Optimal Timing for Portfolio Adjustment Using Aggregated Time Series Data

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Portfolio Management

- Selecting and allocating various assets to achieve specific investment objectives while managing risk.
- Diversification is a key principle of portfolio management.
    - Correlations between assets play a crucial role.
- Example:
    - Negative correlation: stocks vs. government bonds.
    - Diversification helps cushion the impact of stock market losses on the overall portfolio.

Challenges during Global Economic Downturn

- Volatility in financial markets poses challenges for portfolio management.
- During a severe economic recession or market crisis, the flight to safety can override these traditional correlations:
  - Investors indiscriminately sell off assets, including both stocks and government bonds.
  - Simultaneous price declines across asset classes.
  - A weakening or reversal of their traditional negative correlation.
- Timely portfolio adjustment is needed.

The Importance of Identifying Common Breaks

- Conventional approach for portfolio adjustment:
  - Information from each single asset.
  - Predicting the returns of individual assets within the portfolio.
  - Overall portfolio is constructed by aggregating individual return forecasts.
- Overlooks the **correlation changes** among financial assets:
  - a result of global market dynamics.
  - arise from common trends, shocks, or policy changes.
- Identification of **common breaks** in a multivariate system becomes essential as they signify the adjustment times for portfolio allocations.
  - A change occurs in each time series at a common point.

Aggregation of Asset Returns

- In practice, it is often observed that the aggregation of AR time series manifests statistical properties indicative of a long-memory process.
  - The idea of aggregating AR(1) to (possibly) obtain a long memory time series was initially put forward by Robinson (1978) and more deeply explored by Granger (1980).
  - Zaffaroni (2004): aggregation of heterogeneous ARMA time series
- Diebold and Inoue (2001): low-frequency periodogram of the Markov-switching process can be well approximated by a long-memory process.

ARMA(0,0): $y_{it} = \epsilon_{it}$
ARMA(1,1): $(1 - 0.5L)y_{it} = (1 + 0.3L)\epsilon_{it}$

ARMA(0,0) model

	$\rho = 0.1$			$\rho = 0.5$			$\rho = 0.8$		
N\T	100	300	500	100	300	500	100	300	500
10	0.157	0.123	0.123	0.346	0.338	0.326	0.300	0.117	0.084
15	0.167	0.155	0.158	0.386	0.396	0.397	0.368	0.212	0.177
20	0.179	0.185	0.190	0.399	0.431	0.435	0.403	0.291	0.257
30	0.201	0.239	0.258	0.405	0.461	0.461	0.414	0.391	0.393

ARMA(1,1) model

	$\rho = 0.1$			$\rho = 0.5$			$\rho = 0.8$		
N\T	100	300	500	100	300	500	100	300	500
10	0.256	0.217	0.188	0.251	0.139	0.097	0.292	0.213	0.201
15	0.276	0.249	0.221	0.340	0.265	0.242	0.333	0.159	0.124
20	0.285	0.257	0.246	0.393	0.378	0.371	0.358	0.202	0.130
30	0.299	0.288	0.277	0.431	0.466	0.475	0.376	0.327	0.297

Main Goals

1. Introduce an easy-to-implement method to estimate the fractional integrated order d in the aggregated return series.
   - Estimation without specifying orders p and q in an ARFIMA model.
   - Approximate an ARFIMA by an AR(k) model.
   - Consistency and asymptotic normality of $\hat{d}$.

2. Propose an on-line common break test.

$$\hat{t} = \frac{\hat{d}_{t+1} - \hat{d}_t}{SE(\hat{d}_{t+1} - \hat{d}_t)},$$

$\hat{d}$ is computed using a dynamic rolling window scheme.

The Long-Memory Process

Consider an ARFIMA(p,d,q) model of aggregated return y_t:

$$(1-L)^d y_t = \psi(L)/\phi(L)\epsilon_t = u_t,$$

where $\phi(z) = 1 - \sum_{j=1}^{p} \phi_j z^j$ and $\psi(z) = 1 - \sum_{j=1}^{q} \theta_j z^j$ are polynomials with roots outside the unit circle.

The fractional difference is defined by its binomial expansion:

$$(1-L)^d = \sum_{j=0}^{\infty} \Delta_j(d) L^j,$$

where $\Delta_j(d) = \frac{\Gamma(j-d)}{\Gamma(-d)\Gamma(j+1)}$ with $\Gamma(\cdot)$ denoting the gamma (generalized factorial) function.

Existing Methods in Estimating d

- The exact MLE by Sowell (1992)
  - Time-consuming.
  - Inaccurate when sample small and d close to 0.5.
- The generalized minimum distance estimation (GMD) by Mayoral (2007)
  - Requires prior information of parametric setups and weighting matrix.
  - Less efficient than exact MLE.
- The local Whittle estimation (ELW) by Shimotsu and Phillips (2005)
  - Sensitive to bandwidth selection.
- The GPH estimation by Geweke and Porter-Hudak (1983)
  - Biased estimation with small sample and when d is close to 0. (Grato and Ray, 1996)

Estimation of d: FEAR

In the AR(k) appr., v_t can be expressed as a function of d:

$$v_t(d) = y_t - \Delta_j(d) y_{t-j}.$$

The FEAR Estimation of d:

$$\hat{d} = \arg\min_d \sum_{t=1}^{T} \left[v_t(d) - \tilde{v}_t \right]^2,$$

where $\tilde{v}_t = y_t - \sum_{j=1}^{k} \tilde{\beta}_j y_{t-j}$, $\tilde{\beta}_j$ is the estimated AR(k) coefficient.

The On-Line Common Break Test

- For N asset returns, we estimate the long-memory parameter d_t for the aggregated returns up to time t.
  - Initial training window size: $W = \delta T$, $\delta \in (0, 1)$.
  - Dynamic rolling window scheme: d_t, d_{t+1}, ..., d_T.

- The on-line common break test:

$$\hat{t} = \frac{\hat{d}_{t+1} - \hat{d}_t}{SE(\hat{d}_{t+1} - \hat{d}_t)}.$$

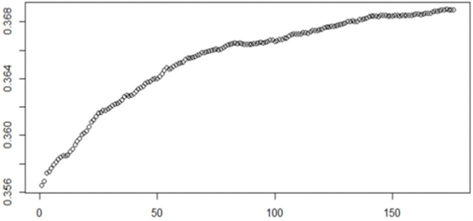
A common break at t is detected if the null of no break is rejected.

Example:

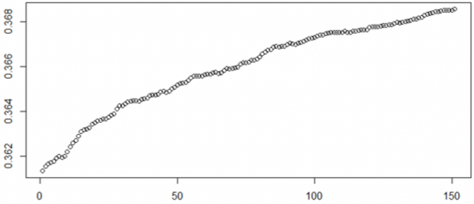
$$
\begin{aligned}
y_{it} &= e_{it},\ t = 1, ..., k_0,\ i = 1, ..., N \\
y_{it} &= \mu + e_{it},\ t = k_0 + 1, ..., T,\ i = 1, ..., N
\end{aligned}
$$

where $e_{it} = 0.8e_{i,t-1} + \epsilon_{it} + 0.7\epsilon_{i,t-1}$, $\epsilon_{it} \sim i.i.d.N(0, 1)$.

- Assume $\mu = 0.5$, $T = 250$, $N = 30$ or 50, $W = 100$, $k_0 = 151$.
- We then perform the on-line common break test on the aggregated time series: $S_t = \sum_{i=1}^{N} y_{it}$.

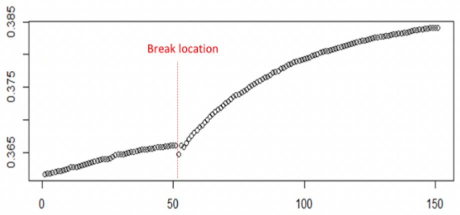


(a) Recursive d estimation (N = 30)

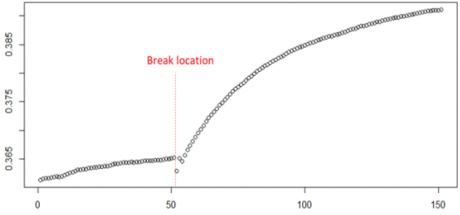


(b) Recursive d estimation (N = 50)

Figure 1: Aggregation without break



(a) Recursive d estimation (N = 30)



(b) Recursive d estimation (N = 50)

Figure 2: Aggregation with break

Application: Portfolio with Cross-Border Equities

- Test for significant changes in the long-memory parameters of the aggregated portfolio returns.
- Identify opportune times to adjust portfolio allocations.
- Daily returns of stock market indices for 19 countries, spanning from January 2000 to May 2022.
  - **Advanced markets**: US, UK, Japan, Switzerland, Europe, Norway, Sweden, South Korea.
  - **Emerging markets**: South Africa, China, Turkey, Thailand, Saudi Arabia, Philippines, India, Mexico, Chile, Brazil, Malaysia.

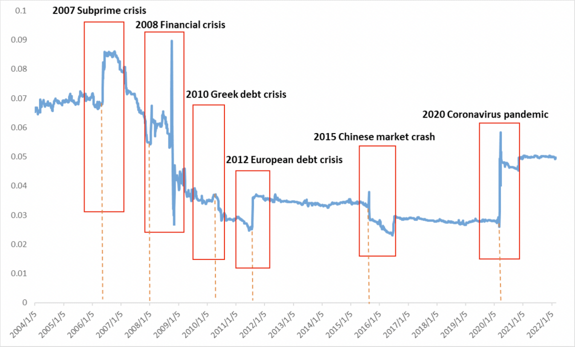


Figure 3: Recursive FEAR-d estimates for aggregated stock market index return from both developed and developing countries

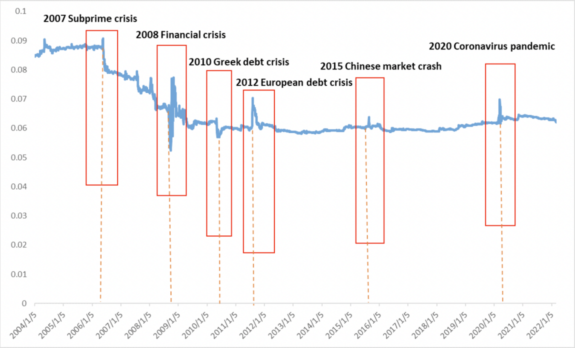


Figure 4: Recursive FEAR-d estimates for aggregated stock market index return from developed countries

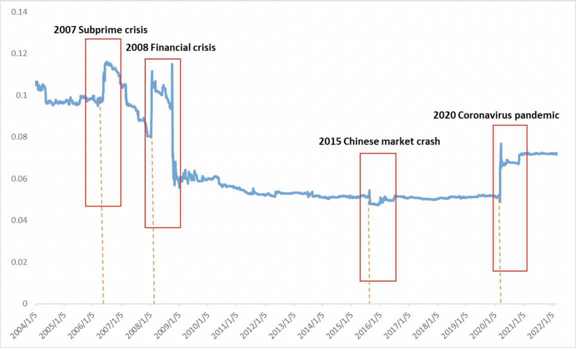


Figure 5: Recursive FEAR-d estimates for aggregated stock market index return from developing countries

Conclusion

1. We propose the FEAR estimation designed for estimating the fractional differencing parameter d in a stationary long-memory process.
2. We also construct an on-line predictive test to detect the common breaks in the aggregated long-memory time series.
3. The test offers valuable insights for portfolio adjustment under volatile market conditions.
4. Both simulation experiments and empirical studies affirm the practicality and robustness of the FEAR estimation and the proposed on-line test.

Thank you!