency.*

Pareto-efficiency: For each $(R, M) \in \mathscr{R}_a^n \times \mathbb{R}_{++}$, there is no $z \in Z(M)$ such that for each $i \in N$, $z_iR_i\varphi_i(R, M)$, and for some $j \in N$, $z_jP_j\varphi_j(R, M)$.

We provide the following fact that will be useful.

[7] See Schummer (2000) for a more detailed discussion.

Fact 1. *If a rule φ on $\mathscr{R}_S^n \times \mathbb{R}_{++}$ is Pareto-efficient, then for each $(R, M) \in \mathscr{R}_S^n \times \mathbb{R}_{++}$, the following properties hold:*
If $\sum_{i\in N} p(R_i) \leq M$, then for each $i \in N$, $p(R_i) \leq \varphi_i(R, M)$. If $\sum_{i\in N} p(R_i) \geq M$, then for each $i \in N$, $p(R_i) \geq \varphi_i(R, M)$.

Proof: Consider the case where $\sum_{i\in N} p(R_i) \leq M$. Suppose, by way of contradiction, that there exists $j \in N$ such that $p(R_j) > \varphi_j(R, M)$. By feasibility, there exists $k \in N \setminus \{j\}$ such that $p(R_k) < \varphi_k(R, M)$. Let $\varepsilon = \min\{\varphi_k(R, M) - p(R_k), p(R_j) - \varphi_j(R, M)\}$. Let $z = (z_1, z_2, \ldots, z_n)$ be such that $z_j = \varphi_j(R, M) + \varepsilon$, $z_k = \varphi_k(R, M) - \varepsilon$, and $z_i = \varphi_i(R, M)$ for each $i \in N \setminus \{j, k\}$. Then the allocation z Pareto-dominates $\varphi(R, M)$. This is a contradiction. By the same way as the above, we can prove the case where $\sum_{i\in N} p(R_i) \geq M$. □

Finally, we introduce the distributional requirement called *symmetry*. This condition requires the rule that no agent be treated unfairly.

***Symmetry*:** For each $(R, M) \in \mathscr{R}_a^n \times \mathbb{R}_{++}$ and each $i, j \in N$ such that $R_i = R_j$, $\varphi_i(R, M) I_i \varphi_j(R, M)$.

3 Main Results

3.1 Characterization

In this subsection, we seek rules satisfying *bribe-proofness* and *symmetry* on the single-peaked domain. Ching (1994) showed that the following rule, known as the uniform rule (Benassy, 1982), is the only rule on the single-peaked domain that satisfies *strategy-proofness*, *Pareto-efficiency*, and *symmetry*.

Definition 1. The *uniform rule* $U = (U_1, U_2, \ldots, U_n)$ is defined as follows: For each $(R, M) \in \mathscr{R}_S^n \times \mathbb{R}_{++}$ and each $i \in N$,

$$U_i(R, M) = \begin{cases} \min\{p(R_i), \lambda(R, M)\} & \text{if } M \leq \sum_{i\in N} p(R_i); \\ \max\{p(R_i), \lambda(R, M)\} & \text{otherwise,} \end{cases}$$

where $\lambda(R, M)$ solves $\sum_{i\in N} U_i(R, M) = M$.

Ching's (1994) characterization holds when M is fixed. The result also holds when M is not fixed. That is, the characterization when M is fixed implies the characterization when M is not fixed. Thus, we first demonstrate our characterization when M is fixed.

Before proceeding, let us introduce additional notation. Given $M \in \mathbb{R}_{++}$, we define $\mathscr{R}_S(M)$ as the set of single-peaked preferences obtained by restricting on $[0, M]$ all preferences in $\mathscr{R}_S$. We denote by $R_i(M)$ a generic element of $\mathscr{R}_S(M)$. When M is fixed and a domain is $\mathscr{R}_S(M)^n$, we consider that agent i's single-peaked preference relation is defined only on $[0, M]$, an economy is represented by a list $R(M) = (R_1(M), R_2(M), \ldots, R_n(M)) \in \mathscr{R}_S(M)^n$, and a rule is a function $\varphi(\cdot, M)\colon \mathscr{R}_S(M)^n \to Z(M)$. *Bribe-proofness*, *strategy-proofness*, *Pareto-efficiency*, and *symmetry* are similarly defined in this setting. Thus, we omit their definitions.

Fact 2. *Let $M \in \mathbb{R}_{++}$. If a rule $\varphi(\cdot, M)$ on the single-peaked domain $\mathscr{R}_S(M)^n$ satisfies* bribe-proofness, *then it satisfies* Pareto-efficiency.

Proof: Suppose, by way of contradiction, that a solution $\varphi(\cdot, M)$ on the single-peaked domain $\mathscr{R}_S(M)^n$ is not *Pareto-efficient*. Then, there exist $R(M) \in \mathscr{R}_S(M)^n$ and $(z_1, z_2, \ldots, z_n) \in Z(M)$ such that $z_i R_i(M) \varphi_i(R(M), M)$ for each $i \in N$ and $z_j P_j(M) \varphi_j(R(M), M)$ for some $j \in N$. Without loss of generality, $z_j > \varphi_j(R(M), M)$. Then, by feasibility, there exists $k \in N \setminus \{j\}$ such that $z_k < \varphi_k(R(M), M)$. By $R_j(M), R_k(M) \in \mathscr{R}_S(M)$, we have $p(R_j(M)) > \varphi_j(R(M), M)$ and $p(R_k(M)) < \varphi_k(R(M), M)$. Let $b = \min\{p(R_j(M)) - \varphi_j(R(M), M), \varphi_k(R(M), M) - p(R_k(M))\}$. Let $R'_j(M) = R_j(M)$. Then we have $(\varphi_j(R(M), M) + b) P_j(M) \varphi_j(R(M), M)$ and $(\varphi_k(R(M), M) - b) P_k(M) \varphi_k(R(M), M)$, which is a contradiction to *bribe-proofness.*

□

Remark 1 and Fact 2 convey that *bribe-proofness* implies *strategy-proofness* and *Pareto-efficiency* on the single-peaked domain. However, it is interesting to note that on the single-peaked domain, *strategy-proofness* and *Pareto-efficiency* do not imply *bribe-proofness* (See Example 1 in Massò and Neme (2006)).

Now we offer a new characterization of the uniform rule on the single-peaked domain.

Theorem 1. *Let $M \in \mathbb{R}_{++}$. A rule $\varphi(\cdot, M)$ on the single-peaked domain $\mathscr{R}_S(M)^n$ satisfies* bribe-proofness *and* symmetry *if and only if $\varphi(R(M), M) = U(R(M), M)$ for each $R(M) \in \mathscr{R}_S(M)^n$.*

Proof: The proof of the "if" part is tedious but not difficult. The proof of the "if" part is in Appendix A. Thus, we prove the "only if" part. Suppose that a rule $\varphi(\cdot, M)$ on the single-peaked domain $\mathscr{R}_S(M)^n$ satisfies *bribe-proofness* and *symmetry.*

By Remark 1 and Fact 2, *bribe-proofness* implies *strategy-proofness* and *Pareto-efficiency.* Hence, by Ching (1994), $\varphi(R(M), M) = U(R(M), M)$ for each $R(M) \in \mathscr{R}_S(M)^n$. $\square$

Theorem 1 states that when M is fixed, the uniform rule is the unique rule satisfying *bribe-proofness* and *symmetry* on the single-peak domain. The characterization when M is not fixed is a corollary of Theorem 1.

Corollary 1. *A rule φ on the single-peaked domain $\mathscr{R}_S^n \times \mathbb{R}_{++}$ satisfies* bribe-proofness *and* symmetry *if and only if $\varphi(R, M) = U(R, M)$ for each $(R, M) \in \mathscr{R}_S^n \times \mathbb{R}_{++}$.*

3.2 Maximal Domain

In subsection 3.1, we characterized the set of all *bribe-proof* and *symmetric* rules when each agent has single-peaked preferences. How much can the single-peaked domain be enlarged to allow for the existence of *bribe-proof* and *symmetric* rules? Following Ching and Serizawa (1998), we introduce the following notion to address this question.

Definition 2. A domain $\mathscr{R}_m^n$ is a *maximal domain* for a list of axioms if

(i) $\mathscr{R}_m^n \subseteq \mathscr{R}^n$;

(ii) there is a rule on $\mathscr{R}_m^n \times \mathbb{R}_{++}$ satisfying the axioms;

(iii) there is no rule satisfying the same axioms on any $\mathscr{R}_a^n \times \mathbb{R}_{++}$ such that $\mathscr{R}_m^n \subset \mathscr{R}_a^n \subseteq \mathscr{R}^n$.

Note that a maximal domain for a list of axioms may not be unique. However, Ching and Serizawa (1998) proved that the single-plateaued domain is the unique maximal domain including the single-peaked one for *strategy-proofness*, *Pareto-efficiency*, and *symmetry.* On the single-peaked domain, based on Remark 1 and Fact 2, *bribe-proofness* implies *strategy-proofness* and *Pareto-efficiency.* Taking account of Ching and Serizawa (1998), one might conjecture that the single-plateaued domain is the unique maximal domain including the single-peaked one for *bribe-proofness* and *symmetry.* However, this conjecture is incorrect. Before demonstrating this, we define the following rule, known as the extended uniform rule.

Definition 3. The *extended uniform rule* $U^e = (U_1^e, U_2^e, \ldots, U_n^e)$ is defined as follows: For each $(R, M) \in \mathscr{R}_C^n \times \mathbb{R}_{++}$ and each $i \in N$,

$$U_i^e(R, M) = \begin{cases} \min\{\underline{p}(R_i), \lambda(R, M)\} & \text{if } M \leq \sum_{i \in N} \underline{p}(R_i); \\ \min\{\overline{p}(R_i), \underline{p}(R_i) + \lambda(R, M)\} & \text{if } \sum_{i \in N} \underline{p}(R_i) < M < \sum_{i \in N} \overline{p}(R_i); \\ \max\{\overline{p}(R_i), \lambda(R, M)\} & \text{otherwise,} \end{cases}$$

where $\lambda(R, M)$ solves $\sum_{i \in N} U_i^e(R, M) = M$.

Example 1 below illustrates that the extended uniform rule on the convex domain is *bribe-proof* but not *Pareto-efficient*. This means that outside the single-plateaued domain, *bribe-proofness* does not imply *Pareto-efficiency*. For this reason, the single-plateaued domain is not a maximal domain including the single-peaked one for *bribe-proofness* and *symmetry*.

Example 1. Let $N = \{1, 2, 3\}$. The extended uniform rule U^e on $\mathscr{R}_C^3 \times \mathbb{R}_{++}$ satisfies *bribe-proofness*, as shown by Claim 1 below. In this example, we want to demonstrate that the extended uniform rule U^e on $\mathscr{R}_C^3 \times \mathbb{R}_{++}$ does not satisfy *Pareto-efficiency*. To demonstrate this, consider $(R, M) \in \mathscr{R}_C^3 \times \mathbb{R}_{++}$ such that $M = 15$ and for each $i \in N$, R_i satisfies the following conditions:

- $[\underline{p}(R_i), \overline{p}(R_i)] = [10, 12]$;
- for each $x, y \in [5, 10]$ such that $x > y$, xP_iy;
- for each $x, y \in [0, 5]$, xI_iy;
- for each $x, y \in [12, 14]$ such that $x < y$, xP_iy;
- for each $x, y \in [14, +\infty) \cup \{+\infty\}$, xI_iy;
- for each $x \in [14, +\infty) \cup \{+\infty\}$ and each $y \in [0, 5]$, xP_iy.

Since U^e satisfies *symmetry*, $U^e(R) = (5, 5, 5)$. Let $x = U^e(R)$. Now, consider an allocation $y = (0, 7.5, 7.5)$. Then, $x_1I_1y_1$, $y_2P_2x_2$, and $y_3P_3x_3$. Therefore, the allocation y Pareto-dominates the allocation x. This denotes that U^e on $\mathscr{R}_C^3 \times \mathbb{R}_{++}$ does not satisfy *Pareto-efficiency*. ■

Theorem 2 below states that although *bribe-proofness* does not imply *Pareto-efficiency* on the convex domain, the convex domain is the unique maximal domain including the single-peaked one for *bribe-proofness* and *symmetry*. The maximal domain result of Ching and Serizawa (1998) states that there is no rule satisfying *strategy-proofness*, *Pareto-efficiency*, and *symmetry* on any domain strictly larger than the single-plateaued domain. However, Theorem 2 below denotes that there is a rule satisfying *bribe-proofness* and *symmetry* on the convex domain. Thus, the assumption of single-peakedness can be weakened if one insists on *bribe-proofness* and *symmetry*.

Theorem 2. *The convex domain is the unique maximal domain including the single-peaked domain for* bribe-proofness *and* symmetry*.*

The basic structure of the proof of Theorem 2 is similar to Ching and Serizawa (1998). However, their proof method does not work for Theorem 2 since *bribe-proofness* does not imply *Pareto-efficiency* outside the single-plateaued domain, as we demonstrated in Example 1.

Before proceeding to the proof of Theorem 2, we present a useful lemma. Lemma 1 below states that if a rule on a domain including the single-peaked domain satisfies *bribe-proofness*, then *symmetry* implies *strong symmetry*, which requires that whenever two agents have the same preference relation, they receive the same consumption level; i.e., for each $(R, M) \in \mathscr{R}_a^n \times \mathbb{R}_{++}$ and each $i, j \in N$ such that $R_i = R_j$, $\varphi_i(R, M) = \varphi_j(R, M)$.

Lemma 1. *Let $\mathscr{R}_S^n \subseteq \mathscr{R}_a^n \subseteq \mathscr{R}^n$. If a rule φ on $\mathscr{R}_a^n \times \mathbb{R}_{++}$ satisfies* bribe-proofness *and* symmetry*, then for each $(R, M) \in \mathscr{R}_a^n \times \mathbb{R}_{++}$ and each $i, j \in N$ such that $R_i = R_j \in \mathscr{R}_S$, $\varphi_i(R, M) = \varphi_j(R, M)$.*

Proof: Suppose, by contradiction, that there exist $(R, M) \in \mathscr{R}_a^n \times \mathbb{R}_{++}$ and $i, j \in N$ such that $R_i = R_j \in \mathscr{R}_S$ and $\varphi_i(R, M) \neq \varphi_j(R, M)$. Since $R_i = R_j \in \mathscr{R}_S$ and $\varphi_i(R, M) \neq \varphi_i(R, M)$, we have $\varphi_i(R, M) \neq p(R_i)$ and $\varphi_j(R, M) \neq p(R_j)$. Otherwise, $\varphi_i(R, M) = p(R_i)$ or $\varphi_j(R, M) = p(R_j)$, and $\varphi_i(R, M) \neq \varphi_j(R, M)$. This is a contradiction to *symmetry*. Without loss of generality, $\varphi_i(R, M) < p(R_i)$. Since $\varphi_i(R, M) \neq \varphi_j(R, M)$, by *symmetry*, we have $\varphi_j(R, M) > p(R_j)$. Let $b = \min\{p(R_i) - \varphi_i(R, M), \varphi_j(R, M) - p(R_j)\}$. Then, by $R_i = R_j \in \mathscr{R}_S$, we have $(\varphi_i(R, M) + b) P_i \varphi_i(R, M)$ and $(\varphi_j(R, M) - b) P_j \varphi_j(R, M)$. This is a contradiction to *bribe-proofness*. □

Proof of Theorem 2: Let $\mathscr{R}_S^n \subseteq \mathscr{R}_a^n \subseteq \mathscr{R}^n$. Suppose that there is a rule φ on $\mathscr{R}_a^n \times \mathbb{R}_{++}$ satisfying *bribe-proofness* and *symmetry*. We will show that $\mathscr{R}_a^n \subseteq \mathscr{R}_C^n$. Suppose, by contradiction, that there is $R_0 \in \mathscr{R}_a \setminus \mathscr{R}_C$. Let $R_0 \in \mathscr{R}_a \setminus \mathscr{R}_C$. Then, there exist three points $x_0 < y_0 < z_0$ such that $x_0 P_0 y_0$ and $z_0 P_0 y_0$. Let

$$x_0^* = \begin{cases} \max\{x_0' \in [x_0, y_0] \colon x_0' I_0 x_0\} \text{ if } z_0 R_0 x_0 \\ \max\{x_0' \in [x_0, y_0] \colon x_0' I_0 z_0\} \text{ otherwise;} \end{cases}$$
$$z_0^* = \begin{cases} \min\{z_0' \in [y_0, z_0] \colon z_0' I_0 x_0\} \text{ if } z_0 R_0 x_0 \\ \min\{z_0' \in [y_0, z_0] \colon z_0' I_0 z_0\} \text{ otherwise.} \end{cases}$$

Since R_0 is continuous, x_0^* and z_0^* are well-defined. Note that $x_0 \leq x_0^* < y_0 < z_0^* \leq z_0$, $x_0^* I_0 z_0^*$ and for each $x_0' \in (x_0^*, z_0^*)$, $z_0^* P_0 x_0'$. Also, since R_0 is continuous and $z_0^* P_0 x_0'$ for each $x_0' \in (x_0^*, z_0^*)$, there exists $r \in (x_0^*, z_0^*)$ such that $x' P_0 x''$ for each $x', x'' \in (x_0^*, r)$ with $x' < x''$. Pick any $\underline{y} \in (x_0^*, r)$. Let $\overline{y} = \min\{y' \in (\underline{y}, z_0^*) \colon \underline{y} I_0 y'\}$. Since R_0 is continuous, $\underline{y}$ and $\overline{y}$ are well-defined. Note that $\overline{y} P_0 r$ and $\overline{y} I_0 \underline{y}$. By definition, $x_0 \leq x_0^* < \underline{y} < r < \overline{y} < z_0^* \leq z_0$. This is illustrated in Figure 1.

Let $M = n\overline{y}$. Let $R_0' \in \mathscr{R}_S$ be such that $p(R_0') \in \left(\overline{y}, \frac{M-\underline{y}}{n-1}\right)$ and $M P_0' \overline{y}$. See Figure 2. Let $R' = (R_1', R_2', \ldots, R_n')$ be such that $R_i' = R_0'$ for each $i \in N$. By Lemma 1, $\varphi(R', M) = (\overline{y}, \overline{y}, \ldots, \overline{y})$. Let $i \in N$ and $R_i = R_0$. Without loss of generality, assume that $i = 1$. We consider the allocation when agent 1 changes her preferences from R_1' to R_1. We can distinguish two cases.

Case 1: $\boldsymbol{\varphi_1(R_1, R_{-1}', M) = \varphi_1(R', M)}$.

Since $\varphi_1(R_1, R_{-1}', M) = \overline{y}$, $\sum_{j \in N \setminus \{1\}} \varphi_j(R_1, R_{-1}', M) = (n-1)\overline{y}$. By Lemma 1, $\varphi_j(R_1, R_{-1}', M) = \overline{y}$ for each $j \in N \setminus \{1\}$. Let $b = \varphi_1(R_1, R_{-1}', M) - x_0^*$ and $i \in N \setminus \{1\}$. Then, $(\varphi_1(R_1, R_{-1}', M) - b) P_1 \varphi_1(R_1, R_{-1}', M)$ and $(\varphi_i(R_1, R_{-1}', M) + b) P_i' \varphi_i(R_1, R_{-1}', M)$, contradicting *bribe-proofness*.

Case 2: $\boldsymbol{\varphi_1(R_1, R_{-1}', M) \neq \varphi_1(R', M)}$.

We can also distinguish three subcases.

Subcase 2-1: $\boldsymbol{\overline{y} < \varphi_1(R_1, R_{-1}', M)}$.

In this subcase, we have $\varphi_1(R_1, R_{-1}', M) P_1' \varphi_1(R', M)$. Thus, agent 1 with preferences R_1' can gain by announcing false preferences R_1, in violation of *strategy-proofness* (hence contradicting *bribe-proofness*).

Subcase 2-2: $\boldsymbol{\underline{y} < \varphi_1(R_1, R_{-1}', M) < \overline{y}}$.

In this subcase, we have $\varphi_1(R', M) P_1 \varphi_1(R_1, R_{-1}' M)$. Thus, agent 1 with preferences R_1 can gain by announcing false preferences R_1', in violation of *strategy-proofness* (hence contradicting *bribe-proofness*).

Subcase 2-3: $\varphi_1(R_1, R'_{-1}, M) \leq \underline{y}$.

Let $R''_1 \in \mathscr{R}_S$ be such that $p(R''_1) \leq \varphi_1(R_1, R'_{-1}, M)$. See Figure 3. By Corollary 1, $\varphi(R''_1, R'_{-1}, M) = U(R''_1, R'_{-1}, M)$. Since $p(R''_1) \leq \underline{y}$ and $p(R'_j) \in \left(\overline{y}, \frac{M-\underline{y}}{n-1}\right)$ for each $j \in N \setminus \{1\}$, $p(R''_1) + \sum_{j \in N \setminus \{1\}} p(R'_j) < M$. By the definition of U, $\lambda(R''_1, R'_{-1}, M) \leq \overline{y}$. Otherwise, since $\overline{y} < \lambda(R''_1, R'_{-1}, M) \leq U_j(R''_1, R'_{-1}, M)$ for each $j \in N \setminus \{1\}$, we have $M < \sum_{j \in N} U_j(R''_1, R'_{-1}, M) = M$, a contradiction. Therefore, $\varphi_1(R''_1, R'_{-1}, M) = M - (n-1)p(R'_0)$ and $\varphi_j(R''_1, R'_{-1}, M) = p(R'_j)$ for each $j \in N \setminus \{1\}$. Thus, since $p(R'_0) \in \left(\overline{y}, \frac{M-\underline{y}}{n-1}\right)$, $\varphi_1(R''_1, R'_{-1}, M) \in (\underline{y}, \overline{y})$. Since $p(R''_1) \leq \varphi_1(R_1, R'_{-1}, M) < \varphi_1(R''_1, R'_{-1}, M)$, $\varphi_1(R_1, R'_{-1}, M) P''_1 \varphi_1(R''_1, R'_{-1}, M)$. See Figure 4. Thus, agent 1 with preferences R''_1 can gain by announcing false preferences R_1, contradicting *strategy-proofness* (hence contradicting *bribe-proofness*).

Since we obtain contradiction in all cases, $\mathscr{R}^n_a \subseteq \mathscr{R}^n_C$.

The proof of Theorem 2 is completed by constructing a rule satisfying *bribe-proofness* and *symmetry* on the convex domain. It can be easily verified that the extended uniform rule defined in Definition 3 satisfies *symmetry*. Thus, to complete the proof, it is sufficient to show that the extended uniform rule on the convex domain satisfies *bribe-proofness*.[8]

Claim 1. *The extended uniform rule U^e on the convex domain $\mathscr{R}^n_C \times \mathbb{R}_{++}$ satisfies bribe-proofness.*

The rather tedious proof of Claim 1 is omitted but available in Appendix A. □

Remark 2. We can easily demonstrate that if there exists a rule defined on a domain satisfying *bribe-proofness* and *strong symmetry*, then that domain is a subdomain of the convex domain. The proof of this result is in Appendix A.

4 Conclusion

This paper concludes with two main results. First, we obtained an alternative characterization of the uniform rule by using *bribe-proofness* instead of *strategy-proofness* and *Pareto-efficiency* (Theorem 1). Secondly, by exploiting our characterization, we showed that the maximal domain including the single-peaked one for *bribe-proofness* and *symmetry* is unique and it is the convex domain that is strictly larger than the

[8] Whether the extended uniform rule is the only rule satisfying *bribe-proofness* and *symmetry* on the convex domain is open.

single-plateaued domain (Theorem 2). Our maximal domain result means that the assumption of single-peakedness can be weakened if one insists on *bribe-proofness* and *symmetry*.

In the following, we discuss three remaining problems. First, to identify maximal domains, we formulate rules as functions of preference profiles and the amounts of the good. Thus, our maximal domain result does not apply when the amount of the good is fixed and the rule is only a function of preference profile. Therefore, our maximal domain result does not exclude the possibility that when the amount of the good is fixed, there exist rules satisfying *bribe-proofness* and *symmetry* on larger domain than the convex domain. Massò and Neme (2001) obtained maximal domain for rules satisfying *strategy-proofness*, *Pareto-efficiency*, and *strong symmetry* when the amount of the good is fixed. Thus, the following question requires an answer: When the amount of the good is fixed, what domain is a maximal domain including the single-peaked one for *bribe-proofness* and *symmetry* (or *strong symmetry*)?

Second, if a rule satisfies *bribe-proofness* on a domain, it also satisfies *bribe-proofness* on any subdomain. In public good economies, Schummer (2000) demonstrated that *bribe-proofness* is so strong on the *rich* domain that only the constant rule can satisfy *bribe-proofness*. Thus, the larger the domain on which rules are required to satisfy *bribe-proofness*, the stronger is the requirement. Similarly, *symmetry* is stronger on larger domains. Therefore, the smaller the domain, the more rules satisfying *bribe-proofness* and *symmetry* potentially exist.[9] Indeed, Schummer (2000) demonstrated that the median voter rule satisfies *bribe-proofness* on the extremely small domain. Mizobuchi and Serizawa (2006) considered minimal domains on which *strategy-proofness*, *Pareto-efficiency*, and *symmetry* imply the uniqueness of the rule. They established that a rule on a *minimally rich domain*[10] satisfies *strategy-proofness*, *Pareto-efficiency*, and *symmetry* if and only if it is the uniform rule. Thus, the following question requires an answer: How much can we shrink the single-peaked domain while preserving the uniform rule as the unique rule satisfying *bribe-proofness* and *symmetry*?

Third, Barberà et al. (1997) demonstrated that many rules on the single-peaked domain satisfy *strategy-proofness* and *Pareto-efficiency*. They identified the func-

[9] Generally speaking, since the degrees of freedom in designing rules increase with the larger domain, there might be an interesting rule satisfying axioms on the larger domain.

[10] A *minimally rich domain* is a small subset of the single-peaked domain satisfying the following two conditions: (i) for each consumption level, there exists only one preference whose peak coincides with the consumption level; (ii) given two distinct consumption levels, say x and y, there exists at least one preference whose peak is between x and y such that x is preferred to y.

tional form of all *strategy-proof*, *Pareto-efficient*, and *replacement monotonic*[11] rules on the single-peaked domain. It is the class of all sequential rules. Massò and Neme (2006) demonstrated that the class of all *strongly bribe-proof* and *peak-only*[12] rules on the single-peaked domain is strictly larger than the class of all sequential rules. This paper does not identify the functional form of all *bribe-proof* rules on the single-peaked domain.

[11]The condition of *replacement monotonicity* states that if an agent receives a larger consumption level after changing her preference relation then all the other agents should receive smaller consumption levels.

[12]The condition of *peak-only* states that the amount allocated to agents depends only on their peaks.

A Appendix: Proofs

A.1 Proof of the "if" Part of Theorem 1

To simplify notation, we denote $\mathscr{R}_S(M)$ by $\mathscr{R}_S$ throughout the proof of the "if" part of Theorem 1. We denote a generic element of $\mathscr{R}(M)$ by R instead of $R(M)$. For the same reason, $\varphi(R, M)$, $U(R, M)$, and $Z(M)$ are replaced by $\varphi(R)$, $U(R)$, and Z, respectively.

Before proceeding to the proof, we introduce additional notation and provide lemmas. For any three numbers $\alpha, \beta, \gamma \in [0, M]$, define $\mathrm{med}\{\alpha, \beta, \gamma\}$ as the number $z \in \{\alpha, \beta, \gamma\}$ such that $\#\{z' \in \{\alpha, \beta, \gamma\}\colon z' \geq z\} \geq 2$ and $\#\{z' \in \{\alpha, \beta, \gamma\}\colon z' \geq z\} \leq 2$.[13]

Lemma 2 (Sprumont, 1991). *Let a rule φ on the single-peaked domain $\mathscr{R}_S^n$ be strategy-proof and Pareto-efficient. Then, for each $i \in N$ and each $R \in \mathscr{R}_S^n$, there exist $\alpha_i\colon \mathscr{R}_S^{n-1} \to [0, M]$ and $\beta_i\colon \mathscr{R}_S^{n-1} \to [0, M]$ such that $\alpha_i(R_{-i}) \leq \beta_i(R_{-i})$ and $\varphi_i(R) = \mathrm{med}\{\alpha_i(R_{-i}), \beta_i(R_{-i}), p(R_i)\}$.*

Proof: See Sprumont (1991). □

Since the uniform rule U on the single-peaked domain $\mathscr{R}_S^n$ satisfies *strategy-proofness* and *Pareto-efficiency* (Ching, 1994), Lemma 2 holds for the uniform rule.

Non-bossiness, which was introduced by Satterthwaite and Sonnenschein (1981), requires that if an agent changes her preferences but her allocation is unchanged, then the allocation of each agent should be unchanged.

Non-bossiness: For each $R \in \mathscr{R}^n$, each $i \in N$, and each $R'_i \in \mathscr{R}$, if $\varphi_i(R) = \varphi_i(R'_i, R_{-i})$, then $\varphi(R) = \varphi(R'_i, R_{-i})$.

Lemma 3. *The uniform rule U on the single-peaked domain $\mathscr{R}_S^n$ satisfies non-bossiness.*

Proof: Let $R \in \mathscr{R}_S^n$, $i \in N$, and $R'_i \in \mathscr{R}_S$ be such that $U_i(R) = U_i(R'_i, R_{-i})$. We only consider the case where $\sum_{j \in N} p(R_j) \geq M$, since a similar argument holds when $\sum_{j \in N} p(R_j) < M$. We can distinguish two cases.

Case 1: $\sum_{j \in N \setminus \{i\}} p(R_j) + p(R'_i) \geq M$.

[13] Given a set B, we denote the cardinality of B by $\#B$.

In this case, by the definition of U, we obtain that for each $j \in N \setminus \{i\}$, $U_j(R) = \min\{p(R_i), \lambda(R)\}$ and $U_j(R'_i, R_{-i}) = \min\{p(R_i), \lambda(R'_i, R_{-i})\}$. Thus, we have either

$$U_j(R) \geq U_j(R'_i, R_{-i}) \text{ for each } j \in N \setminus \{i\}, \text{ or}$$
$$U_j(R) \leq U_j(R'_i, R_{-i}) \text{ for each } j \in N \setminus \{i\}.$$

If there exists $k \in N \setminus \{i\}$ such that $U_k(R) > U_k(R'_i, R_{-i})$ or $U_k(R) < U_k(R'_i, R_{-i})$, then $\sum_{j \in N} U_j(R) > \sum_{j \in N} U_j(R'_i, R_{-i})$ or $\sum_{j \in N} U_j(R) < \sum_{j \in N} U_j(R'_i, R_{-i})$. This is a contradiction to feasibility. Hence, $U(R) = U(R'_i, R_{-i})$.

Case 2: $\sum_{j \in N \setminus \{i\}} p(R_j) + p(R'_i) < M$.

In this case, by the definition of U, we have $U_j(R) = \min\{p(R_i), \lambda(R)\} \leq \max\{p(R_i), \lambda(R'_i, R_{-i})\} = U_j(R'_i, R_{-i})$ for each $j \in N \setminus \{i\}$. If there exists $k \in N \setminus \{i\}$ such that $U_k(R) < U_k(R'_i, R_{-i})$, then $\sum_{j \in N} U_j(R) < \sum_{j \in N} U_j(R'_i, R_{-i})$, which is a contradiction to feasibility. Hence, $U(R) = U(R'_i, R_{-i})$. □

Lemma 4. *The uniform rule U on the single-peaked domain $\mathscr{R}_S^n$ satisfies bribe-proofness.*

Proof: Suppose, by contradiction, that the uniform rule U on the single-peaked domain $\mathscr{R}_S^n$ is not *bribe-proof*. Then, there exist $R \in \mathscr{R}_S^n$, $i, j \in N$, $R'_i \in \mathscr{R}_S$, and $b \in \mathbb{R}$ such that $(U_i(R'_i, R_{-i}) + b) P_i U_i(R)$ and $(U_j(R'_i, R_{-i}) - b) P_j U_j(R)$.

Now, assume that $\sum_{j \in N} p(R_j) \geq M$. By the definition of the uniform rule U, $p(R_k) \geq U_k(R)$ for each $k \in N$. Then $(U_i(R'_i, R_{-i}) + b) P_i U_i(R)$ and $(U_j(R'_i, R_{-i}) - b) P_j U_j(R)$ together imply $U_i(R'_i, R_{-i}) + b > U_i(R)$ and $U_j(R'_i, R_{-i}) - b > U_j(R)$. Thus, by feasibility, we have

$$U_i(R'_i, R_{-i}) + U_j(R'_i, R_{-i}) > U_i(R) + U_j(R); \tag{1}$$
$$\sum_{k \in N \setminus \{i,j\}} U_k(R'_i, R_{-i}) < \sum_{k \in N \setminus \{i,j\}} U_k(R). \tag{2}$$

Agent i might have an incentive to be bribed only if $p(R_i) > U_i(R)$. Thus, we have $U_i(R) = \min\{p(R_i), \lambda(R)\} = \lambda(R)$.

If agent i reports $R'_i \in \mathscr{R}_S$ such that $p(R'_i) \geq p(R_i)$, she gets $U_i(R'_i, R_{-i}) = \min\{p(R'_i), \lambda(R'_i, R_{-i})\}$. Since $U_i(R) < p(R_i)$, it follows from Lemma 2 that $U_i(R) = \text{med}\{\alpha_i(R_{-i}), \beta_i(R_{-i}), p(R_i)\} = \beta_i(R_{-i})$. Also, since $p(R'_i) \geq p(R_i)$, it follows from Lemma 2 that $U_i(R'_i, R_{-i}) = \text{med}\{\alpha_i(R_{-i}), \beta_i(R_{-i}), p(R'_i)\} = \beta_i(R_{-i})$. Therefore, we obtain $U_i(R) = U_i(R'_i, R_{-i})$. Thus, by *non-bossiness* (Lemma 3), $U(R) = U(R'_i, R_{-i})$, which is a contradiction to (1) and (2).

Let $R'_i \in \mathscr{R}_S$ be such that $p(R_i) > p(R'_i)$. There are two cases.

Case 1: $\sum_{j\in N\setminus\{i\}} p(R_j) + p(R'_i) \geq M$.

Then, agent i gets $\min\{p(R'_i), \lambda(R'_i, R_{-i})\}$. We can also distinguish two subcases.

Subcase 1-1: $p(R'_i) \geq \lambda(R)$.

Since $U_i(R) < p(R_i)$, by the definition of U and Lemma 2, $\lambda(R) = U_i(R) = \text{med}\{\alpha_i(R_{-i}), \beta_i(R_{-i}), p(R_i)\} = \beta_i(R_{-i})$. Since $p(R'_i) \geq \lambda(R) = \beta_i(R_{-i})$, it follows from Lemma 2 that $U_i(R'_i, R_{-i}) = \text{med}\{\alpha_i(R_{-i}), \beta_i(R_{-i}), p(R'_i)\} = \beta_i(R_{-i})$. Therefore, we obtain $U_i(R) = U_i(R'_i, R_{-i})$. Thus, by *non-bossiness* (Lemma 3), we have $U(R) = U(R'_i, R_{-i})$, which is a contradiction to (1) and (2).

Subcase 1-2: $p(R'_i) < \lambda(R)$.

In this subcase, we have $U_i(R) = \lambda(R) > \min\{p(R'_i), \lambda(R'_i, R_{-i})\} = U_i(R'_i, R_{-i})$. By feasibility, there exists $h \in N \setminus \{i\}$ such that $U_h(R) < U_h(R'_i, R_{-i})$, that is, $\min\{p(R_h), \lambda(R)\} < \min\{p(R_h), \lambda(R'_i, R_{-i})\}$. This implies that $\lambda(R) < \lambda(R'_i, R_{-i})$. Then, $U_k(R'_i, R_{-i}) \geq U_k(R)$ for each $k \in N \setminus \{i\}$, which implies that $U_i(R'_i, R_{-i}) + U_j(R'_i, R_{-i}) \leq U_i(R) + U_j(R)$ and $\sum_{k\in N\setminus\{i,j\}} U_k(R'_i, R_{-i}) \geq \sum_{k\in N\setminus\{i,j\}} U_k(R)$, contradicting (1) and (2).

Case 2: $\sum_{k\in N\setminus\{i\}} p(R_j) + p(R'_i) < M$.

In this case, $U_i(R'_i, R_{-i}) = \max\{p(R'_i, R_{-i}), \lambda(R'_i, R_{-i})\}$. By Lemma 2, $U_i(R'_i, R_{-i}) = \text{med}\{\alpha_i(R_{-i}), \beta_i(R_{-i}), p(R'_i)\} \leq \beta_i(R_{-i}) = U_i(R)$. Thus, we obtain $U_i(R) = \lambda(R) \geq \max\{p(R'_i), \lambda(R'_i, R_{-i})\} = U_i(R'_i, R_{-i})$. There are also two subcases.

Subcase 2-1: $U_i(R) = U_i(R'_i, R_{-i})$.

By *non-bossiness* (Lemma 3), $U(R) = U(R'_i, R_{-i})$, contradicting (1) and (2).

Subcase 2-2: $U_i(R) > U_i(R'_i, R_{-i})$.

In this subcase, we have $\lambda(R) > \max\{p(R'_i), \lambda(R'_i, R_{-i})\}$, that is, $\lambda(R) > \lambda(R'_i, R_{-i})$. This implies that for each $k \in N \setminus \{i\}$, $U_k(R'_i, R_{-i}) = \max\{p(R_k), \lambda(R'_i, R_{-i})\} \geq \min\{p(R_k), \lambda(R)\} = U_k(R)$. Since $U_i(R) > U_i(R'_i, R_{-i})$, it follows from feasibility that $U_i(R'_i, R_{-i}) + U_j(R'_i, R_{-i}) \leq U_i(R) + U_j(R)$ and $\sum_{k\in N\setminus\{i,j\}} U_k(R'_i, R_{-i}) \geq \sum_{k\in N\setminus\{i,j\}} U_k(R)$, contradicting (1) and (2).

A similar argument holds when $\sum_{j\in N} p(R_j) < M$, thus completing the proof. □

Let us prove the "if" part of Theorem 1.

Proof of the "if" Part of Theorem 1: By Lemma 4 and Ching (1994), the uniform rule U on $\mathscr{R}_S^n$ satisfies *bribe-proofness* and *symmetry.* $\square$

A.2 Proof of Claim 1

We distinguish three cases.

Case 1: $\sum_{k\in N} \underline{p}(R_k) \geq M$.

Suppose to the contrary that the extended uniform rule U^e on the convex domain $\mathscr{R}_C^n \times \mathbb{R}_{++}$ is not *bribe-proof.* Then, there exist $(R, M) \in \mathscr{R}_C^n \times \mathbb{R}_{++}$, $i, j \in N$, $R'_i \in \mathscr{R}_C$, and $b \in \mathbb{R}$ such that $(U_i^e(R'_i, R_{-i}, M) + b) P_i U_i^e(R, M)$ and $(U_j^e(R'_i, R_{-i}, M) - b) P_j U_j^e(R, M)$. Then, by the definition of U^e, $\underline{p}(R_k) \geq U_k^e(R, M)$ for each $k \in N$. Thus, $(U_i^e(R'_i, R_{-i}, M)+b) P_i U_i^e(R, M)$ and $(U_j^e(R'_i, R_{-i}, M)-b) P_j U_j^e(R, M)$ together imply $U_i^e(R'_i, R_{-i}, M) + b > U_i^e(R, M)$ and $U_j^e(R'_i, R_{-i}, M) - b > U_j^e(R, M)$. Then, by feasibility, we have

$$U_i^e(R'_i, R_{-i}, M) + U_j^e(R'_i, R_{-i}, M) > U_i^e(R, M) + U_j^e(R, M); \quad (3)$$

$$\sum_{k\in N\setminus\{i,j\}} U_k^e(R'_i, R_{-i}, M) < \sum_{k\in N\setminus\{i,j\}} U_k^e(R, M). \quad (4)$$

Agent i might have an incentive to be bribed only if $\underline{p}(R_i) > U_i^e(R, M)$. Thus, $U_i^e(R, M) = \min\{\underline{p}(R_i), \lambda(R, M)\} = \lambda(R, M)$.

If agent i reports $R'_i \in \mathscr{R}_C$ such that $\underline{p}(R'_i) \geq \underline{p}(R_i)$, then $\sum_{k\in N\setminus\{i\}} \underline{p}(R_k) + \underline{p}(R'_i) \geq M$. Thus, $U_i^e(R'_i, R_{-i}, M) = \min\{\underline{p}(R'_i), \lambda(R'_i, R_{-i}, M)\}$.

Now we have either

(a) $U_i^e(R'_i, R_{-i}, M) = \underline{p}(R'_i)$ or

(b) $U_i^e(R'_i, R_{-i}, M) = \lambda(R'_i, R_{-i}, M)$.

Suppose (a) holds. Then, we have $U_i^e(R'_i, R_{-i}, M) > U_i^e(R, M)$. This means that $\lambda(R'_i, R_{-i}, M) \geq \underline{p}(R'_i) \geq \underline{p}(R_i) > \lambda(R, M)$. Note that for each $k \in N \setminus \{i\}$,

$$U_k^e(R, M) = \min\{\underline{p}(R_k), \lambda(R, M)\};$$
$$U_k^e(R'_i, R_{-i}, M) = \min\{\underline{p}(R_k), \lambda(R'_i, R_{-i}, M)\}.$$

Thus $U_k^e(R'_i, R_{-i}, M) \geq U_k^e(R, M)$ for each $k \in N \setminus \{i\}$. This implies that $M = \sum_{k\in N} U_k^e(R'_i, R_{-i}, M) > \sum_{k\in N} U_k^e(R, M) = M$, a contradiction.

Suppose (b) holds. If $\lambda(R'_i, R_{-i}, M) \geq \lambda(R, M)$, then $U^e_k(R'_i, R_{-i}, M) \geq U^e_k(R, M)$ for each $k \in N$. If $U^e_j(R'_i, R_{-i}, M) > U^e_j(R, M)$ for some $j \in N$, then $M = \sum_{k\in N} U^e_k(R'_i, R_{-i}, M) > \sum_{k\in N} U^e_k(R, M) = M$, a contradiction. Thus, for each $k \in N$, $U^e_k(R'_i, R_{-i}, M) = U^e_k(R, M)$. However, this is a contradiction to (3) and (4). If $\lambda(R'_i, R_{-i}, M) < \lambda(R, M)$, then $U^e_i(R'_i, R_{-i}, M) < U^e_i(R, M)$ and for each $k \in N\setminus\{i\}$, $U^e_k(R'_i, R_{-i}, M) \leq U^e_k(R, M)$. This implies that $M = \sum_{k\in N} U^e_k(R'_i, R_{-i}, M) < \sum_{k\in N} U^e_k(R, M) = M$, a contradiction.

Next, consider that agent i reports $R'_i \in \mathscr{R}_C$ such that $\underline{p}(R'_i) < \underline{p}(R_i)$. We also distinguish three subcases.

Subcase 1-1: $\boldsymbol{\sum_{k\in N\setminus\{i\}} \underline{p}(R_k) + \underline{p}(R'_i) \geq M}$.

Note that $U^e_i(R'_i, R_{-i}, M) = \min\{\underline{p}(R'_i), \lambda(R'_i, R_{-i}, M)\}$. We further distinguish three subsubcases.

Subsubcase 1-1-1: $\boldsymbol{\underline{p}(R'_i) > \lambda(R, M)}$.

In this subsubcase, we obtain that $\lambda(R, M) = \lambda(R'_i, R_{-i}, M)$. Otherwise, $M = \sum_{k\in N} U^e_k(R'_i, R_{-i}, M) \neq \sum_{k\in N} U^e_k(R, M) = M$, a contradiction. Since $\lambda(R, M) = \lambda(R'_i, R_{-i}, M)$, it follows from feasibility that for each $k \in N$, $U^e_k(R'_i, R_{-i}, M) = U^e_k(R, M)$. However, this is a contradiction to (3) and (4).

Subsubcase 1-1-2: $\boldsymbol{\underline{p}(R'_i) = \lambda(R, M)}$.

We have either

(a) $U^e_i(R'_i, R_{-i}, M) = \underline{p}(R'_i)$ or

(b) $U^e_i(R'_i, R_{-i}, M) = \lambda(R'_i, R_{-i}, M)$.

Suppose (a) holds. Then, $U^e_i(R'_i, R_{-i}, M) = U^e_i(R, M)$, that is, $\lambda(R'_i, R_{-i}, M) \geq \underline{p}(R'_i) = \lambda(R, M)$. This implies that $U^e_k(R'_i, R_{-i}, M) \geq U^e_k(R, M)$ for each $k \in N \setminus \{i\}$. If there exists $h \in N \setminus \{i\}$ such that $U^e_h(R'_i, R_{-i}, M) > U^e_h(R, M)$, then we have $M = \sum_{k\in N} U^e_k(R'_i, R_{-i}, M) > \sum_{k\in N} U^e_k(R, M) = M$, a contradiction. Thus, it must be that $U^e_k(R'_i, R_{-i}, M) = U^e_k(R, M)$ for each $k \in N \setminus \{i\}$. However, this is a contradiction to (3) and (4).

Suppose (b) holds. Then, $U^e_i(R'_i, R_{-i}, M) \leq U^e_i(R, M)$, that is, $\lambda(R'_i, R_{-i}, M) \leq \underline{p}(R'_i) = \lambda(R, M)$. If $\lambda(R'_i, R_{-i}, M) < \lambda(R, M)$, then $U^e_i(R'_i, R_{-i}, M) < U^e_i(R, M)$ and $U^e_k(R'_i, R_{-i}, M) \leq U^e_k(R, M)$ for each $k \in N \setminus \{i\}$, which imply that $M = \sum_{k\in N} U^e_k(R'_i, R_{-i}, M) < \sum_{k\in N} U^e_k(R, M) = M$, a contradiction. If $\lambda(R'_i, R_{-i}, M) = \lambda(R, M)$, $U^e(R, M) = U^e(R'_i, R_{-i}, M)$, which is a contradiction to (3) and (4).

Subsubcase 1-1-3: $\underline{p}(R'_i) < \lambda(R, M)$.

In this subsubcase, we have $U_i^e(R, M) = \lambda(R, M) > \min\{\underline{p}(R'_i), \lambda(R'_i, R_{-i}, M)\} = U_i^e(R'_i, R_{-i}, M)$. By feasibility, there exists $h \in N \setminus \{i\}$ such that

$$U_h^e(R, M) < U_h^e(R'_i, R_{-i}, M). \tag{5}$$

Now we have either

(a) $U_h^e(R, M) = \underline{p}(R_h)$ or

(b) $U_h^e(R, M) = \lambda(R, M)$.

Suppose (a) holds. Then, we have $U_h^e(R'_i, R_{-i}, M) = \min\{\underline{p}(R_h), \lambda(R'_i, R_{-i}, M)\} \leq U_h^e(R, M)$, contradicting (5).

Suppose (b) holds. By (5), $\lambda(R, M) < \lambda(R'_i, R_{-i}, M)$. Then, $U_k^e(R'_i, R_{-i}, M) \geq U_k^e(R, M)$ for each $k \in N \setminus \{i\}$, which implies that $\sum_{k \in N \setminus \{i,j\}} U_k^e(R'_i, R_{-i}, M) \geq \sum_{k \in N \setminus \{i,j\}} U_k^e(R, M)$ and $U_i^e(R'_i, R_{-i}, M) + U_j^e(R'_i, R_{-i}, M) \leq U_i^e(R, M) + U_j^e(R, M)$. This is a contradiction to (3) and (4).

Subcase 1-2: $\sum_{k \in N \setminus \{i\}} \overline{p}(R_k) + \overline{p}(R'_i) \leq M$.

In this subcase, $U_i^e(R'_i, R_{-i}, M) = \max\{\overline{p}(R'_i), \lambda(R'_i, R_{-i}, M)\}$ and $U_k^e(R'_i, R_{-i}, M) = \max\{\overline{p}(R_k), \lambda(R'_i, R_{-i}, M)\}$ for each $k \in N \setminus \{i\}$. Note that for each $k \in N \setminus \{i\}$,

$$U_k^e(R, M) = \min\{\underline{p}(R_k), \lambda(R, M)\} \leq \underline{p}(R_k);$$
$$U_k^e(R'_i, R_{-i}, M) = \max\{\overline{p}(R_k), \lambda(R'_i, R_{-i}, M)\} \geq \overline{p}(R_k),$$

that is,

$$U_k^e(R'_i, R_{-i}, M) \geq U_k^e(R, M) \text{ for each } k \in N \setminus \{i\}. \tag{6}$$

Now we have either

(a) $U_i^e(R'_i, R_{-i}, M) = \overline{p}(R'_i)$ or

(b) $U_i^e(R'_i, R_{-i}, M) = \lambda(R'_i, R_{-i}, M)$.

Suppose (a) holds. We further have either

(a-1) $U_i^e(R'_i, R_{-i}, M) = \overline{p}(R'_i) > \lambda(R, M) = U_i^e(R, M)$ or

(a-2) $U_i^e(R'_i, R_{-i}, M) = \overline{p}(R'_i) \leq \lambda(R, M) = U_i^e(R, M)$.

Suppose (a-1) holds. By (6), $M = \sum_{k\in N} U_k^e(R_i', R_{-i}, M) > \sum_{k\in N} U_k^e(R, M) = M$, a contradiction.

Suppose (a-2) holds. Then, $\lambda(R, M) \geq \lambda(R_i', R_{-i}, M)$. Thus, by feasibility and (6), $\sum_{k\in N\setminus\{i,j\}} U_k^e(R_i', R_{-i}, M) \geq \sum_{k\in N\setminus\{i,j\}} U_k^e(R, M)$ and $U_i^e(R_i', R_{-i}, M) + U_j^e(R_i', R_{-i}, M) \leq U_i^e(R, M) + U_j^e(R, M)$, contradicting (3) and (4).

Suppose (b) holds. We further have either

(b-1) $U_i^e(R_i', R_{-i}, M) = \lambda(R_i', R_{-i}, M) > \lambda(R, M) = U_i^e(R, M)$ or

(b-2) $U_i^e(R_i', R_{-i}, M) = \lambda(R_i', R_{-i}, M) \leq \lambda(R, M) = U_i^e(R, M)$.

The proof of (b-1) is similar to that of (a-1). Also, by an argument similar to (a-2), we can prove (b-2). Thus, we omit the details.

Subcase 1-3: $\sum_{k\in N\setminus\{i\}} \underline{p}(R_k) + \underline{p}(R_i') < M < \sum_{k\in N\setminus\{i\}} \overline{p}(R_k) + \overline{p}(R_i')$.

In this subcase, we obtain that

$$\text{for each } k \in N \setminus \{i\},\ U_k^e(R_i', R_{-i}, M) = \min\{\overline{p}(R_k), \underline{p}(R_k) + \lambda(R_i', R_{-i}, M)\};$$
$$U_i^e(R_i', R_{-i}, M) = \min\{\overline{p}(R_i'), \underline{p}(R_i') + \lambda(R_i', R_{-i}, M)\}.$$

Then, we have $U_k^e(R_i', R_{-i}, M) \geq U_k^e(R, M)$ for each $k \in N \setminus \{i\}$, which implies that $\sum_{k\in N\setminus\{i,j\}} U_k^e(R_i', R_{-i}, M) \geq \sum_{k\in N\setminus\{i,j\}} U_k^e(R, M)$ and $U_i^e(R_i', R_{-i}, M) + U_j^e(R_i', R_{-i}, M) \leq U_i^e(R, M) + U_j^e(R, M)$, contradicting (3) and (4).

Case 2: $\sum_{k\in N} \overline{p}(R_k) \leq M$.

Since the proof of Case 2 is similar to that of Case 1, we omit the details.

Case 3: $\sum_{k\in N} \underline{p}(R_k) < M < \sum_{k\in N} \overline{p}(R_k)$.

It follows from the definition of U^e that for each $k \in N$, $\underline{p}(R_k) \leq U_k^e(R, M) \leq \overline{p}(R_k)$. Thus, no agent $k \in N$ has an incentive to be bribed. □

A.3 Proof of Remark 2

Let $\mathscr{R}_a^n \subseteq \mathscr{R}^n$. Suppose that there is a rule φ on $\mathscr{R}_a^n \times \mathbb{R}_{++}$ satisfying *bribe-proofness* and *strong symmetry*. We will show that $\mathscr{R}_a^n \subseteq \mathscr{R}_C^n$. Suppose, by contradiction, that there is $R_0 \in \mathscr{R}_a \setminus \mathscr{R}_C$. Let $R_0 \in \mathscr{R}_a \setminus \mathscr{R}_C$. Then, there exist $x_0 < y_0 < z_0$ such that $x_0 P_0 y_0$ and $z_0 P_0 y_0$. Without loss of generality, assume that $x_0 R_0 z_0$. Let $x_0^* = \max\{x_0' \in [x_0, y_0] \colon x_0' I_0 z_0\}$ and $z_0^* = \min\{z_0' \in [y_0, z_0] \colon z_0' I_0 z_0\}$. Note that $x_0 \leq x_0^* < y_0 < z_0^* \leq z_0$, $x_0^* I_0 z_0^*$, and for each $x_0' \in (x_0^*, z_0^*)$, $x_0^* P_0 x_0'$.

Let $R = (R_0, R_0, \ldots, R_0)$ and $M = n \cdot \left(\frac{x_0^* + z_0^*}{2}\right)$. By *strong symmetry*, $\varphi_i(R, M) = \frac{x_0^* + z_0^*}{2}$ for each $i \in N$. Let $i, j \in N$ and $b = \varphi_i(R, M) - x_0^*$. Then, $(\varphi_i(R, M) - b) P_i \varphi_i(R, M)$ and $(\varphi_j(R, M) + b) P_j \varphi_j(R, M)$. This is a contradiction to *bribe-proofness*.

As we proved in Theorem 2, the extended uniform rule on the convex domain satisfies *bribe-proofness*. Also, it is straightforward to verify that the extended uniform rule on the convex domain satisfies *strong symmetry*. □

B Appendix: The Relation Between Bribe-proofness and Effective Pairwise Strategy-proofness

In this paper, we explore the possibility of designing a non-trivial rule that is immune manipulation by pairs of agents. It is natural to attempt to rule out situations where two agents *jointly* misrepresent. Thus, let us introduce the following axiom.

Pairwise Bribe-proofness: For each $(R, M) \in \mathscr{R}_a^n \times \mathbb{R}_{++}$ and each $i, j \in N$, there exist no $(R'_i, R'_j) \in \mathscr{R}_a \times \mathscr{R}_a$ and $b \in \mathbb{R}$ such that $(\varphi_i(R'_i, R'_j, R_{N\setminus\{i,j\}}, M) + b) P_i \varphi_i(R, M)$ and $(\varphi_j(R'_i, R'_j, R_{N\setminus\{i,j\}}, M) - b) P_j \varphi_j(R, M)$.

Serizawa (2006) examined the consequences of disallowing pairs of agents to jointly misrepresent types with *no transfers*. He introduced the notion of *effective pairwise strategy-proofness*; it rules out only unilateral manipulation and *self-enforcing* pairwise manipulation. This axiom is weaker than *pairwise strategy-proofness*, which requires that the rule must be immune against misrepresentation by pairs of agents; i.e., for each $(R, M) \in \mathscr{R}_a^n \times \mathbb{R}_{++}$, each $i, j \in N$, and each $(R'_i, R'_j) \in \mathscr{R}_a \times \mathscr{R}_a$, if $\varphi_i(R'_i, R'_j, R_{N\setminus\{i,j\}}, M) P_i \varphi_i(R, M)$, then $\varphi_j(R, M) R_j \varphi_j(R'_i, R'_j, R_{N\setminus\{i,j\}}, M)$. To define *effective pairwise strategy-proofness* formally, we introduce the additional notion. Let a rule φ, a preference profile and an amount $(R, M) \in \mathscr{R}_a^n \times \mathbb{R}_{++}$, and a pair of agents $\{i, j\} \subseteq N$ be given. Then a preference profile of the pair $(R'_i, R'_j) \in \mathscr{R}_a \times \mathscr{R}_a$ is a *self-enforcing manipulation* if

(i) $\varphi_i(R'_i, R'_j, R_{N\setminus\{i,j\}}, M) R_i \varphi_i(R, M)$ and $\varphi_j(R'_i, R'_j, R_{N\setminus\{i,j\}}, M) R_j \varphi_j(R, M)$;

(ii) $\varphi_i(R'_i, R'_j, R_{N\setminus\{i,j\}}, M) P_i \varphi_i(R, M)$ or $\varphi_j(R'_i, R'_j, R_{N\setminus\{i,j\}}, M) P_j \varphi_j(R, M)$;

(iii) for each $\widehat{R}_i \in \mathscr{R}_a$, $\varphi_i(R'_i, R'_j, R_{N\setminus\{i,j\}}, M) R_i \varphi_i(\widehat{R}_i, R'_j, R_{N\setminus\{i,j\}}, M)$;

(iv) for each $\widehat{R}_j \in \mathscr{R}_a$, $\varphi_j(R'_i, R'_j, R_{N\setminus\{i,j\}}, M) R_j \varphi_j(R'_i, \widehat{R}_j, R_{N\setminus\{i,j\}}, M)$.

Effective Pairwise Strategy-proofness: φ is *strategy-proof* and no pair of agents has a self-enforcing manipulation.

Remark 3. By definitions, *pairwise strategy-proofness* is weaker than *pairwise bribe-proofness*. Thus, *effective pairwise strategy-proofness* is weaker than *pairwise bribe-proofness*.

This section shows that *bribe-proofness* and *effective pairwise strategy-proofness* are logically independent on the single-peaked domain. So, *bribe-proofness* is not equivalent to *pairwise bribe-proofness* on the single-peaked domain.

First, we demonstrate that there exists an *effectively pairwise strategy-proof* rule on the single-peaked domain that does not satisfy *bribe-proofness*. To observe this, we introduce the following rule.

Definition 4. The *egalitarian rule* $E = (E_1, E_2, \dots, E_n)$ is defined as follows: For each $(R, M) \in \mathscr{R}_a \times \mathbb{R}_{++}$ and each $i \in N$,

$$E_i(R, M) = \frac{M}{n}.$$

It is easy to verify that the egalitarian rule on the single-peaked domain is *effectively pairwise strategy-proof* but not *bribe-proof*.

Next, we describe an example of a rule on the single-peaked domain that is *bribe-proof* but not *effectively pairwise strategy-proof*.

Example 2. Let $N = \{1, 2, 3\}$. Define the rule $\varphi\colon \mathscr{R}_S^3 \times \mathbb{R}_{++} \to \mathbb{R}_+^n$ as follows: For each $(R, M) \in \mathscr{R}_S^3 \times \mathbb{R}_{++}$,

$$\varphi_1(R, M) = \min\{p(R_1), M\};$$
$$\varphi_2(R, M) = \begin{cases} \min\{p(R_2), M - \varphi_1(R, M)\} & \text{if } 0R_1 1; \\ \max\{0, M - \varphi_1(R, M) - \varphi_3(R, M)\} & \text{otherwise;} \end{cases}$$
$$\varphi_3(R, M) = \begin{cases} \min\{p(R_3), M - \varphi_1(R, M)\} & \text{if } 1P_1 0; \\ \max\{0, M - \varphi_1(R, M) - \varphi_2(R, M)\} & \text{otherwise.} \end{cases}$$

We will check that the rule φ on $\mathscr{R}_S^3 \times \mathbb{R}_{++}$ is *bribe-proof*. Note that for each $(R, M) \in \mathscr{R}_S^3 \times \mathbb{R}_{++}$, $\varphi_1(R, M) = \min\{p(R_1), M\}$. If $\varphi_1(R, M) = M$, $\varphi_1(R) < p(R_1)$, and for each $i \in \{2, 3\}$, $\varphi_i(R, M) = 0$. Then, for each $i, j \in N$, there exist no $R'_i \in \mathscr{R}_S$ and $b \in \mathbb{R}$ such that $(\varphi_i(R'_i, R_{-i}, M) + b) P_i \varphi_i(R, M)$ and $(\varphi_j(R'_i, R_{-i}, M) - b) P_j \varphi_j(R, M)$. If $\varphi_1(R, M) = p(R_1)$, then agent 1 does not have an incentive to be bribed. Without loss of generality, assume that $0R_1 1$. If $\varphi_2(R,M) = p(R_2)$, then $\varphi_3(R,M) = \varphi_3(R'_3, R_{-3}, M)$ for each $R'_3 \in \mathscr{R}_S$. If $\varphi_2(R,M) < p(R_2)$, then $\varphi_3(R) = \varphi_3(R'_3, R_{-3}, M) = 0$ for each $R'_3 \in \mathscr{R}_S$. Thus, for each $i, j \in N$, there exist no $R'_i \in \mathscr{R}_S$ and $b \in \mathbb{R}$ such that $(\varphi_i(R'_i, R_{-i}, M)+b)P_i\varphi_i(R,M)$ and $(\varphi_j(R'_i, R_{-i}, M)-b)P_j\varphi_j(R,M)$. Hence, we can conclude that the rule φ on $\mathscr{R}_S^3 \times \mathbb{R}_{++}$ is *bribe-proof.*

Observe that the rule φ on $\mathscr{R}_S^3 \times \mathbb{R}_{++}$ does not satisfy *effective pairwise strategy-proofness.* To see this, let $(R,M) \in \mathscr{R}_S^3 \times \mathbb{R}_{++}$ be such that $M = 1$, $1P_1 0$, and $(p(R_1), p(R_2), p(R_3)) = \left(\frac{1}{5}, \frac{4}{5}, \frac{4}{5}\right)$. Let $R'_1 \in \mathscr{R}_S$ be such that $p(R'_1) = \frac{1}{5}$ and $0P'_1 1$. Then $\varphi(R,1) = \left(\frac{1}{5}, 0, \frac{4}{5}\right)$ and $\varphi(R'_1, R_{-1}, 1) = \left(\frac{1}{5}, \frac{4}{5}, 0\right)$. Hence, we have

$$\varphi_1(R'_1, R_{-1}, 1)I_1\varphi_1(R,1) \text{ and } \varphi_2(R'_1, R_{-1}, 1)P_2\varphi_2(R,1);$$
$$\varphi_1(R'_1, R_{-1}, 1)R_1\varphi_1(\widehat{R}_1, R_{-1}, 1) \text{ for each } \widehat{R}_1 \in \mathscr{R}_S;$$
$$\varphi_2(R'_1, R_{-1}, 1)R_2\varphi_2(R'_1, \widehat{R}_2, R_3, 1) \text{ for each } \widehat{R}_2 \in \mathscr{R}_S.$$

This indicates that φ on $\mathscr{R}_S^3 \times \mathbb{R}_{++}$ does not satisfy *effective pairwise strategy-proofness.* ■

By Remark 3, the solution φ defined in Example 2 above does not satisfy *pairwise strategy-proofness* and *pairwise bribe-proofness.* This can be interpreted that on the single-peaked domain, *bribe-proofness* is not equivalent to *pairwise bribe-proofness.*

C Appendix: Independence of Axioms

We will check the independence of axioms in Theorem 1. In what follows, we exhibit a rule on the single-peaked domain $\mathscr{R}_S(M)^n$ that does not satisfy either *bribe-proofness* or *symmetry.*

Example 3 (Dropping *bribe-proofness*). Let $M \in \mathbb{R}_{++}$. The egalitarian rule $E(\cdot, M)$ on $\mathscr{R}_S(M)^n$ satisfies *symmetry*, but not *bribe-proofness*: Let $i, j \in N$ with $i \neq j$. Consider any preference $(R_i(M), R_j(M)) \in \mathscr{R}_S(M) \times \mathscr{R}_S(M)$ such that $p(R_i(M)) = 0$ and $p(R_j(M)) = M$. Let $b = \frac{M}{n}$. Then $(E_i(R(M), M) - b)P_i(M)E_i(R(M), M)$ and $(E_j(R(M), M) + b)P_j(M)E_j(R(M), M)$. ■

Example 4 (Dropping *symmetry*). Let $M \in \mathbb{R}_{++}$. The *queuing rule* $Q(\cdot, M) = (Q_1(\cdot, M), Q_2(\cdot, M), \ldots, Q_n(\cdot, M))$ is the rule such that there is a permutation π of N, and for each $R(M) \in \mathscr{R}_a(M)^n$ and each $i \in N$,

$$Q_{\pi(1)}(R(M), M) = p(R_{\pi(1)}(M));$$

$$
\begin{aligned}
Q_{\pi(2)}(R(M), M) &= \min\left\{p\left(R_{\pi(2)}(M)\right), M - Q_{\pi(1)}(R(M), M)\right\};\\
Q_{\pi(3)}(R(M), M) &= \min\left\{p\left(R_{\pi(3)}(M)\right), M - \sum_{j\in\{\pi(1),\pi(2)\}} Q_j(R(M), M)\right\};\\
&\vdots\\
Q_{\pi(n)}(R(M), M) &= M - \sum_{j\in N\setminus\{\pi(n)\}} Q_j(R(M), M).
\end{aligned}
$$

Then the rule $Q(\cdot, M)$ on $\mathscr{R}_S(M)^n$ satisfies *bribe-proofness*, but not *symmetry.* ■

References

Barberà, S., M. O. Jackson, and A. Neme (1997): "Strategy-proof Allotment Rules," *Games and Economic Behavior*, **18**, 1–21.

Barberà, S., H. Sonnenschein, and L. Zhou (1991): "Voting by Committees," *Econometrica*, **59**, 595–609.

Benassy, J-P. (1982): *The Economics of Market Disequilibrium*. New York: Academic Press.

Berga, D. and S. Serizawa (2000): "Maximal Domain for Strategy-proof Rules with One Public Good," *Journal of Economic Theory*, **90**, 39–61.

Ching, S. and S. Serizawa (1998): "A Maximal Domain for the Existence of Strategy-proof Rules," *Journal of Economic Theory*, **78**, 157-166.

Ching, S. (1994): "An Alternative Characterization of the Uniform Rule," *Social Choice and Welfare*, **11**, 131–136.

Ehlers, L. (2002): "Coalitional Strategy-Proof House Allocation," *Journal of Economic Theory*, **105**, 298–317.

Massò, J. and A. Neme (2001): "Maximal Domain of Preferences in the Division Problem," *Games and Economic Behavior*, **37**, 367–387.

Massò, J. and A. Neme (2006): "Bribe-proof Rules in the Division Problem," mimeo, Universitat Autonoma de Barcelona.

Mizobuchi, H. and S. Serizawa (2006): "Maximal Domain for Strategy-proof Rules in Allotment Economies," *Social Choice and Welfare*, **27**, 195–210.

Mizukami, H. (2003): "On the Constancy of Bribe-proof Solutions," *Economic Theory*, **22**, 211–217.

Satterthwaite, M. A. and H. Sonnenschein (1981): "Strategy-proof Allocation Mechanisms at Differentiable Points," *Review of Economic Studies*, **48**, 587–597.

Schummer, J. (2000): "Manipulation thorough Bribes," *Journal of Economic Theory*, **91**, 180–198.

Serizawa, S. (1995): "Power of Voters and Domain of Preferences when Voting by Committees is Strategy-proof," *Journal of Economic Theory*, **67**, 599–519.

Serizawa, S. (2006): "Pairwise Strategy-Proofness and Self-Enforcing Manipulation," *Social Choice and Welfare*, **26**, 305–331.

Sprumont, Y. (1991): "The Division Problem with Single-peaked Preferences: A Characterization of the Uniform Allocation Rule," *Econometrica*, **59**, 509–519.

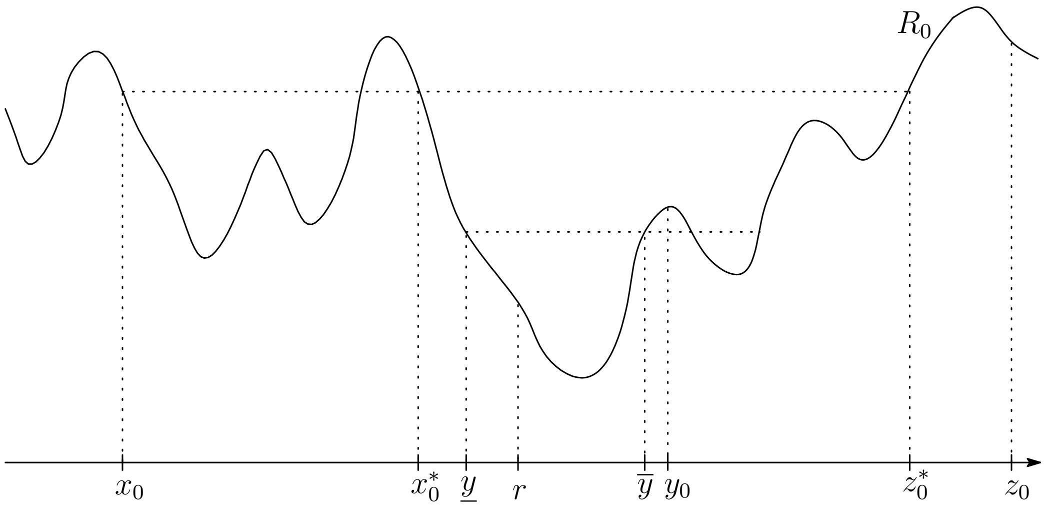

Figure 1: Illustration of eight points in the proof of Theorem 2.

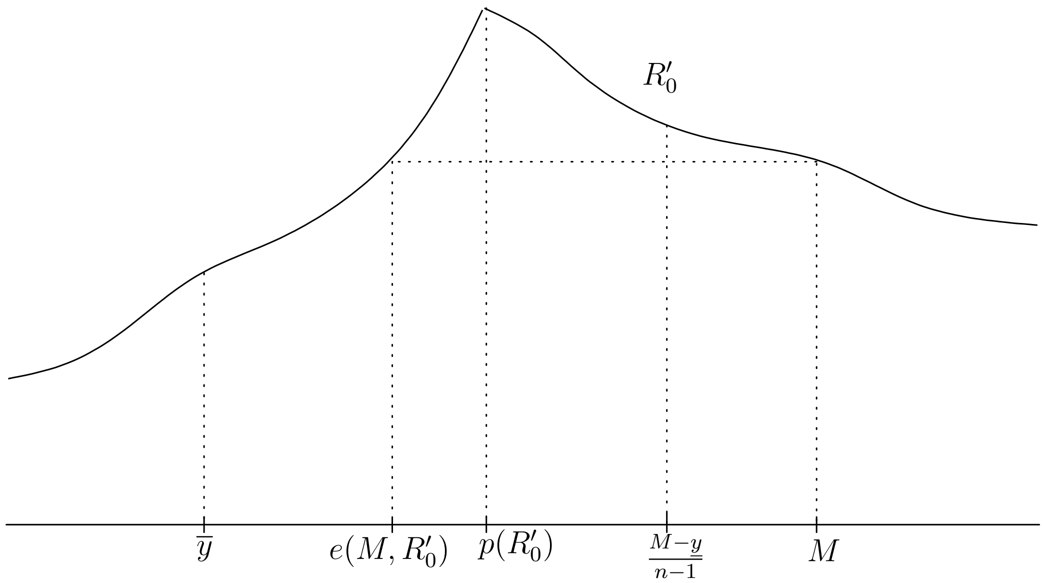

Figure 2: Illustration of $R_0' \in \mathscr{R}_S$ in the proof of Theorem 2. Given $R_0 \in \mathscr{R}_S$ and $z_0 \in (\mathbb{R}_+ \cup \{\infty\}) \setminus \{p(R_0)\}$, there is at most one element $\widehat{z_0} \in (\mathbb{R}_+ \cup \{\infty\}) \setminus \{z_0\}$ such that $\widehat{z_0} I_0 z_0$ and we denote the element $e(z_0, R_0)$ if it exists. Since $M P_0' \overline{y}$ implies $e(M, R_0') P_0' \overline{y}$, $e(M, R_0') > \overline{y}$.

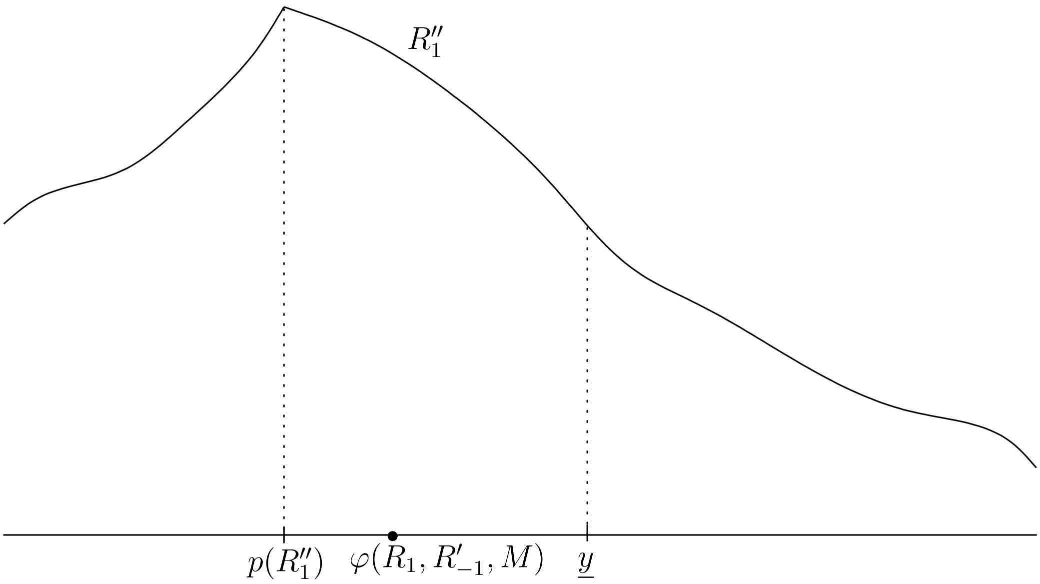

Figure 3: Illustration of $R''_1 \in \mathscr{R}_S$ in the proof of Theorem 2.

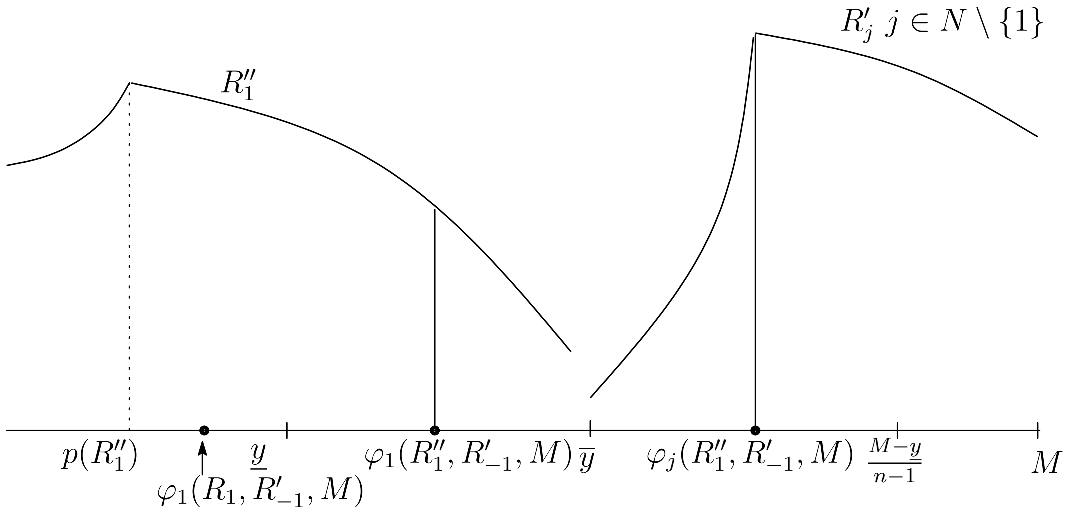

Figure 4: Subcase 2-3 in the proof of Theorem 2.