TMU Workshop on Financial Mathematics and Statistics 2016

November 29–30, 2016

Akihabara Campus, Tokyo Metropolitan University

Program & Abstracts

November 29, 2016 (Tuesday)

13:20-13:30	Opening address
13:30-14:30	**Shigeo Kusuoka** (University of Tokyo) "Naive Computation for Greeks and Its Justification"
14:30-15:00	Break
15:00-15:45	**Daisuke Nagakura** (Keio University) "State Space Method for Quadratic Estimator of Integrated Variance in the Presence of Market Microstructure Noise"
15:45-16:30	**Teppei Ogihara** (Institute of Statistical Mathematics) "Parameter Estimation for Diffusion Processes with Noisy, Nonsynchronous Observations"
16:30-16:45	Break
16:45-17:30	**Takaki Hayashi** (Keio University) "Wavelet-Based Methods for High-Frequency Lead-Lag Analysis"

November 30, 2016 (Wednesday)

10:30-11:15	**Yuta Koike** (Tokyo Metropolitan University) “Statistical Lead-Lag Effects”
11:15-12:00	**Kensuke Ishitani** (Tokyo Metropolitan University) “Computation of the Greeks for Barrier Options Using Chain Rules for Wiener Path Integrals between Two Curves”
12:00-13:30	Lunch break
13:30-14:30	**Freddy Delbaen** (ETH Zurich) “Risk Measures on Orlicz Spaces: Some New Characterisation of Convex Closed Sets”
14:30-15:00	Break
15:00-15:45	**Hideatsu Tsukahara** (Seijo University) “Some Applications of Distortion Risk Measures”
15:45-15:55	Closing address

Naive computation for Greeks and its justification

KUSUOKA Shigeo

Let $(\Omega, \mathcal{F}, P)$ be a probability space, $B(t) = (B^1(t), \ldots, B^d(t))$, $t \geqq 0$, be a d-dimensinal standard Wiener process. Let $B^0(t) = t$, $t \geqq 0$.

Let $V_k \in C_b^\infty(\mathbf{R}^N \times \mathbf{R}^M; \mathbf{R}^N)$, $k = 0, 1, \ldots, d$, and we think of Stratonovich type SDE on $\mathbf{R}^N$

$$dX(t, x, \theta) = \sum_{k=0}^{d} V_k(X(t, x), \theta) \circ dB^k(t)$$

$$X(0, x, \theta) = x \in \mathbf{R}^N$$

Let $b \in C_b^\infty(\mathbf{R}^N \times \mathbf{R}^M; \mathbf{R}^N)$ be given by

$$b(x, \theta) = V_0(x; \theta) + \frac{1}{2} \sum_{k=1}^{d} \sum_{i=1}^{N} V_k^i(x, \theta) \frac{\partial}{\partial x^i} V_k(x; \theta).$$

Let us think of Euler-Maruyama approximation $X_h : [0, \infty) \times \mathbf{R}^N \times \mathbf{R}^M \times \Omega \to \mathbf{R}^N$, $h > 0$, given by

$$X_h(0, x, \theta) = x,$$

$$X(t, x, \theta) = X(nh, x, \theta) + \sum_{k=1}^{d} V_k(X(nh, x, \theta))(B^k(t) - B^k(nh)) + b(X_h(nh, x, \theta))(t - nh),$$

for $t \in (nh, (n+1)h]$, $n = 0, 1, \ldots$

Then the following result is known.

Theorem 1 If $\{V_0, V_1, \ldots, V_d\}$ satisfies (UH) (Uniform Hörmander condition), then for any $T > 0$ there is a $C \in (0, \infty)$ such that

$$\sup_{(x,\theta) \in \mathbf{R}^N \times \mathbf{R}^M} |E[f(X(t, x, \theta)] - E[f(X_{T/n}(T, x, \theta)]| \leqq \frac{C}{n} ||f||_{L^\infty}$$

for any $n \geqq 1$. and $f \in C_b^\infty(\mathbf{R}^N; \mathbf{R})$.

We will show the following extended result.

Theorem 2 If $\{V_0, V_1, \ldots, V_d\}$ satisfies (UH) , then for any $T > 0$ and any multi-index $\gamma \in \mathbf{Z}_{\geqq 0}^M$, there is a $C \in (0, \infty)$ such that

$$\sup_{(x,\theta) \in \mathbf{R}^N \times \mathbf{R}^M} |\frac{\partial^{|\gamma|}}{\partial \theta^\gamma} E[f(X(t, x, \theta)] - \frac{\partial^{|\gamma|}}{\partial \theta^\gamma} E[f(X_{T/n}(T, x, \theta)]| \leqq \frac{C}{n} ||f||_{L^\infty}$$

for any $n \geqq 1$. and $f \in C_b^\infty(\mathbf{R}^N; \mathbf{R})$.

This result justifies the naive computation of Greeks in a certain situation.

We also show a similar result for Gaussian K-scheme as Ninomiya-Victoir method.

State Space Method for the Quadratic Estimators of the Integrated Variance in the Presence of Market Microstructure Noise

Daisuke Nagakura† and Toshiaki Watanabe‡

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Abstract

Nagakura and Watanabe (2015) develop a state space method for realized variance (RV) under the existence of market microstructure noise (MMN) in observed prices, where they derive a state space representation of the RV and apply the Kalman filter and smoother to calculate the linear projections of the integrated variance (IV) on the RVs. Recently, several authors have considered a class of IV estimators, called the *quadratic estimator* (QE), that takes a quadratic form in observed returns. The QE includes many existing IV estimators, such as the RV, realized kernel (RK), etc. In this paper, we develop a state space method for the QE. Furthermore, while Nagakura and Watanabe (2015) consider only the cases with no leverage effects, we deal with the cases with leverage effects and derive the multivariate state space representation of QEs, from which we can calculate the linear projection of the IV on QEs of arbitrary number. Although, even in the presence of MMNs in observed prices, some special cases of the QE are consistent and hence do not have the bias due to MMN asymptotically, they still have the bias in finite samples. Our state space method can filter out the finite sample bias in the QE due to the MMN and reduce the finite sample MSE of QEs, where the reduction of the MSE increase as the variance of the MMN increases.

Key Words: Realized Variance; Quadratic Estimator; Realized Kernel; Integrated Variance; Market Microstructure Noise; State Space; Filtering; Leverage effects.

References

Nagakura, D., and T. Watanabe. 2015. "A State Space Approach to Estimating the Integrated Variance under the Existence of Market Microstructure Noise." *Journal of Financial Econometrics*, 13(1), 45–82.

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Parameter estimation for diffusion processes with noisy, nonsynchronous observations

Teppei Ogihara (The Institute of Statistical Mathematics, JST PRESTO)

We study statistical inference for security prices modeled by diffusion processes with high-frequency observations. In particular, we focus on two specific problems on analysis of high-frequency data, that is, nonsynchronous observations and the presence of observation noise called market microstructure noise.

We consider a two-dimensional diffusion process $X = \{X_t\}_{0\leq t\leq T}$ satisfying following SDE:

$$dX_t = \mu(t, X_t, \sigma_*)dt + b(t, X_t, \sigma_*)dW_t, \quad t \in [0, T],$$

where W_t is a two-dimensional standard Wiener process, μ is a two-dimensional random variable, b is an $\mathbb{R}^2 \otimes \mathbb{R}^2$-valued Borel function, $\sigma_* \in \Lambda$ is an unknown parameter, $\Lambda \subset \mathbb{R}^d$ is an open set. Let $X = (X^1, X^2)$, $\{S_i^{n,k}\}_i \subset [0, T]$ be observation times of X^k. We consider a high-frequency limit: $\max_{i,k} |S_i^{n,k} - S_{i-1}^{n,k}| \to^p 0$. Market microstructure noise $\{\epsilon_i^{n,k}\}_{k=1,2,i\in\mathbb{Z}_+}$ is an i.i.d. sequence with $E[\epsilon_i^{n,k}] = 0$ and $E[\epsilon_i^{n,k}\epsilon_j^{n,l}] = v_{k,*}\delta_{k,l}\delta_{i,j}$, where $v_{k,*}$ is an unknown positive constant. Observations are given by $\{X_{S_i^{n,k}} + \epsilon_i^{n,k}\}_{k,i}$.

We construct a maximum-likelihood-type estimator of parameters, and study their asymptotic mixed normality. We also study the local asymptotic mixed normality property and asymptotic efficiency of our estimator when diffusion coefficients are constants and observation noise is normal.

Wavelet-Based Methods for High-Frequency Lead-Lag Analysis

Takaki Hayashi[*,†,‡]   Yuta Koike[†,§,‡]

We propose a novel framework to investigate lead-lag relationships between two financial assets. Our framework bridges a gap between continuous-time modeling based on Brownian motion and the existing wavelet methods for lead-lag analysis based on discrete-time models and enables us to analyze the multi-scale structure of lead-lag effects. We also present a statistical methodology for the scale-by-scale analysis of lead-lag effects in the proposed framework and develop an asymptotic theory applicable to a situation including stochastic volatilities and irregular sampling. Finally, we report several numerical experiments to demonstrate how our framework works in practice.

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Statistical Lead-Lag Effects

Yuta Koike[*†‡] Nakahiro Yoshida[§†‡]

A lead-lag effect is a relationship between two time series which describes a situation where one is correlated to anther with a time lag. One can visualize such a phenomenon by depicting the cross-correlogram between two time series. In high-frequency financial data there are some cases where the cross-correlograms between the intra-daily returns of two assets fluctuate day by day, while they exhibit a regular pattern once they are averaged across days. We call such phenomenons statistical lead-lag effects. In this talk we aim at developing a model having the ability to describe statistical lead-lag effects. We also propose a statistical methodology to estimate statistical characteristics of the statistical lead-lag effects between two time series from their high-frequency observation data. Finally, we report several numerical experiments to demonstrate how the proposed model and methodology work in practice.

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Computation of the Greeks for Barrier Options Using Chain Rules for Wiener Path Integrals between Two Curves

Kensuke Ishitani (Tokyo Metropolitan University)

Barrier options are exotic options whose payoff depends on whether or not the underlying asset price has reached certain barrier levels (also called trigger levels) prior to the maturity. Since barrier options are widely used financial products, computation of their prices and Greeks is an important issue in mathematical finance. There exist previous works on computation of barrier options and their Greeks under the specific framework of some payoff functions (e.g., European type or Lookback type) with constant trigger levels. In this presentation, we propose a new method for the computation of the Greeks for barrier options under the general framework. Our objective is to compute the first-order Greeks for barrier options under the framework of path dependent payoff functions, including European type, Lookback type, Asian type etc., with time-dependent trigger levels. In particular, we develop chain rules (CR) for Wiener path integrals between two curves. The boundary terms of CR, concentrated on the set of paths that touch one of the curves once, are specified. We also apply our CR method to compute first-order Greeks for barrier options under the Black-Scholes market model, and illustrate the effectiveness of our method through numerical examples.

Risk Measures on Orlicz spaces: some new characterisation of convex closed sets

Freddy Delbaen (ETH-Zurich, Uni-Zurich, on visit at Tokyo Metropolitan University)
Keita Owari (Ritsumeikan University, Kusatsu)

The usual definition for monetary utility functions is given on the space L^∞. For dual spaces, L^Φ, of Orlicz$-\Delta_2$ spaces, L^Ψ, there are two generalisations. One uses norm bounded sets, the other one uses order intervals. We show that a monetary utility function has a dual representation with a penalty function defined on L^Φ, if the utility function is upper semi continuous for the convergence in probability on order intervals. More precisely we show that a convex set $C \subset L^\Phi$ is $\sigma(L^\Phi, L^\Psi)$ closed if for each order interval, $[-\eta, \eta] = \{\xi \mid -\eta \le \xi \le \eta\}$ $(0 \le \eta \in L^\Phi)$, the intersection $C \cap [-\eta, \eta]$ is closed for the convergence in probability. The result is based on the following technical lemma. For a norm bounded sequence ξ_n in L^Φ, which converges in probability to 0, there exist *forward* convex combinations $\zeta_n \in conv\{\xi_n, \xi_{n+1}, \dots\}$ as well as an element $\eta \in L^\Phi$, such that $\zeta_n \to 0$, almost surely and $|\zeta_n| \le \eta$.

Some Applications of Distortion Risk Measures

Hideatsu Tsukahara (Seijo University)

The class of distortion risk measures is the one we should use if we stick to the law invariance and comonotonic additivity in addition to the four axioms of coherence, and is broad enough to fully express agents' subjective assessment of risk. If one is to apply this risk measure to practical financial risk management problems, it is necessary to pick one distortion function suitable for each purpose. One's choice from among these distortion risk measures should be based on his/her attitude towards risk, but we need some quantitative guidelines helping our understanding of the characteristics of those risk measures. There are many known parametric families of distortion functions to construct distortion risk measures (including the renowned expected shortfall, and the author's proposals in Tsukahara (2009)). In the present paper, we give a comprehensive comparative study of several families of distortion risk measures in some aspects. The value-at-risk (VaR) is also included as a target for comparison in view of its practical popularity. Our findings are presently as follows. Statistical estimation of distortion risk measures is possible with weakly dependent time series data (the case of long range dependence is now under study), but for some risk measures, we do not get stable asymptotic properties; proportional odds risk measure has some nice features. Backtesting procedure can be performed with distortion risk measures, but we may still need more rigorous procedures. The Euler capital allocation based on distortion risk measures are easy to compute and widely applicable, and we evaluate numerically how differently they perform under several dependence scenarios using copulas.